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Emission Standard versus Tax under Oligopoly:

The Role of Free Entry

By

Sajal Lahiri [§] and Yoshiyasu Ono [‡]

Abstract

We compare the effects of an emission tax, and those of an emission standard, on welfare and pollution levels under oligopolistic market structures. We consider the cases where the number of firms is fixed and where there is free entry and exit of firms. When the number of firms is fixed (i) an emission standard is welfare-superior to a pollution-equivalent emission tax, and (ii) an emission tax is emission-superior to welfare-equivalent emission standard. Under free entry and exit, the results are just the opposite when the inverse demand function is concave.

JEL Classifications: F2, H2
Keywords: Pollution, Environment, Emission tax, Emission standard, Welfare.

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1 Introduction

The literature on environmental policies is vast and an extensive survey of the literature is given in Cropper and Oates (1992). In particular, there is a large literature on environmental policies in models with imperfect competition. For example, Conrad (1993) considers strategic rent-shifting use of emission taxes where consumers' surplus has no role. Kennedy (1994) extends this analysis to allow for domestic consumption. Barrett (1994) and Kayalica and Lahiri (2005) consider strategic use of emission standards. Ulph (1992, 1996) compares different environmental policies under different types of oligopolistic competitions. Conrad and Wang (1993) examines the effect of emission taxes and abatement subsidies on market structure in the presence and absence of free entry and exit of firms. Gokturk (1979), Markusen et al. (1993), Rauscher (1995) and Levinson (1997) examine the effect of environmental policies on firms' location decision.

In contrast to the above literature, we consider an oligopolistic market for a non-tradeable commodity and analyze two scenarios, viz. the case of a fixed number of firms and that of free entry and exit. All firms emit pollution when producing commodities and possess the same technology for abating the pollution. Under the above specification we compare an emission tax with a standard and derive conditions under which the emission tax is welfare-superior to the 'pollution-equivalent' emission standard, and the emission tax is emission-superior to the 'welfare-equivalent' emission standard.[1] Changes in the two instruments are pollution- (or welfare-) equivalent if they result in an equal change in pollution (or welfare, respectively). We examine the effects of pollution- (or welfare-) equivalent changes in the two instruments on welfare (or pollution) levels. We find that the ranking of the two instruments can be opposite depending on whether free entry and exit are allowed or not.

[1] As Helfand (1991) points out an emission standard itself can take a variety of forms such as an emission quantity restriction per unit of output, an emission restriction per unit of certain input, restrictions on the use of a particular input, or mandated use of a particular pollution-control technology. In this paper we shall compare an emission tax with a quantity restriction on emission in the form of a restriction on emission *per unit of output*, and for expositional simplicity call the latter simply an emission standard. An emission standard is typically not marketable, i.e., it is imposed by environmental authorities as a command.

To make the analysis tractable, we carry out our analysis by starting from an equilibrium in which there is no restriction on emission, and examine the effect on welfare and pollution of a small change in the emission standard or the tax. This approach may also have some policy relevance since environmental restrictions are generally resisted by firm owners (interested in profits) and workers (interested in employment), as seen from various negotiations related to the Kyoto Protocol. Because of these political constraints, the restriction level tends to be much smaller than the optimal one.

There are may papers that attempt to rank environmental policy instruments. The classic paper by Weitzman (1974) shows such a possibility in the presence of uncertainty. Stavins (1996) extends Weitzman's analysis by considering correlated uncertainties.

Helfand (1991) in a different framework finds that an emission standard may 'raise' pollution because firms can comply with a stricter emission standard by raising output rather than by lowering pollution. This means that pollution can be higher under an emission standard than under an emission tax. Ulph (1992 and 1996) compares a pollution quota with an emission tax in the presence of strategic interactions among two firms and two governments, where each government has an exogenous target for emission. Using a specific production technology he finds that quotas are Pareto-superior to taxes (Ulph, 1992) but that the comparison becomes less clear-cut in a more general setting (Ulph, 1996).

This paper contributes to this literature by comparing an emission tax with a standard in an oligopolistic model. We consider the case of a fixed number of firms and that of free entry and exit, and compare the working of the two instruments in the two cases. This will allow us to examine the role of free entry and exit for the results.

The basic structure of the model is spelt out in the following section. Section 3 then compares the two instruments when the number of firms is fixed. The case of free entry and exit of firms is taken up in section 4. An alternative interpretation of our model is provided in section 5. Finally, some concluding remarks are made in section 6.

2 The Basic Framework

We consider a market of a non-tradeable homogeneous commodity in which N firms have the same technology,[2] face the same policy environment, and compete with each other under Cournot oligopolistic conditions.[3] We examine two scenarios *vis-a-vis* the market structure: (i) the number of firms is fixed, and (ii) there is free entry and exit of firms so that N is endogenously determined.

Given the inverse demand function:

$$p = f(D), \quad \text{with } f' < 0, \tag{1}$$

where p and D are respectively the price, and the total demand, of the commodity, a firm that has cost function $c(x, \cdot)$ determines output x so as to maximize profits π:

$$\begin{aligned} \pi &= px - c(x, \cdot), \qquad (2) \\ \text{where } c_x &> 0 \text{ and } c_{xx} > 0. \end{aligned}$$

A part of $c(x, \cdot)$ is determined by technology and factor market conditions, and the other part is policy induced, which will be spelt out later on.

The first order profit maximizing condition under Cournot oligopoly is

$$p - c_x(x, \cdot) = -f'x. \tag{3}$$

Throughout the paper we make the following assumption.

$$f'(X) + f''(X)y < 0 \text{ for all } 0 \le y \le X. \tag{4}$$

[2] In section 5 we provide an alternative interpretation of our model in which some of the firms are foreign.

[3] Although we here use a partial equilibrium framework, we can easily provide a general equilibrium interpretation of the model as follows. There is a competitive numeraire good and households have a quasi-linear utility function in which the utility level is linearly related to the consumption level of the numeraire good. In this case demand for the oligopolistic commodity depends on only its own price and the income effect falls entirely on the numeraire good. Also, assume that there is only one factor of production (e.g., labor) which is freely mobile between sectors. With constant returns to scale technologies in the competitive sector, the wage rate is determined in that sector and the marginal cost in the oligopolistic sector is constant. In this way the present partial equilibrium setting can be regarded as a general equilibrium model.

This assumption corresponds to the 'normal' case in Seade (1980, pp.483-484) and also to strategic substitutes in Bulow, Geanakoplos and Klemperer (1985) and Dixit (1986).

The amount of pollution generated (before any abatement) by the firm is

$$\theta = \theta(x), \quad \text{where} \quad \theta'(x) > 0, \quad \theta''(x) \geq 0, \quad \theta(0) = 0. \tag{5}$$

The restrictions imposed on the θ-function also imply

$$\theta'(x) \geq \frac{\theta(x)}{x}. \tag{6}$$

The cost of abating pollution by amount a is

$$\gamma = \gamma(a), \quad \text{where} \quad \gamma(0) = 0, \quad \gamma'(0) = 0, \quad \gamma'(a) > 0 \text{ for } a > 0, \quad \gamma''(a) > 0. \tag{7}$$

3 Fixed Number of Firms

In this section we consider the case where the total number of firms N is exogenously given.

3.1 Emission Standard

Let us consider the case where the government specifies emission standard z, viz. the maximum allowance of pollution per unit of output. Since the amount of pollution generated by a firm is $\theta(x)$, the level of abatement a is given by $\theta - zx$. In this case total cost $c^Q(x, z)$ consists of $\bar{c}(x)$, the part of the total cost that is determined by technological and factor-market conditions, and abatement cost γ.[4] Thus,

$$c^Q(x, z) = \bar{c}(x) + \gamma(\theta(x) - zx), \tag{8}$$

$$\text{where} \quad c^Q_{xx} = \bar{c}_{xx} + \gamma'\theta'' + \gamma''(\theta' - z)^2, \quad c^Q_{xz} = -(\gamma' + (\theta' - z)\gamma'' x), \quad c^Q_z = -\gamma' x.$$

[4]We introduce the superscript 'Q' below to refer to the present policy regime. For the tax regime, we shall use the superscript 'T'. This will help us to compare the two regimes.

Welfare W^Q is given by

$$W^Q = N\pi + \mathrm{CS} - \phi(Z^Q), \tag{9}$$

where the first term in (9) denotes profits of the firms, the second term is consumers' surplus, and the third term represents disutility from pollution Z^Q. Total pollution level Z^Q and consumers' surplus CS satisfy

$$\begin{aligned} Z^Q &= Nxz = Dz, && (10) \\ d\mathrm{CS} &= -Ddp. && (11) \end{aligned}$$

Differentiating (1)-(3) and (8), we obtain

$$\begin{aligned} \frac{dx}{dz} &= \frac{c^Q_{xz}}{\Phi^Q} > 0, && (12) \\ \frac{dD}{dz} &= \frac{Nc^Q_{xz}}{\Phi^Q} > 0, && (13) \end{aligned}$$

$$\text{where} \;\; \Phi^Q = N(f' + f''x) + f' - c^Q_{xx} < 0.$$

That is, an increase in z — which means a less strict emission standard — reduces the marginal cost of production and increases each firm's output x and total demand D.

Differentiating (2) and (10) and using (3) and (8) yield

$$\begin{aligned} \frac{dZ^Q}{dz} &= D + \frac{Nzc^Q_{xz}}{\Phi^Q} > 0, && (14) \\ \frac{d\pi}{dz} &= \frac{(N-1)xf'c^Q_{xz}}{\Phi^Q} - c^Q_z = \frac{x[\gamma'(2f' + f''D - \bar{c}_{xx}) - (N-1)(\theta' - z)\gamma''xf']}{\Phi^Q}. && (15) \end{aligned}$$

An increase in z increases total emission level Z^Q, whereas the effect on profits is ambiguous. An increase in z reduces abatement costs and thus raises profits (the second term in (15)) but it also reduces the marginal costs of rival firms, lowering the firm's profits (the first term). The sign of the sum of the two effects is generally ambiguous. However, if condition

(4) is valid and function $\gamma(a)$ is almost linear, we get the normal result that an increase in z raises each firm's profits.[5]

By differentiating (9) and substituting (13)-(15) we obtain

$$dW^Q = -N\left\{\frac{f'xc^Q_{xz}}{\Phi^Q} + c^Q_z\right\}dz - \phi' dZ^Q. \tag{16}$$

A reduction in z unambiguously reduces total pollution Z^Q (equation (14)) and thus the disutility from pollution (the second term on the right hand side of (16)). However, because of the Cournot oligopoly distortion an increase in production benefits the country. Thus, a decrease in production owing to a reduction in z harms the country. The total welfare effect of the emission standard depends on the relative magnitude of these two effects. We shall come back to this equation in section 3.3.

3.2 Emission Tax

Having analyzed the effects of emission standard z on pollution and welfare, we now turn to pollution tax t. It generates two associated costs to each firm: (i) the tax paid, and (ii) the cost of pollution abatement. Denoting by e the total post-abatement emission level of a firm, the total emission of the economy and each firm's total costs are given by

$$Z^T = Ne, \tag{17}$$

$$c^T(x,\cdot) = \bar{c}(x) + \gamma(\theta(x) - e) + te, \tag{18}$$

where $\bar{c}(x)$ is as before, $\gamma(\theta - e)$ is the total abatement cost, and te the total tax paid.

In this case, the firm decides on e and x. The optimal behaviour on pollution emission

[5]It may well arise in a general Cournot oligopoly with the number of firms to be N. For example, suppose that the marginal cost is constant at c and that the inverse demand function is $p = D^{-\varepsilon}$ where $\varepsilon > 0$. In this setting, if $N > \varepsilon > 1$, the profit function is concave and $d\pi/dc > 0$ so that a reduction in the marginal cost lowers each firm's profits. For example, under duopoly ($N = 2$) it occurs if $2 > \varepsilon > 1$. Note that the perverse result can never occur under monopoly ($N = 1$) or under condition (4).

gives

$$\begin{aligned}
\frac{\partial c}{\partial e} &= t - \gamma'(\theta(x) - e) = 0 \implies e = e(x,t) = \theta(x) - \gamma'^{-1}(t), \qquad (19)\\
\text{where } e_x &= \theta' \text{ and } e_t = -1/\gamma'',
\end{aligned}$$

i.e., the firm abates pollution up to the point where the marginal abatement cost equals the emission tax rate.[6] Substituting (19) into (18) yields

$$\begin{aligned}
c^T(x,t) &= \bar{c}(x) + \gamma(\theta(x) - e(x,t)) + te(x,t), \qquad (20)\\
\text{where } c^T_{xx} &= \bar{c}_{xx} + t\theta'', \;\; c^T_{xt} = \theta', \;\; c^T_t = e.
\end{aligned}$$

Welfare in this case is given by

$$W^T = N\pi + \mathrm{CS} + tZ^T - \phi(Z^T), \qquad (21)$$

In addition to the three terms in (9), we have an additional term in emission tax revenue (the third term).

From (3) and (20) we obtain

$$\begin{aligned}
\frac{dx}{dt} &= \frac{c^T_{xt}}{\Phi^T} < 0, \qquad (22)\\
\frac{dD}{dt} &= \frac{Nc^T_{xt}}{\Phi^T} < 0, \qquad (23)
\end{aligned}$$

$$\text{where } \Phi^T = N(f' + f''x) + f' - c^T_{xx} < 0.$$

Thus, an increase in emission tax t, by raising the marginal cost of production, decreases output x and total demand D. From (2), (3), (17), (19), (21) and (20) we find

$$\begin{aligned}
\frac{dZ^T}{dt} &= N\left\{\frac{\theta' c^T_{xt}}{\Phi^T} - \frac{1}{\gamma''}\right\} < 0, \qquad (24)\\
\frac{d\pi}{dt} &= \frac{(N-1)xf'c^Q_{xt}}{\Phi^T} - c^T_t. \qquad (25)
\end{aligned}$$

[6]We assume the existence an interior solution, i.e., $\gamma'(0) < t < \gamma'(\theta(x))$.

Hence, an increase in t reduces pollution emission. But, for reasons similar to the case of an emission standard, the effect of an increase in t on profits is ambiguous. Differentiating (21) and using (11) and (23)-(25), we get

$$dW^T = -\left\{ \frac{Nf'xc_{xt}^T}{\Phi^T} + Nc_t^T - Z^T \right\} dt - (\phi' - t)dZ^T. \tag{26}$$

3.3 Comparing Emission Standard with Tax

In this section we compare the effects of the two instruments on pollution and welfare. For this, we assume that in the initial equilibrium neither policy is in place, i.e., $t = 0$ and $zx = \theta(x)$,[7] and then consider the effects of 'equivalent' changes in the two instruments. Equivalence is defined in two different ways. In our first definition equivalent changes in the two instruments mean that they have the same effect on welfare. We call this *welfare-equivalent* changes. In the second definition, equivalent changes in the two instruments have the same effects on the total emission level; we call them *emission-equivalent* changes. We carry out two comparisons: (i) the effects of welfare-equivalent changes in the two instruments on total emission, and (ii) the effects of emission-equivalent changes in the two instruments on welfare. These two comparisons are taken up in the following two subsection.

We focus on the case where some environmental restriction benefits the country, whether it is a tax or an emission standard. Thus,

$$\left.\frac{dW^Q}{dz}\right|_{\theta=zx} < 0, \left.\frac{dW^T}{dt}\right|_{t=0} > 0. \tag{27}$$

[7]This assumption is needed for the initial equilibrium to be the same under the two policy regimes; otherwise the analysis becomes intractable. Note that the assumption that $\gamma'(0) = 0$ (See (7)) ensures that the equilibrium is the same under a non-restrictive tax ($t = 0$) and under a non-restrictive emission standard $zx = \theta(x)$.

3.3.1 Emission-equivalent Changes

From (16) and (26) the difference between dW^Q and dW^T in the initial equilibrium is derived as

$$\left[dW^Q - dW^T\right]\Big|_{t=0,\ zx=\theta} = -N\left\{\frac{f'xc^Q_{xz}}{\Phi^Q} + c^Q_z\right\}dz + \left\{\frac{Nf'xc^T_{xt}}{\Phi^T} + Nc^T_t - Z^T\right\}dt. \quad (28)$$

We evaluate this value when dt and dz are such that dZ^Q from (14) equals dZ^T from (24), that is,

$$\left[\frac{(\theta')^2}{\Phi^T} - \frac{1}{\gamma''}\right]dt = x\cdot\frac{\Phi^T - \gamma''(\theta' - z)\theta'}{\Phi^T - \gamma''(\theta' - z)^2}\cdot dz. \quad (29)$$

Substituting (29) into (28) yields

$$\left.\frac{dW^Q - dW^T}{dt}\right|_{t=0,\ zx=\theta} = \frac{Z^T f'}{\Phi^T - \gamma''(\theta' - z)\theta'} > 0,$$

i.e., a tightening of emission standard z increases welfare by more than an emission-equivalent increase in emission tax t does. Formally,

Proposition 1 *Consider a Cournot oligopolistic model with a fixed number of firms. A lowering of the emission standard increases welfare by more than an emission-equivalent increase in the emission tax.*

In order to explain the result clearly, let us consider the special case when $\theta(x) = \theta' x$ so that in our initial equilibrium $\theta' = z$. It then can be verified that in the initial equilibrium $0 = c^Q_{xz} < c^T_{xt} = \theta'$.[8] Thus, an increase in t, by increasing the marginal cost, reduces production and hence increases the oligopolistic distortion, whereas a decrease in z has no effect on the marginal cost and thus does not affect the oligopolistic distortion. As for the pollution distortion, the effect of the two instruments on this distortion is the same by construction. Hence a reduction in z is better for welfare than an increase in t in this case.

[8]When $\theta(x) \neq \theta' x$, from (8), (20) and (29) it can be verified that a lowering of the emission standard increases the marginal cost by less than an emission-equivalent increase in the emission tax.

3.3.2 Welfare-equivalent Changes

From (14) and (24), we obtain

$$\phi' \left[dZ^Q - dZ^T\right]\Big|_{t=0,\ zx=\theta} = \frac{Nf'x^2(\theta' - z)\gamma''}{\Phi^Q} \cdot dz + \frac{Nf'x\theta'}{\Phi^T} \cdot dt, \tag{30}$$

where dt and dz are such that dW^Q from (16) equals dW^T from (26), i.e.,

$$-\left[\frac{f'x\theta'}{\Phi^T} + \phi'\left(\frac{(\theta')^2}{\Phi^T} - \frac{1}{\gamma''}\right)\right] dt = \left[\frac{f'x^2(\theta' - z)\gamma''}{\Phi^Q} - \frac{\phi' x\{\Phi^T - \gamma''(\theta' - z)\theta'\}}{\Phi^Q}\right] dz.$$

Substituting the above equation into (30) gives

$$\left.\frac{dZ^Q - dZ^T}{dt}\right|_{t=0,\ zx=\theta} = -\frac{DZ^T f'\phi'}{\Phi} > 0,$$

where $\Phi = N[f'(\theta' - z)x\gamma'' - \phi'(\Phi^T - \gamma''(\theta' - z)\theta')] > 0$ since we focus on the case that satisfies (27) and hence $(dW^Q/dz)|_{\theta=zx} = \Phi/\Phi^Q < 0$.

That is, a tightening of the emission standard increases emission by more than an welfare-equivalent increase in the emission tax. Formally,

Proposition **2** *Consider a Cournot oligopolistic model with a fixed number of firms. A tightening of the emission standard increases emission by more than an welfare-equivalent increase in the emission tax.*

The above result can be explained as follows. Proposition 1 shows that a stricter emission standard increases welfare by more than an emission-equivalent increase in the emission tax. Given this, equality of welfare levels (which welfare-equivalent changes require) can only be achieved by relaxing the emission standard, which results in more pollution emission than under the welfare-equivalent tax.

4 Free Entry and Exit

Under free entry and exit, firms would move into (out of) the market if π is positive (negative). In equilibrium, therefore, one must have

$$\pi = 0. \tag{31}$$

4.1 Emission Standard

Since firms do not make any profits under free entry and exit, the host country's welfare under emission standard z given by (9) reduces to

$$W^Q = \mathrm{CS} - \phi(Z^Q), \tag{32}$$

where (10) and (11) are still valid. Thus, welfare has only two parts: (i) consumers' surplus, and (ii) disutility from pollution.

Since the total cost of a firm under the emission standard is (8), applying (31) to (2), totally differentiating (1)-(3) and using (8) give

$$f'\Delta^Q dx = [\gamma' f'' x - (\theta' - z)x\gamma'' f']dz, \tag{33}$$

$$f'\Delta^Q dD = [-\gamma'(2f' - c^Q_{xx}) - (\theta' - z)x\gamma'' f']dz, \tag{34}$$

$$\text{where } \; \Delta^Q = 2f' + f''x - c^Q_{xx} < 0, \;\; c^Q_{xx} = \bar{c}_{xx} + \gamma'\theta'' + \gamma''(\theta' - z)^2.$$

The negativity of Δ^Q implies the concavity of profit function (2). Using (6), (33) and (34) we find

$$dx/dz > 0 \quad \text{if} \quad f'' > 0,$$

$$dD/dz > 0.$$

Thus, an increase in z — which implies a relaxation of the emission standard — reduces the marginal production cost including the required abatement cost and hence increases total output (= consumption) D.

Turning to output x, the effect through a reduction in the marginal cost is given by the second term in (33). Under free entry and exit of firms, there is another effect (the first term in (33)) which works via a change in the number of firms. The sign of this effect depends on the convexity/concavity of the inverse demand function. If the inverse demand function is convex, a decrease in the marginal cost raises each firm's profits, causing new entry, and eventually the price settles at a less inclined part of the demand curve. Thus, in order to satisfy the zero-profit condition, each firm has to expand output more under a convex demand curve than what the initial decrease in the marginal cost would entail. If the inverse demand function is concave, the second effect goes in the opposite direction to the first effect, and the net effect on x may be negative.

From (10), (11) and (32)-(34), we obtain

$$\frac{dZ^Q}{dz} = D - \frac{z[\gamma'(2f' - c_{xx}^Q) + (\theta' - z)x\gamma'' f']}{f'\Delta^Q} > 0, \tag{35}$$

$$\frac{dW^Q}{dz} = \frac{D[\gamma'(2f' - c_{xx}^Q) + (\theta' - z)x\gamma'' f']}{\Delta^Q} - \phi' \cdot \frac{dZ^Q}{dz}. \tag{36}$$

4.2 Emission Tax

By totally differentiating (1)-(3) and using (19), (20) and (31), we obtain the effects of emission tax t on D and x:

$$\frac{dD}{dt} = \frac{\theta'(x) + \frac{(f' - c_{xx}^T)e}{f'x}}{\Delta^T} < 0, \tag{37}$$

$$\frac{dx}{dt} = \frac{\theta'(x) - \frac{(f' + f''x)e}{f'x}}{\Delta^T}, \tag{38}$$

$$\text{where} \quad \Delta^T = 2f' + f''x - c_{xx}^T < 0, \quad c_{xx}^T = \bar{c}_{xx} + t\theta''.$$

Intuitively, an increase in t raises the marginal cost and thus increases the commodity price, causing total consumption D to decrease. However, the effect of a rise in t on each firm's output x depends on the sign of f'', as in the case of the emission standard. Since $\theta'(x)$ gives the effect of t on the marginal cost, the first term in (38) represents the effect on x for a given N, and this effect is negative. However, it also reduces the firm's profits and its magnitude depends on the shape of the demand function. If the demand function is concave ($f'' < 0$), the reduction in profits is large as compared to the case of a convex demand function. Thus, in order to recover profits to zero x should increase more under concave demand than under convex demand. The output recovery is represented by the second term of the right-hand side in (38). The latter may dominate the former and then a more restrictive tax raises each firm's output. In particular, if $\theta(x)/x$ is constant so that $\theta'(x)x = \theta(x)$, we have $\theta'(x)x = e$ in the initial equilibrium. In this case, from (38), we derive

$$dx/dt \gtreqless 0 \Leftrightarrow f'' \lesseqgtr 0 \quad \text{when} \quad \theta(x)/x = \text{constant}. \tag{39}$$

Turning to the other variables, using (18), (19), (37) and (38) we obtain

$$\frac{de}{dt} = \frac{\theta'\left(\theta' - \frac{(f'+f''x)e}{f'x}\right)}{\Delta^T} - \frac{1}{\gamma''}, \tag{40}$$

$$\frac{\Delta^T}{N}\cdot\frac{dN}{dt} = \frac{e\left[(N+1)f' + f''D - c^T_{xx}\right]}{f'xD} + \left(\frac{1}{D} - \frac{1}{x}\right)\theta' \tag{41}$$

$$\frac{\Delta^T}{N}\cdot\frac{dZ^T}{dt} = (f' - c^T_{xx})\cdot\left(\frac{e^2}{f'x^2N} - \frac{1}{\gamma''}\right) - \frac{f'+f''x}{\gamma''} + \frac{(\theta'x - e)^2}{x^2} + \frac{e\theta'}{D} - \frac{f''e(\theta'x - e)}{f'x}, \tag{42}$$

Since each firm's output x can rise with emission tax t when $f'' < 0$ (see (39)), a rise in t can also increase net emission by each firm (see (40)) under the same condition. From (42) it is also clear that when the demand function is linear ($f'' = 0$), a rise in t will reduce the total amount of emission, as has also been shown by Conrad and Wang (1993).

Since firms do not make any profit under free entry and exit, welfare under the tax, given by (9), reduces to

$$W^T = \mathrm{CS} + tZ^T - \phi(Z^T). \tag{43}$$

Differentiating (43)) totally and using (11), (17) and (37) yield

$$\frac{dW^T}{dt} = -\frac{N[-(f' + f''x)e + f'x\theta']}{\Delta^T} - (\phi' - t) \cdot \frac{dZ^T}{dt}. \tag{44}$$

4.3 Comparing Emission Standard with Tax

As in section 3.3, we consider the cases of emission-equivalent and welfare-equivalent changes in the two instruments.

4.3.1 Emission-equivalent Changes

For this exercise, we consider dt and dz such that dZ^Q given by (35) equals dZ^T given by (42), which leads to

$$\begin{aligned} \Omega \cdot dz \quad = \quad & \left[\frac{eZ^T\{\Delta^T + (N-1)(f' + f''x)\}}{f'x} - (N-1)Z^T\theta' - \frac{ND\Delta^T}{\gamma''} \right. \\ & \left. + N\theta'\{D\theta' - (1 + \frac{f''x}{f'})Z^T\} \right] \ dt, \end{aligned} \tag{45}$$

$$\text{where} \quad \Omega = \frac{\Delta^T D\{D(\Delta^T - \gamma''(\theta' - z)^2) - z(\theta' - z)x\gamma''\}}{\Delta^T - \gamma''(\theta' - z)^2} < 0.$$

By substituting (45) into (36) and (44) and calculating the difference between them in the initial equilibrium we obtain

$$\left. \frac{dW^Q - dW^T}{dt} \right|_{t=0,\ \theta=zx} = \frac{Z^T\left[z(\theta' - z)\gamma'' - f''D\right]}{N\left[\Delta^T - \gamma''(\theta' - z)^2\right] - z(\theta' - z)\gamma''}. \tag{46}$$

From (46) we derive the following result:

$$\left. \frac{dW^Q}{dt} \right|_{t=0,\ \theta=zx} < \left. \frac{dW^T}{dt} \right|_{t=0,\ \theta=zx} \quad \text{if} \ \ f'' \leq 0 \ \ \text{and} \ \ \theta'' > 0.$$

That is, an increase in t increases welfare by more than a pollution-equivalent increase in z when $f'' \leq 0$ and $\theta'' > 0$.

If $\theta(x)/x$ is constant, i.e., if $\theta(x) = \theta' x$, we have $\theta' = z$ and then

$$\left.\frac{dW^Q - dW^T}{dt}\right|_{t=0,\ \theta=zx} = -\frac{Z^T x f''}{\Delta^T},$$

and thus have a stronger result:

$$\left.\frac{dW^T}{dt}\right|_{t=0,\ \theta=zx} \gtreqless \left.\frac{dW^Q}{dt}\right|_{t=0,\ \theta=zx} \iff f'' \lesseqgtr 0.$$

The above results are summarized in the following proposition.

PROPOSITION **3** *Consider a Cournot Oligopolistic model with free entry and exit of firms. (i) Suppose $f'' \leq 0$ and $\theta' > \theta/x$. Staring from the equilibrium without any restriction on pollution, a rise in t increases welfare more than a pollution-equivalent decrease in z.*
(ii) Suppose $\theta(x)/x$ is constant. A rise in t increases welfare by more than, less than, or equal to, a pollution-equivalent decrease in z according as f'' is negative, positive, or equal to, zero.

Comparing the above results with those for the case where the number of firms is fixed, the following interesting points are worth noting. First, whereas the emission standard yields higher welfare than the emission-equivalent tax in the case where the number of firms is fixed (see proposition 1), the results here depend on the concavity/convexity of the demand function. In particular, when the inverse demand function is concave, the results here are just the opposite compared to the case where the number of firms is fixed. Furthermore, if the demand function is linear ($f'' = 0$) and the gross-pollution function $\theta(x)$ is proportional to x so that $\theta' = z$ in the initial equilibrium, an increase in t or an emission-equivalent decrease in z have the same effect on welfare.

The above-mentioned difference in the results for the case of free entry and exit from that of a fixed number of firms can be tracked down to the fact that a higher emission tax in the present case does not necessarily reduce each firm's output x but that the effect depends on the concavity/convexity of the inverse demand function for the reasons given before. In order to understand the result more clearly, let us start with the benchmark case where $f'' = 0$ and $\theta(x)/x$ is constant.

In this case an increase in t has no effect on each firm's output, but reduces their profits. In order to maintain the zero-profit condition, some firms flows out of the market, reducing total output and thus total demand, and increasing price. In fact, the increase in p is exactly the same as the increase in the marginal cost. (When the demand is not linear, the increase in p is higher (lower) than the increase in marginal costs if the inverse demand function is convex (concave).) Furthermore, the reduction in consumers' surplus resulting from the increase in p exactly offsets the increase in the emission tax revenue. Thus, the increase in the emission tax affects only the disutility from pollution, but not the other components of welfare.

As in the case where the number of firms is fixed, a decrease in z from the initial equilibrium has no effect on marginal costs. Hence, it only affects disutility from pollution irrespective of whether the demand function is concave, convex or linear. Since the changes in the policy instruments are emission-equivalent, the two instruments have the same effect on welfare in the case where $f'' = 0$ and $\theta(x)/x$ is constant.

If the demand function is not linear, however, it should be clear from the above discussion that the convexity/concavity of the demand function creates an wedge between consumers' surplus and the emission tax revenue. In contrast, a decrease in z only affects pollution disutility irrespective of the shape of the demand curve, as mentioned above. Thus, the two instruments have different effects on welfare, as mentioned in proposition 3.

4.3.2 Welfare-equivalent Changes

Here we consider dt and dz such that dW^Q given by (36) equals dW^T given by (44), which gives

$$\frac{D(\theta' - z)x\gamma'' f'}{\Delta^Q} \cdot dz = \frac{Z^T(f' + f''x) - f'D\theta'}{\Delta^T} \cdot dt - \phi'(dZ^T - dZ^Q). \tag{47}$$

Substituting (47) into (35) and (42) yields

$$\Omega_2 \cdot \left.\frac{dZ^T - dZ^Q}{dt}\right|_{t=0,\ \theta=zx} = \frac{\Delta^T\{z\gamma''(\theta' - z) - f''x\}}{\gamma''(\theta' - z)} + \frac{(N-1)^2 z(f' + f''x)}{N}, \tag{48}$$

where

$$\begin{aligned} \Omega_2 &= \frac{\Delta^T f' \Omega_3}{Nz} < 0, \\ \Omega_3 &= 1 - \phi' \cdot \frac{D\Delta^Q - z(\theta' - z)x\gamma''}{D(\theta' - z)x\gamma'' f'} < 0. \end{aligned}$$

Note that Ω_3 is negative because we consider the case where lowering emission standard z from no restriction benefits the country.

$$\left.\frac{dW^Q}{dz}\right|_{\theta=zx} = \frac{\Omega_3 D(\theta' - z)x\gamma'' f'}{\Delta^Q} < 0.$$

From (48) we derive

$$\left.\frac{dZ^T}{dt}\right|_{t=0,\ \theta=zx} > \left.\frac{dZ^Q}{dt}\right|_{t=0,\ \theta=zx} \quad \text{if} \quad f'' \leq 0.$$

That is, an increase in t reduces pollution by more than a welfare-equivalent decrease in z provided $f'' \leq 0$.

If $\theta(x)/x$ is constant, i.e., if $\theta(x)/x = \theta' =$ constant so that $\theta' = z$ in the initial equilibrium, we obtain

$$\left.\frac{dZ^T - dZ^Q}{dt}\right|_{t=0,\ \theta=zx} = \frac{Z^T x f''}{\phi' \Delta^T},$$

and thus derive the following stronger result:

$$\left.\frac{dZ^T}{dt}\right|_{t=0,\ \theta=zx} \gtreqless \left.\frac{dZ^Q}{dt}\right|_{t=0,\ \theta=zx} \quad \Longleftrightarrow \quad f'' \lesseqgtr 0 \ \text{ when } \ \theta(x)/x = \text{constant}.$$

The above results are summarized in the following proposition.

PROPOSITION 4 *Consider a Cournot oligopolistic model with free entry and exit of firms.*
(i) Suppose $f'' \leq 0$ and $\theta' > \theta(x)/x$. Staring from the equilibrium without any restriction on pollution, a rise in t reduces pollution by less than a welfare-equivalent decrease in z.
(ii) Suppose $\theta(x)/x$ is constant. A rise in t reduces pollution by less than, more than, or equal to, a welfare-equivalent decrease in z according as f'' is negative, positive, or equal to, zero.

By comparing propositions 4 with 3 we see that when a stricter emission standard increases welfare by more than an emission-equivalent increase in emission tax, a stricter emission standard also increases pollution by more than a welfare-equivalent increase in emission tax. This is because the equality of welfare levels (which welfare-equivalent changes require) can only be achieved by relaxing emission standards which will result in more pollution emission under the policy regime of emission standards.

5 An Alternative Interpretation

In the preceding sections we assume that all the firms are domestically owned. In this section we reinterpret our results in an international context.[9] We assume that domestic and foreign firms compete with each other in a Cournot oligopolistic market of a non-tradeable commodity in the host country. We also assume that the number of domestic firms is m which is fixed since managerial resources are limited in this country, and that the

[9]We ignore cross-border pollution. For an analysis of cross-border pollution see, for example, Copeland (1996), Kiyono and Ishikawa (2004) and Hatzipanayotou et al. (2002, 2005).

host country is small in the market for FDI so that under free entry and exit domestic and foreign firms move in and out of the country until their profits equal zero.[10]

In this setting we first consider the case where the government of the host country prohibits any foreign firm to enter the country and restricts pollution emission by using the two instruments.[11] It has exactly the same structure as that of section 3 by making m equal N and thus propositions 1 and 2 hold.

Next, we consider that the government allows foreign firms to enter the country directly (FDI) without any restriction. By taking N to be the total number of firms in the domestic market and m to be that of domestically-owned firms irrespective of their location, we can directly apply the same model as that of section 4 to the present context.[12] Thus, propositions 3 and 4 still apply. The comparison between the two cases can be interpreted as the comparison between the case of total prohibition against FDI and that of free entry of FDI.

Note that even if domestic and foreign firms can earn some positive reservation profits F under free entry and exit, all our results hold true. In this case in (32) there is an additional term mF representing profits by domestic firms. However since both m and F are exogenous, it has no effect on any of our results.

6 Conclusion

We develop a model in which a number of firms compete in a Cournot oligopolistic market. We consider two situations depending on whether the number of firms is fixed or there is free

[10]As compared to the exiting literature on FDI where the 'outside option' of the foreign firms is to export rather than to make FDI in a particular country, in this paper the outside option is implicitly to take FDI to some other country. This approach of endogenising the number of foreign firms is borrowed from Lahiri and Ono (1998, 2003, 2004) which do not consider pollution at all.

[11]Here the government does not restrict outflows of the domestic firms and yet there is no incentive for them to go out. It is because whereas they earn excess profits in their home country, they would earn zero profits if they went out abroad.

[12]It is possible that all domestic firms locate themselves abroad.

entry/exit of firms. When producing the commodity, they emit pollution and the government imposes an emission tax or standard on the emission. They affect the country's welfare via the following channels: (i) consumers' surplus, (ii) the level of pollution, and (in the case where the number of firms is fixed) (iii) producers' surplus. In the case of the emission tax, there is an additional effect that arises via emission tax revenue.

In this framework we compare the effects of the two instruments for pollution control. For the case where the number of firms is fixed, we find the emission standard to be welfare-superior to the 'pollution-equivalent' emission tax, and the emission tax is pollution-superior to the 'welfare-equivalent' emission standard. For the case of free entry and exit of firms, the results are just the opposite when the inverse demand function is concave. In particular, if the gross pollution emitted by a firm is proportional to its output level, the emission standard is welfare-inferior to the 'pollution-equivalent' emission tax (and the emission tax is pollution-inferior to the 'welfare-equivalent' emission standard) if and only if the inverse demand function is concave.

The overall conclusion therefore is that the ranking of the two instruments crucially depends on the convexity/concavity of the demand function as well as on the existence or otherwise of free entry and exit of firms. An important lesson from this paper is that the assumption of linearity of demand function — an assumption very frequently made in the literature — may be innocuous when the number of firms is fixed, but not so when there is free entry and exit of firms.

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