

2-1985

# Nonparametric Receiver for FH-MFSK Mobile Radio

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## Recommended Citation

Viswanathan, R. and Gupta, S. C., "Nonparametric Receiver for FH-MFSK Mobile Radio" (1985). *Articles*. Paper 22. [http://opensiuc.lib.siu.edu/ece\\_articles/22](http://opensiuc.lib.siu.edu/ece_articles/22)

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noted the harmonics in the form of  $\cos n\{\omega_c t + \omega_c f^{-1}[x(t)]\}$  where  $f^{-1}[x(t)]$  is equal to  $\cos^{-1}[x(t)]$  in sine wave crossings, while in the notation of the paper<sup>1</sup> the cosine function is expanded in terms of  $\cos[n \cos^{-1}(x)]$ , i.e., the Chebyshev polynomial. The latter approach is nice to show the band-limitedness of the in-phase modulation. However, my approach shows that the harmonics are the phase modulated signals that can be extracted, provided that the aliasing is minimized by increasing  $\omega_c$ . Clearly, my method shows that by bandpass filtering rather than low-pass filtering, one can take advantage of noise immunity of PM signals.

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### Nonparametric Receiver for FH-MFSK Mobile Radio

R. VISWANATHAN AND S. C. GUPTA

**Abstract**—Various parametric receivers such as the maximum likelihood and the hard-limiter have been analyzed for their performance in decoding the frequency hopped multilevel FSK (FH-MFSK) messages in mobile environment. Here, some nonparametric receivers such as the maximum rank sum receiver (MRSR) and the reduced rank sum receiver (RRR) are considered. RRR and MRSR are nearly identical in performance but the former is much simpler to implement. The results indicate that RRR is a competing alternative to the parametric receivers.

#### I. INTRODUCTION

In recent years, considerable interest has been shown in finding a spectrally efficient modulation scheme for mobile radio [1], [2]. A spread-spectrum modulation scheme known as frequency hopped multilevel FSK (FH-MFSK) has been considered as a possible modulation method [3], [4]. Some new receivers based on a nonparametric statistical approach are presented here, to decode the FH-MFSK messages. Such an approach has some advantages, such as the robustness of the receiver performance against any changes in the probability model and the absence of any adaptive scheme, usually required with a parametric approach.

The receivers discussed still employ a noncoherent envelope analyzer to estimate the energy in each time-frequency slot, but employ a postdetection combining scheme based on the nonparametric approach [5]. Although the performance of

these receivers has been analyzed in the following sections with specific reference to mobile radio constraints, the scheme is useful in any multiple-access FH-MFSK system. Application of nonparametric detection in spread-spectrum systems has received attention in the recent past [15]. In Section II the maximum rank sum receiver (MRSR) is formulated. In Section III, a reduced rank sum receiver (RRR) is presented, followed by some simulation results. Section IV discusses the performance estimate of these receivers based on an asymptotic theory. In Section V, a discussion on the choice of number of bits in a transmitted word is presented. Section VI concludes with a discussion on the usefulness of this receiver for mobile radio.

#### II. MAXIMUM RANK SUM RECEIVER

Before we discuss the receiver, we describe briefly the FH-MFSK modulation scheme. A detailed description can be found in [3]. Each user in a multiuser mobile radio system is assigned a unique address  $a$  of  $L$  symbols. The user data at rate  $R_b$  bits/s are grouped into  $K$  bits of duration  $T$  seconds. Denote the address vector of a user  $u$  as  $a_u = (a_{u1}, a_{u2}, \dots, a_{uL})$  and the data vector as  $D = (d, d, \dots, L \text{ times})$ ,  $d \in (1, 2, \dots, 2^K)$ ,  $a_{ui} \in (1, 2, \dots, 2^K)$ . Modulation is performed by obtaining a vector  $= a_u + D = (Y_1, Y_2, \dots, Y_L)$ , where  $+$  denotes modulo  $2^K$  addition. Therefore, if we have  $2^K$  orthogonal tones spanning an available  $W$  Hz bandwidth, then for each  $Y_i$ , a tone will be transmitted for a duration of  $\tau (= T/L)$  seconds. At the receiver over each  $\tau$  seconds, a spectrum analysis will be done to find out the energy content of each one of the  $2^K$  frequency slots. When the procedure is repeated  $L$  times, we obtain the received spectrum as shown in Fig. 1. By performing a modulo  $2^K$  subtraction with the address vector, each entry in a column of the received spectrum matrix is shifted into a different position in the same column, in the decoded matrix (Fig. 1).

Let us consider a simplified Rayleigh fading channel and the FH-MFSK scheme as described above. Also, we shall assume that the tone spacing in FH-MFSK modulation exceeds the coherence bandwidth of the mobile channel. This implies that the tones would experience independent fading. Then, by considering the base-to-mobile transmission, along with the ideal conditions described above, the entries in the decoded matrix of a user  $u$  can be characterized statistically [4]. Among the  $2^K$  rows in the decoded matrix, only one row is the correct row, due to the intended signal plus noise. In each of the rest of the  $(2^K - 1)$  spurious rows, the samples (entries) have contribution partly from interfering users plus noise and partly from the receiver noise alone. Therefore, a sample  $Z$  in a row has the following density function:

spurious row

$$Z \sim p\lambda_1 e^{-\lambda_1 z} + (1-p)\lambda_0 e^{-\lambda_0 z}$$

correct row

$$Z \sim \lambda_1 e^{-\lambda_1 z} \quad (1)$$

with

$$\left(\frac{1}{\lambda_1}\right) > \left(\frac{1}{\lambda_0}\right)$$

Here,  $(1/\lambda_1)$  represents the mean energy in a signal plus noise (either intended or interfering),  $(1/\lambda_0)$  represents the mean energy in the noise at the receiver and  $p$  is the proportion which accounts for the interference due to other users.  $p$  is estimated using a random assignment argument. If each of the  $M$  users transmits one of the  $2^K$  frequency tones at ran-

Paper approved by the Editor for Communication Theory of the IEEE Communications Society for publication after presentation at IEEE GLOBE-COM, San Diego, CA, November 1983. Manuscript received July 5, 1983; revised September 3, 1984. This work was supported by the Air Force Office of Scientific Research under Grant 82-0309.

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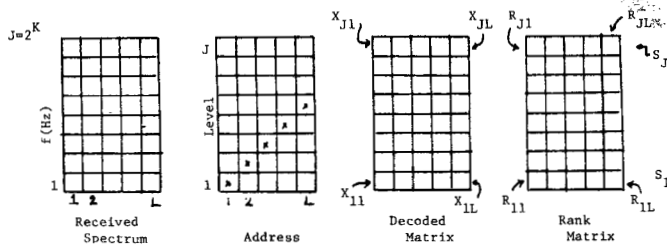


Fig. 1. Maximum rank sum receiver operation. Notes: 1)  $R_{im}$  = rank of  $X_{im}$  among  $\{X_{pq}; p = 1, 2, \dots, l, \dots, J, q = 1, 2, \dots, m, \dots, L\}$ . 2) The ranking is done by assigning the largest rank of  $JL$  to the largest sample, the next largest rank of  $(JL - 1)$  to the next largest sample, and so on. The smallest sample gets the rank of 1. 3)  $S_i$  = rank sum of  $i$ th row ( $i = 1, 2, \dots, J$ ) =  $\sum_{m=1}^L R_{im}$ .

dom, the probability that a particular frequency tone is not being transmitted by one specific user is  $(1 - 2^{-K})$ . Therefore, the probability that none of the  $(M - 1)$  interferers transmits a particular frequency tone over a slot duration equals  $(1 - 2^{-K})^{M-1}$ , or the probability that at least one interferer would transmit a specific tone in a slot, is given by

$$p = 1 - (1 - 2^{-K})^{M-1} \quad (2)$$

where  $K$  is the number of bits in a transmitted word and  $M$  is the number of users in the system. By normalizing the sample with mean energy in a signal plus noise, we have the following density function:

spurious row

$$X \sim pe^{-x} + (1 - p)\beta e^{-\beta x} \quad (3)$$

correct row

$$X \sim e^{-x}$$

where  $\beta = (1/\lambda_0)/(1/\lambda_1)$  represents the signal-plus-noise-to-noise ratio.

A random variable  $x$  is said to be stochastically larger than another random variable  $y$  if the cumulative distribution functions of the two variables satisfy  $F_x(\eta) < F_y(\eta)$  for all  $\eta$  [14]. It is clear that in the above situation, the correct row samples are stochastically larger than the spurious row samples. There will be deviations from this model due to several reasons like the effect of adjacent cell interference in a cellular system, the departure from the "idealness" assumed in arriving at the model, the presence of impulsive noise due to vehicle ignition, and so on. However, although the exact distribution is unknown, under these conditions, the correct row samples would still be stochastically larger than the spurious row samples. The problem of identifying the correct row with stochastically larger samples among a pool of  $(2^K - 1)$  spurious rows is similar to the statistical problem known as the "slippage problem" [6], [7].

If the parametric model (3) is perfectly valid, then the maximum likelihood receiver would be the best receiver [4], [8]. The equivalent test in the nonparametric domain would be to pick the row having the maximum rank sum. Therefore, the idea behind a maximum rank sum receiver (MRSR) is to rank order the samples in the decoded matrix by considering the entire  $(2^K \cdot L)$  samples. By summing these rank orders across each row, the row with the largest sum is picked as the correct row. It is possible that more than one row might possess the same maximum rank sum. In such an event, the ties can be broken by randomization. Intuitively, such a scheme appears to be the best [5]. In Fig. 1, the various matrices pertaining to the operation of the receiver are shown.

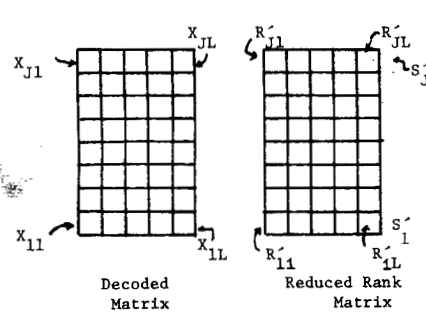


Fig. 2. Reduced rank sum receiver operation. Notes:  $R'_{im}$  = rank of  $X_{im}$  among  $\{X_{pm}; p = 1, 2, \dots, l, \dots, J\}$ .  $S'_i$  = reduced rank sum of  $i$ th row ( $i = 1, \dots, J$ ) =  $\sum_{m=1}^L R'_{im}$ .

### III. REDUCED RANK SUM RECEIVER

With the values of  $K = 8$ ,  $L = 19$  (which are optimum for the parametric receivers when the bandwidth equals 20 MHz and the bit rate  $R_b$  equals 32 kbits/s [3], [4]), it can be observed that over each  $L\tau (=T)$  seconds,  $(2^8 \cdot 19)$  samples will have to be ranked. With 32 kbit/s data rate, this amounts to ranking 4864 samples in 250  $\mu$ s. Since this may imply considerable complexity, we consider a reduced ranking method. In this method, the ranking will be done by considering the samples in each column only (Fig. 2). Since  $L$  columns of samples arrive sequentially in time, ranking of 256 samples will be done in  $\tau (=13 \mu$ s) duration.

#### A. Simulation Results

By generating the samples based on the model (3), using the IMSL (International Mathematical and Statistical Library) routine GGEXP, it is straightforward to simulate the receiver performance. On each simulation trial, 255  $\times$  19 spurious samples and 19 correct samples are generated. Without loss of generality, the first row contains the correct samples. Then the ranks and the sums are computed to simulate the operation of MRSR and RRR. Tables I and II show the performance of MRSR and RRR. As can be seen, both the receivers are roughly similar in performance. At SNR of 25 dB, each could accommodate about 135 users at an estimated probability of bit error of  $P_b \cong 2 \times 10^{-3}$ . By simulating the samples which take into account the effect of adjacent cell interference [12], the MRSR is tested under this condition. The probability of bit error  $P_b$  remains practically the same at  $2 \times 10^{-3}$  (with a controlled average SNR of 25 dB and when the user is at about halfway toward the cell corner). Some robustness in the performance of MRSR against a changing probability model is thus indicated. It should be mentioned that an extensive simulation study could not be carried out because of excessive simulation time requirement.

### IV. ERROR RATE ESTIMATE BASED ON ASYMPTOTIC THEORY

It has been shown that the  $J$  ( $J = 2^K$ ) rank sums are asymptotically jointly normal, for large values of  $L$  [7], [10]. For values of  $L$  of the order of 20, we expect the asymptotic theory to be only approximately true. However, the error estimates based on the asymptotic theory show reasonable agreement with the simulation results obtained earlier. Actually, the asymptotic estimates of error rate are slightly on the higher side. This approach allows us to estimate the performance of the receivers under different conditions (for example, for different values of  $M$ ).

#### A. Maximum Rank Sum Receiver (MRSR)

For the maximum rank sum receiver, the asymptotic procedure to find the probability of correct selection is

TABLE I

SNR = 25 dB		
M	$P_b$	# Simulation Trials
130	$2.12 \times 10^{-3}$	4000
140	$3 \times 10^{-3}$	4500
170	$2.3 \times 10^{-2}$	1000

TABLE II

SNR = 25 dB		
M	$P_b$	# Simulation Trials
140	$2.12 \times 10^{-3}$	8000
160	$1.25 \times 10^{-2}$	2000

readily available in the literature [7]. Denoting

$$g(X, Y) = \begin{cases} 1 & X \leq Y \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

we write the rank sum for the  $p$ th row as

$$S_p = \frac{(L+1)L}{2} + \sum_{s=1}^J \sum_{\substack{l,m=1 \\ s \neq p}}^L g(X_{sl}, X_{pm}) \quad p = 1, 2, \dots, J. \quad (5)$$

Here,  $X_{ij}$  denotes the entry in the  $i$ th row and  $j$ th column of the decoded matrix of the user. The row counting is from the bottom upward, as shown in Figs. 1 and 2. For large  $L$ , it is possible to find  $E(S_p)$ ,  $\text{var}(S_p)$ , and  $\text{cov}(S_p, S_q)$  and, hence, characterize the random variables  $(S_1, \dots, S_J)$ . Without loss of generality, assume the  $j$ th row as the correct row. Then, the probability of correct selection (or decision) is

$$P_C = \Pr [S_j = \text{Max}_i (S_i), i = 1, 2, \dots, j, j+1, \dots, J] \\ = \Pr [S_j - S_i \geq 0, i = 1, \dots, J, i \neq j]. \quad (6)$$

The above equation can be shown to reduce to [7]

$$P_C = \int_{-\infty}^{\infty} \Phi^{J-1}((\sqrt{L}a + \sqrt{c}x)(b-c)^{-1/2}) d\Phi(x) \\ + 0(1/\sqrt{L}) \quad (7)$$

where

$$a = J(\eta - \frac{1}{2}) \quad (8)$$

$$b = (J^2 - 15J - 22)/12 + \eta(3J+2) - \eta^2(J^2 + J + 2) \\ + \theta(J^2 - J + 2) + \psi(J+2) \quad (9)$$

$$c = \eta(1+2J) - \eta^2(1+J+J^2) + \theta(1+J^2) \\ + \psi(1+J) - (11+13J)/12 \quad (10)$$

$\Phi$  the cdf of the standard univariate normal

$$\eta = \int_0^{\infty} F_i(X) dF_j(X) \quad (11)$$

$$\theta = \int_0^{\infty} F_i^2(X) dF_j(X) \quad (12)$$

$$\psi = \int_0^{\infty} F_j^2(X) dF_i(X). \quad (13)$$

$F_j$  is the cdf of the samples from the correct row and  $F_i$  ( $i \neq j$ ) is the cdf of the samples from the spurious rows.

If we assume that  $F_j$  and  $F_i$  satisfy the model (3), we have

$$F_j(X) = \begin{cases} 1 - e^{-X} & X > 0 \\ 0 & \text{elsewhere} \end{cases} \quad (14)$$

$$F_i(X) = \begin{cases} p(1 - e^{-X}) + (1-p)(1 - e^{-\beta X}) & X > 0 \\ 0 & \text{elsewhere.} \end{cases} \quad (15)$$

Therefore we evaluate  $\eta$ ,  $\theta$ , and  $\psi$  as follows:

$$\eta = 1 - p/2 - \frac{(1-p)}{(\beta+1)} \quad (16)$$

$$\theta = \frac{p^2}{3} + (1-p)^2 \left( 1 + \frac{1}{2\beta+1} - \frac{2}{\beta+1} \right) + p(1-p) \\ \cdot \left( 1 - \frac{2}{\beta+1} + \frac{2}{\beta+2} \right) \quad (17)$$

$$\psi = \frac{p}{3} + (1-p) \left( 1 + \frac{\beta}{\beta+2} - \frac{2\beta}{\beta+1} \right). \quad (18)$$

The probability of bit error  $P_b$  can be evaluated as

$$P_b = \frac{2^{K-1}}{(J-1)} (1 - P_C) \quad (19)$$

using (7)–(13) and (16)–(18).

#### B. Reduced Rank Sum Receiver (RRR)

For this receiver, the rank sums are given by

$$S_p' = L + \sum_{m=1}^L \sum_{\substack{s=1 \\ s \neq p}}^J g(X_{sm}, X_{pm}) \quad p = 1, \dots, J. \quad (20)$$

Proceeding along similar lines, we derive the probability of correct selection  $P_C'$  for the reduced rank receiver as

$$P_C' = \int_{-\infty}^{\infty} \Phi^{J-1}((\sqrt{L}a' + \sqrt{c'}x)(b'-c')^{-1/2}) \\ \cdot d\Phi(x) + 0(1/\sqrt{L}) \quad (21)$$

where

$$a' = J(\eta - 1/2) \quad (22)$$

$$b' = \frac{J^2 - J - 2}{12} + \eta(2J) - \eta^2(J^2) + \theta(J^2 - 2J) \quad (23)$$

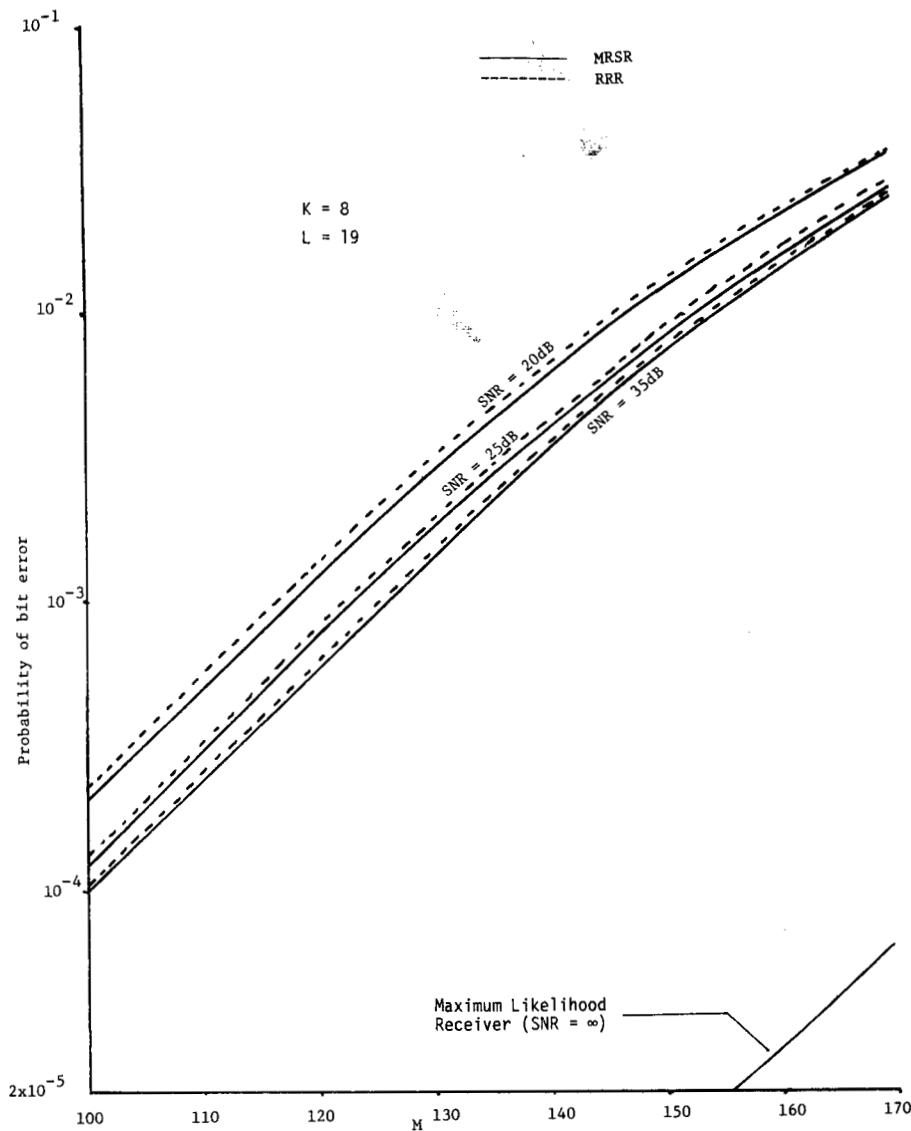


Fig. 3. Asymptotic error rate versus  $M$ .

$$c' = \eta(J) - \eta^2(J^2) + \theta(J^2 - J) - \frac{J}{12} \quad (24)$$

Therefore, the probability of bit error  $P_b'$  for the RRR can be computed as

$$P_b' = \frac{2^{K-1}}{(J-1)} (1 - P_{C'}) \quad (25)$$

The error estimates of these two receivers are plotted in Fig. 3, using  $K = 8$ ,  $L = 19$ . From Fig. 3 we observe that both the receivers have similar performance. This is not surprising when we observe that large  $J$  ( $J = 256$ ) implies that  $b' \cong b$  and  $c' \cong c$ , and therefore, the multivariates  $\{S_j - S_i; i \neq j\}$  and  $\{S_j' - S_i'; i \neq j\}$  have nearly identical distribution. From the information theoretic point of view, the divergence between the two distributions tends to zero [11]. In other words, the reduced ranking possesses nearly as much information as the full ranking. Also, increasing signal-to-noise ratio above 25 dB achieves only a marginal reduction in the bit error rate. Essentially the performance becomes interference limited. For comparison, we also show in Fig. 3 the error rate of a maximum likelihood receiver, when  $SNR \rightarrow \infty$  [4].

Although the maximum likelihood receiver is superior to a rank receiver in its performance in an isolated cellular cell, the performance of the likelihood receiver is bound to degrade when there is adjacent cell interference. A hard-limited parametric receiver, which is only slightly inferior to the likelihood receiver [4], accommodates a significantly smaller number of users when the adjacent cell interference is taken into consideration [9]. However, MRSR (or RRR) shows no such degradation due to adjacent cell interference, as explained earlier.

### V. CHOICE OF $K$

It is difficult to arrive at an optimum value of  $K$  which would maximize the performance of MRSR (or RRR) under all probability models. It is not easier, even if the parametric model (3) is satisfied. However, through some indirect assessment, the value of  $K = 8$  can be justified. Assuming that (3) is the underlying probability model, we compute some form of distance measure between two samples that are obtained under the hypotheses of correct and incorrect selection. The value of  $K$  which maximizes the distance is found. Another method is to observe the asymptotic error rate (Section IV) as a function of  $K$ .

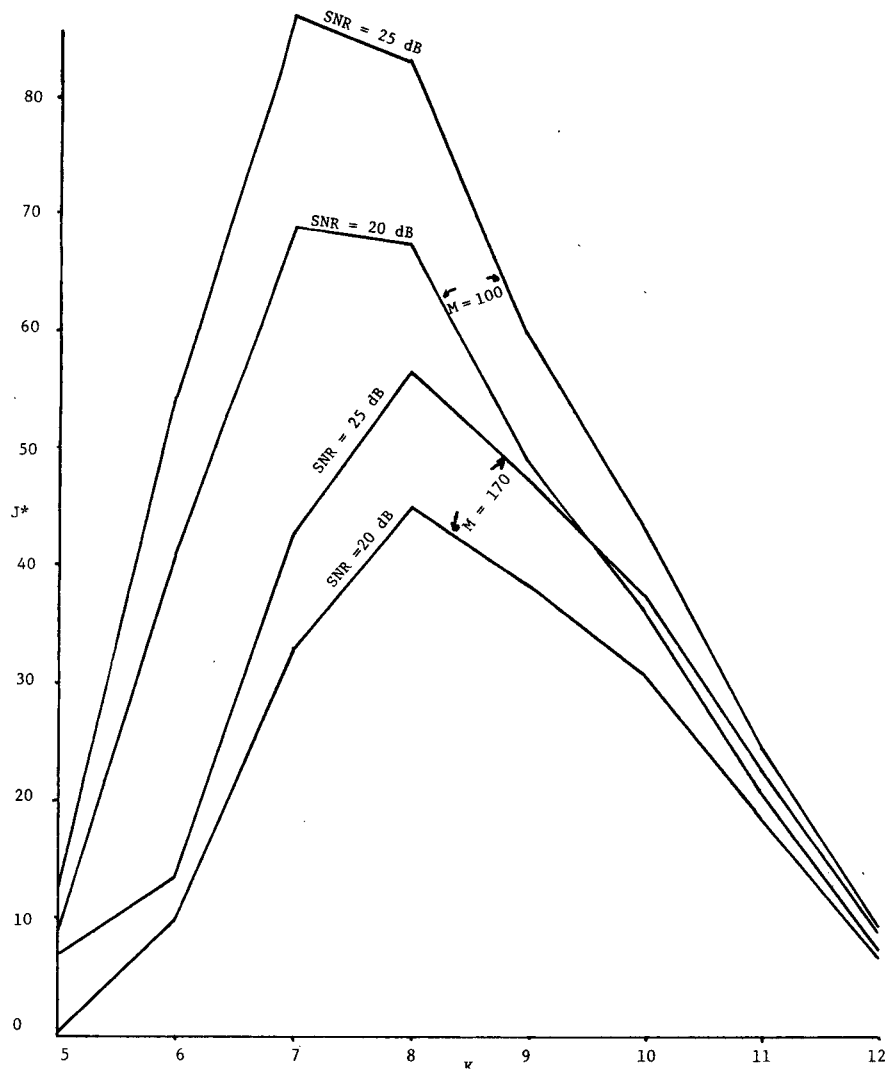


Fig. 4. Divergence  $J^*$  versus  $K$ .

A. Distance Measures

Consider the received matrix of size  $(2^K \cdot L)$ . The parameters  $K$  and  $L$  are related by [3]

$$L = \langle rK/2^K \rangle \tag{26}$$

$$r = \frac{W}{R_b} = 625. \tag{27}$$

Here,  $\langle \rangle$  denotes the largest integer operation,  $W$  the one-way bandwidth, assumed to be 20 MHz, and  $R_b$  the bit rate.

Assume that the samples from the correct row have the density function  $f$  and those from the spurious rows have the density function  $g$ . Then, the situation corresponding to the correct and the incorrect row selection can be depicted as follows:

- $H$  correct selection, A number  $L$  of samples selected from  $f$  identified
  - $N$  incorrect selection, A number  $L$  of samples selected from  $g$  identified.
- (28)

Therefore, any of the known distance measures [11], [13] can be computed for the density functions under  $H$  and  $N$ . We present here only the divergence  $J^*$  and the Bhattacharyya distance  $B$ .

B. Divergence

The divergence  $J^*$  can be written as a sum of two components called the directed divergences [11].

$$J^* = I(H, N) + I(N, H) \tag{29}$$

where

$$I(H, N) = \int_x f_H(x) \ln \left( \frac{f_H(x)}{f_N(x)} \right) dx \tag{30}$$

and  $I(N, H)$  is obtained by interchanging  $H$  and  $N$  in the above equation.

Since all the samples are independent, it is easy to observe that

$$I(H, N) = LI(f, g) \tag{31}$$

where  $I(f, g)$  is the directed divergence between the densities  $f$  and  $g$ . That is,

$$\begin{aligned} I(f, g) &= \int_0^\infty f(x) \ln \left( \frac{f(x)}{g(x)} \right) dx \\ &= - \int_0^\infty e^{-x} \ln (p + (1-p)\beta e^{-(\beta-1)x}) dx. \end{aligned} \tag{32}$$

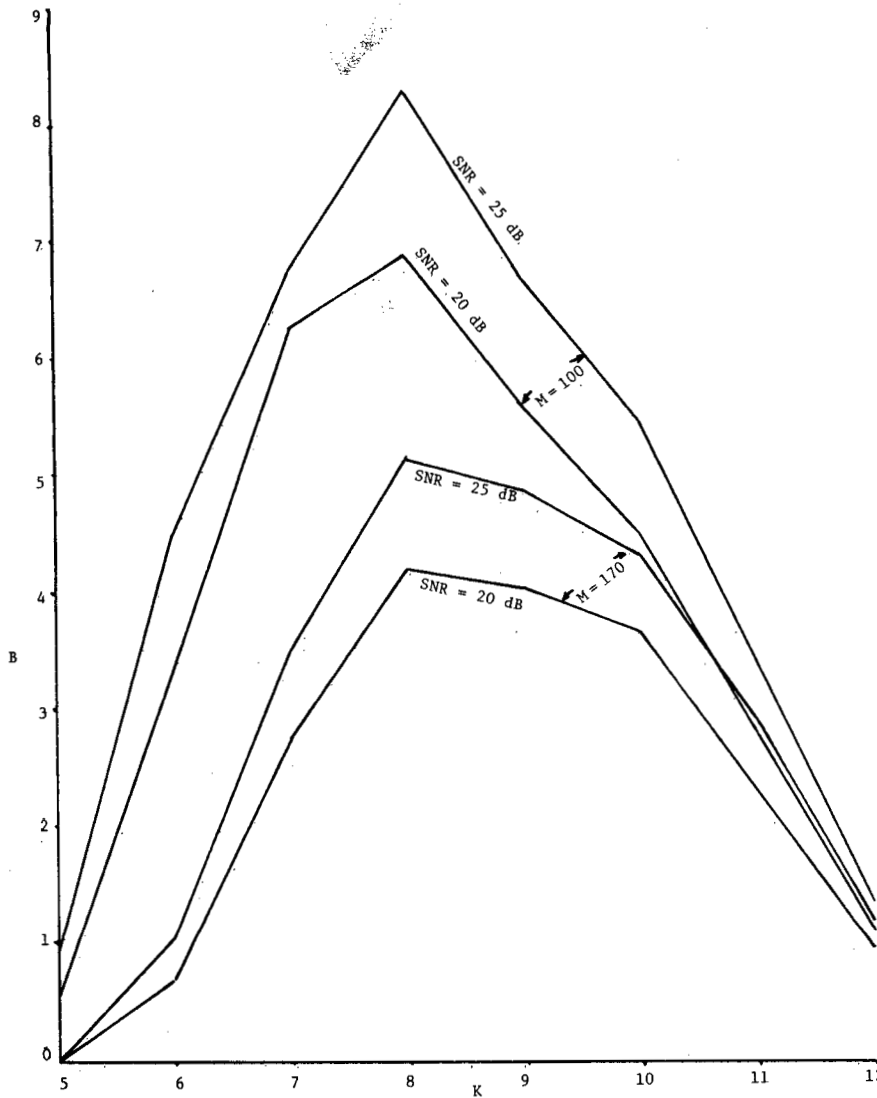


Fig. 5. Bhattacharyya distance versus  $K$ .

Therefore,

$$J^* = L(I(f, g) + I(g, f)). \tag{33}$$

When  $f$  and  $g$  satisfy (3), we can compute  $J^*$  as a function of  $K$ . The results are shown in Fig. 4.

C. Bhattacharyya Distance

The Bhattacharyya distance  $B$  between the two densities  $f_H$  and  $f_N$  is given by

$$B = -\ln \left[ \int_x \sqrt{f_H(x)f_N(x)} dx \right]. \tag{34}$$

Because of sample independence, this reduces to

$$B = -L \ln \left[ \int_0^\infty \sqrt{f(x)g(x)} dx \right]. \tag{35}$$

If  $f$  and  $g$  satisfy (3),  $B$  can be computed as a function of  $K$ . The results are shown in Fig. 5.

As an alternative method, we can observe the effect of  $K$

on the asymptotic error rate (see Fig. 6). The value of  $L$  is constrained because  $r = W/R_b$  is fixed. By observing Figs. 4-6, it can be seen that  $K = 8$  is nearly optimum under any of these performance measures.

The optimization procedure based on distances is normally employed in parametric situations, when the probability of error cannot be easily found [13]. We assumed that such procedure could also be applied to nonparametric tests operating under a known probability model. This is partially justifiable since the ranking does carry some information contained in the original samples.

VI. CONCLUSION

Considering base-to-mobile transmission, it is found that MRSR or RRR could accommodate about 135 users at  $P_b \approx 2 \times 10^{-3}$  and at an average SNR of 25 dB. With the simulated adjacent cell interference, the performance of MRSR remains practically the same (i.e.,  $P_b \approx 2 \times 10^{-3}$  at a controlled SNR of 25 dB, with the receiver about halfway toward the base station). Thus, MRSR (or RRR) shows some robustness against changes in the probability model. Moreover, the adaptive parametric hard-limited receiver accommodates only about the same number of users as the MRSR, when adjacent

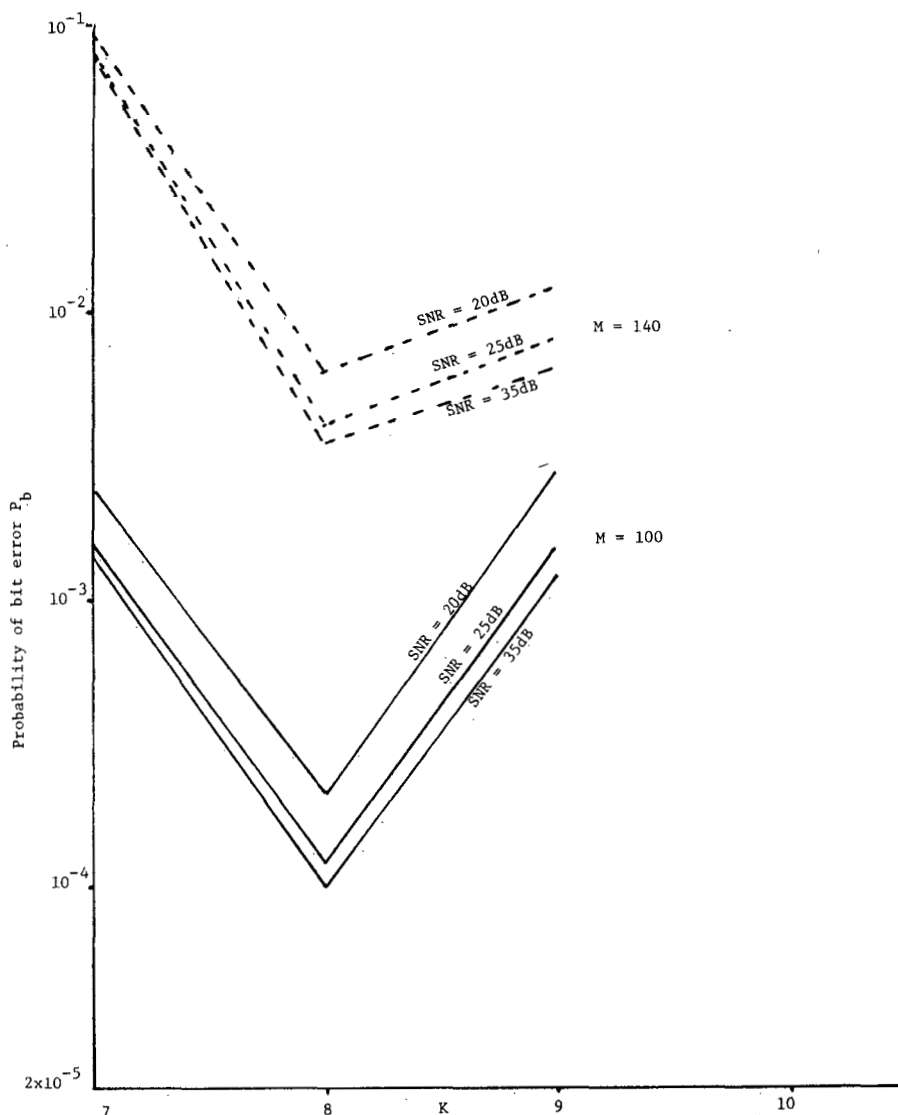


Fig. 6. Asymptotic estimate versus  $K$  (maximum rank sum receiver).

cell interference is taken into consideration [9]. Also, the limited simulation study and asymptotic theory reveal the nearly identical performances of MRSR and RRR. As has been said earlier, it is much simpler to implement the reduced rank sum receiver than to implement MRSR or a parametric receiver. Therefore, one concludes that RRR is a possible competitor to the parametric receivers for FH-MFSK mobile radio.

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