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Diversity Combining in FH/BFSK Systems to Combat Partial Band Jamming

R. Viswanathan
Southern Illinois University Carbondale, viswa@engr.siu.edu

Kashfieh Taghizadeh
Southern Illinois University Carbondale

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Diversity Combining in FH/BFSK Systems to Combat Partial Band Jamming

R. VISWANATHAN, MEMBER, IEEE, AND KASHFIEH TAGHIZADEH

Abstract—**For a FH/BFSK system, a new type of combiner termed the product combining receiver (PCR) is investigated. The performance of the PCR is evaluated for the cases of on/off partial band noise with optimum jamming fraction, and worst case partial band tone jamming. The performance of PCR is shown to be comparable to that of the clipper receiver. The effect of diversity combining along with convolutional coding and ratio threshold technique is also analyzed. Whereas the clipper requires the knowledge of signal-to-noise ratio for threshold adjustments, the PCR does not require this knowledge for its operation.**

I. INTRODUCTION

IN this paper, an L hops per bit frequency-hopped binary frequency shift keyed (FH/BFSK) system is considered. Here, each bit is divided into L independent transmissions of τ s duration ($1/L$th of bit time T_b s) by means of a frequency hopping scheme. Therefore, the hop rate is L times the bit rate R_b. For each data bit, a set of mark tones or a set of space tones would be transmitted during the L hops, depending on whether the bit is a 1 or a 0, respectively. The mark and space tones in each hop can be either adjacent (separated in frequency by $1/\tau$, called parallel FH) or can be randomly dispersed across the spread-spectrum band (called the independent FH) [1]. While considering partial band noise jamming, we assume a parallel FH model (independent FH model is treated in another paper [2]) and while considering the effect of tone jamming, we assume the independent FH model. For the tone jammer, the independent FH system is investigated because the analysis in this case is more involved than the other model. At the receiver, after dehopping with an ideal synchronized frequency-synthesizer, noncoherent energy detection is employed to detect the energy in mark and space frequencies over each of the τ s intervals (Fig. 1). The process is repeated over L diversity slots to obtain $2L$ energy samples. Depending on the type of combining scheme used to utilize these samples, we get different types of receivers. A combining scheme, based on the rankings of the energy samples, has been found useful in a mobile radio system [8]. However, rank type receivers do not perform well in partial band jammed FH/BFSK systems [16].

A. Partial Band Noise Jamming

When the samples are combined linearly, $L > 1$ leads to poor performance [4], [5]. However, if the samples are passed through a soft limiter before the summing operation, we get a clipper receiver [4], [15]. In the case of a clipper receiver, for moderate signal-to-jamming ratios, small L values lead to less probability of error.

In partial band noise analysis, we also account for the presence of thermal noise with two sided power spectral density of $N_o/2$. It is assumed that the jammer has a total of J W, but chooses to jam a fraction γ of the transmission band for the purpose of effective jamming [4], [5]. Under this condition, the jammer noise power in the jammed cell (see Fig. 2) is

$$\sigma_J^2=\frac{B}{\gamma W}J=BN_J/\gamma \tag{1}$$

where W denotes the entire spread-spectrum bandwidth and B is the bandwidth of a single hop. Each hopped tone is then jammed with probability γ or not jammed with probability $(1-\gamma)$. In the following analysis on partial band jamming, assume, without loss of generality, that the space tone is transmitted over $0 \leq t < T_b$. The $2L$ samples $Y_{11}, Y_{12}, \cdots Y_{1L}$ and $Y_{21}, Y_{22}, \cdots Y_{2L}$ at the input to the combiner can be written conveniently in a matrix form

$$\begin{pmatrix} \underline{Y}_1 \\ \underline{Y}_2 \end{pmatrix}=\begin{pmatrix} Y_{11} & Y_{12} & \cdots & Y_{1L} \\ & Y_{21} & \cdots & Y_{2L} \end{pmatrix}. \tag{2}$$

Therefore, any receiver commits an error in the decision, if it chooses the mark (i.e., the second row). The above samples can be shown to have the following density functions [14]:

$$f(y_{1j})=\frac{1}{2N_i}e^{-y_{1j}/2N_i}e^{-A^2/2N_i}I_O(\sqrt{y_{1j}}A/N_i) \quad y_{1j}\geq 0$$
$$f(y_{2j})=\frac{1}{2N_i}e^{-y_{2j}/2N_i} \quad y_{2j}\geq 0. \tag{3}$$

Here, A denotes the amplitude of the received tone, $i = 1$ denotes jamming and $i = 2$ denotes no jamming, and j takes values from $1, 2, \cdots L$. Within a normalizing constant, the density $f(y_{1j})$ in (3) represents a noncentral chi-square distribution with two degrees of freedom [10]. Equation (3) is based on the parallel FSK model in the sense that the entire BFSK subband is either jammed or unjammed. The parameters N_1 and N_2 in (3) are given by

$$N_1=B(N_O+N_J/\gamma),$$
$$N_2=BN_O,$$
$$B=1/\tau. \tag{4}$$

The signal bit energy to noise density ratio (E_b/N_O) and the signal bit energy to jamming density ratio (E_b/N_J) are as follows:

$$\left(\frac{E_b}{N_O}\right)=\frac{\alpha_2 L}{2}$$
$$\left(\frac{E_b}{N_J}\right)=\frac{L}{2\gamma(1/\alpha_1-1/\alpha_2)} \tag{5}$$

where

$$\alpha_1=A^2/N_1,$$
$$\alpha_2=A^2/N_2. \tag{6}$$

In Section II, we evaluate the performance of the product combining receiver under partial band noise jamming. Let l, 0

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The authors are with the Department of Electrical Engineering, Southern Illinois University at Carbondale, Carbondale, IL, 62901.

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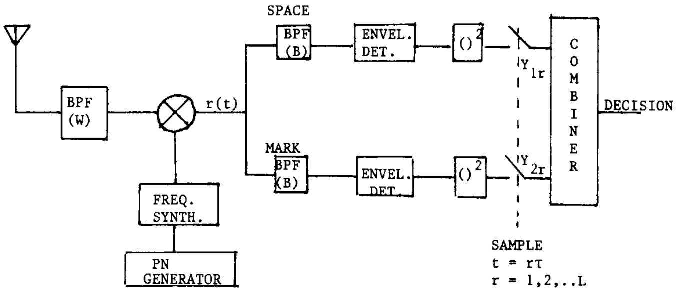

Fig. 1. Receiver for FH/BFSK system.

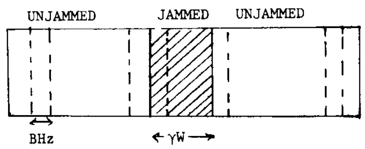

Fig. 2. Partial band jamming model.

$\leq l \leq L$, denote the number of slots jammed. Then the average probability of error, conditioned on the fact that the jammer uses the fraction γ, is

$$P(e;\gamma)=\sum_{l=0}^{L} P(e;\gamma\,|\,l)\gamma^{l}(1-\gamma)^{L-l}\binom{L}{l}. \tag{7}$$

By numerical computation, γ is varied to locate the largest $P(e)$. That is,

$$P(e)=\max_{\gamma} P(e;\gamma). \tag{8}$$

The value of γ which gives the largest $P(e)$ will be called the optimum jamming fraction. In this paper, only a binary FSK system is considered. Using the union bound, an upperbound on the probability of error for an M-ary system employing diversity combining is easily obtained. However, maximizing the bound with respect to the jamming fraction could yield a pessimistic estimate of the actual worst case error rate [5].

B. Tone Jamming

In this simplistic analysis on tone jamming, the presence of thermal noise is neglected. The following simple model is assumed. The jammer knows the exact tone frequencies available to the communicator, and the jammer transmits at random a number K of the tones with frequencies chosen from the set employed by the communicator. Also, the jammer sends at most one tone per BFSK subband. When a transmitted tone is hit, the arrival phase difference between the intended tone and the jammer tone at the receiver is accounted [6]. In this section, the approach to the analysis is formulated for $L = 2$, and the results are presented in Section III. Extension of analysis for higher L is straight forward but is not presented as it would not lead to any additional insight.

The event that a frequency tone corresponding to either a mark or space being transmitted by the jammer is denoted by a "1." Similarly, a "0" denotes the complement of the above event. Then for $L = 2$, the 16 basic event matrices are obtained as follows:

$$\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}\cdots\begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix}.$$

As in the partial band noise case, without any loss of generality, the samples in the first row correspond to the transmitted signal. Since the performance of the receiver depends on how many of the space and/or mark samples are jammed and not on the particular ones jammed, it is possible to group the 16 basic events into nine events E_1 through E_9 (Fig. 3). Then the average probability of error can be computed by averaging the conditional probability of error. That is,

$\longleftarrow$ W Hz $\longrightarrow$

$$\begin{array}{llll}
E_1\begin{bmatrix}1 & 0\\ 1 & 0\end{bmatrix} & E_1\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix} & E_1\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} & E_1\begin{bmatrix}0 & 1\\ 0 & 1\end{bmatrix}\\
E_2\begin{bmatrix}0 & 0\\ 1 & 0\end{bmatrix} & E_2\begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix} & E_3\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix} & E_3\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\\
E_4\begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix} & E_4\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix} & E_5\begin{bmatrix}0 & 1\\ 1 & 1\end{bmatrix} & E_5\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}\\
E_6\begin{bmatrix}0 & 0\\ 1 & 1\end{bmatrix} & E_7\begin{bmatrix}1 & 1\\ 0 & 0\end{bmatrix} & E_8\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix} & E_9\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}
\end{array}$$

Fig. 3. Event matrices.

$$P(e)=\sum_{i=1}^{9} P(e\,|\,E_i)P(E_i). \tag{9}$$

Results from the evaluation of (9) are examined in Section III.

If there are N possible frequencies in the communicator set and if the jammer chooses K of these at random during every diversity slot, then the probabilities of subevents such as $\binom{1}{1}$, etc., can be calculated [7]. These probabilities are given by

$$\begin{aligned}
p_1&=\Pr\left\{\begin{pmatrix}1\\0\end{pmatrix}\right\}=\Pr\left\{\begin{pmatrix}0\\1\end{pmatrix}\right\}=\frac{K(N-K)}{N(N-1)}\\
p_2&=\Pr\left\{\begin{pmatrix}0\\0\end{pmatrix}\right\}=\frac{(N-K)(N-K-1)}{N(N-1)}\\
p_3&=\Pr\left\{\begin{pmatrix}1\\1\end{pmatrix}\right\}=\frac{K(K-1)}{N(N-1)}.
\end{aligned} \tag{10}$$

By independence of jamming from one diversity slot to another, the probability of the events $P(E_i)$, $i = 1, \cdots, 9$, can be calculated. For example, when $i = 1$, $P(E_1) = 2(p_1^2 + p_2p_3)$. It is also assumed that the amplitude of the intended received tone in each diversity slot equals 1, and the jammer tone amplitude equals A. Therefore, the bit energy to jammer density (corresponding to spreading the power uniformly over W Hz) ratio is

$$\frac{E_b}{N_J}=\frac{2N}{KA^2}. \tag{11}$$

II. Product Combining Receiver (PCR)

The product combiner is the result of guessing a good combining scheme. The PCR performs favorably as the theoretical results derived below show. The receiver chooses row 1 as the signal row when the product $Y_{11}, Y_{12} \cdots Y_{1L}$ is greater than the product $Y_{21}, Y_{22} \cdots Y_{2L}$ and chooses row 2 when the converse is true. A salient property of this receiver is that when thermal noise is small, and if at least one of the diversity slots is unjammed, the receiver makes nearly a perfect decision, since the product of the samples in the nonsignal row will be extremely small.

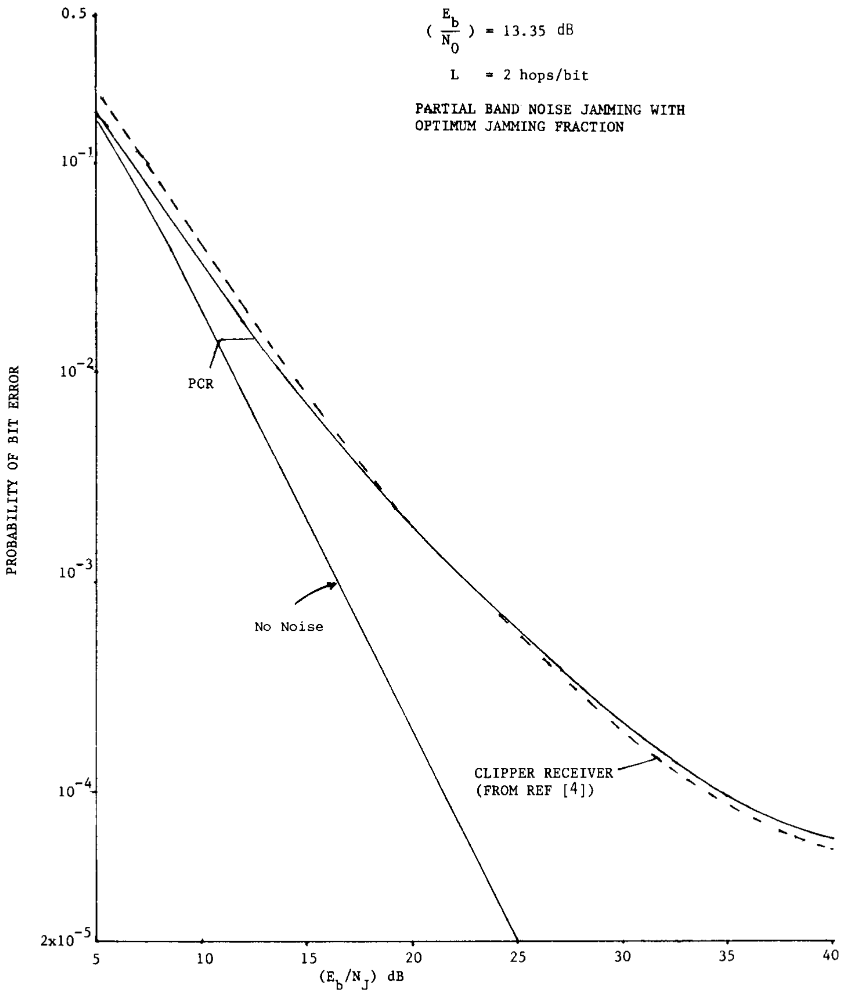

Fig. 4. FH/BFSK receiver performance comparison.

A. Error Rate Analysis for Diversity of Order Two

Let $l = 1$. Without loss of generality assume that the first slot is jammed. Consider the following random variables:

$Y_{11} \sim f_1$, f_1 is the density of Y_{1j} in (3) with parameter N_1.
$Y_{12} \sim f_2$, f_2 is the density of Y_{1j} in (3) with parameter N_2.
$Y_{21} \sim f_3$, f_3 is the density of Y_{2j} in (3) with parameter N_1.
$Y_{22} \sim f_4$, f_4 is the density of Y_{2j} in (3) with parameter N_2.
Define $X = Y_{11}/Y_{21}$ and

$$Y = Y_{12}/Y_{22}. \tag{12}$$

Therefore,

$$\begin{aligned} P(e;\, \gamma \,|\, l=1) &= \Pr\,(Y_{11}Y_{12} < Y_{21}Y_{22}) \\ &= \int_0^\infty F_Y(1/\epsilon) f_X(\epsilon)\, d\epsilon \end{aligned} \tag{13}$$

$F_Y(y)$ is expressed as [10]

$$F_Y(y) = \left(\frac{y}{1+y}\right) e^{-\alpha_2/2(1+y)}, \; y>0. \tag{14}$$

By using a series expansion for $f_X(\ )$, (13) is evaluated as

$$P(e;\, \gamma \,|\, l=1) = \sum_{j=0}^{\infty} (j+1)\, \frac{e^{-\alpha_2/2}\left(\dfrac{\alpha_1}{2}\right)^j}{j!} \left[\int_0^\infty \frac{e^{-\alpha_2(x/1+x)}}{(x+1)^{3+j}}\, x^j\, dx\right]. \tag{15}$$

Upon evaluating the integral,

$$P(e;\, \gamma \,|\, l=1) = 2\left(1+\frac{\alpha_1}{2}\right)\frac{e^{-\alpha_2/2}}{\alpha_2} + (FA_1 - FA_2) \tag{16}$$

where

$$FA_1 = 2\,\frac{e^{-(\alpha_1+\alpha_2)/2}}{\alpha_2}\, G \tag{17}$$

$$G = \frac{(\alpha_2)^2}{(\alpha_2-\alpha_1)^2}\left(e^{\alpha_2/2} + e^{\alpha_1/2}\left(\frac{\alpha_1^2}{2\alpha_2} - \frac{\alpha_1}{2} - 1\right)\right) \tag{18}$$

$$FA_2 = 4\,\frac{e^{-(\alpha_1+\alpha_2)/2}}{\alpha_2^2}\left(G + \alpha_1\,\frac{dG}{d\alpha_1}\right). \tag{19}$$

Similarly,

$$P(e;\, \gamma \,|\, l=2) = \left(\frac{1}{2} + \frac{\alpha_1}{12}\right) e^{-\alpha_1/2} \tag{20}$$

$$P(e;\, \gamma \,|\, l=0) = \left(\frac{1}{2} + \frac{\alpha_2}{12}\right) e^{-\alpha_2/2}. \tag{21}$$

Using the above conditional probabilities and (7), the average error rate $P(e)$ is evaluated. The results are shown in Fig. 4.

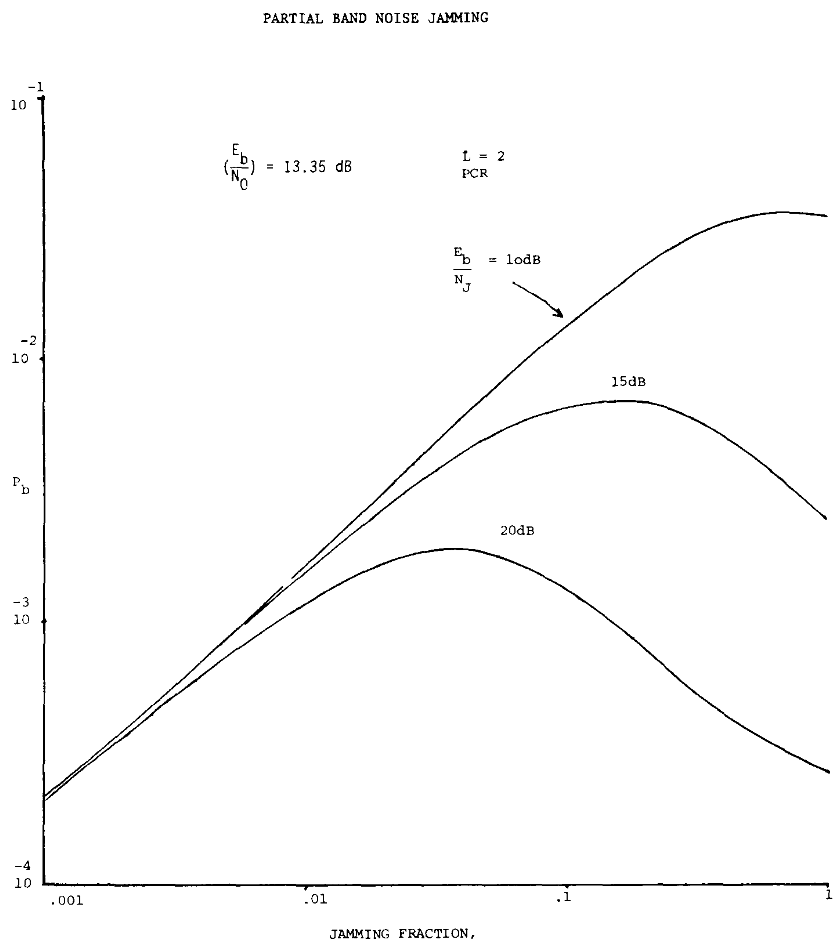

Fig. 5. Probability of bit error versus jamming fraction.

For $L = 2$, the PCR is nearly as good as the clipper receiver, whereas the clipper receiver is of adaptive type, the PCR needs no adaption. In Fig. 5, we plot the probability of bit error as a function of the jamming fraction, γ, for a fixed (E_b/N_O) of 13.35 dB. The optimum jamming fraction, for the worst case bit error rate, decreases as the signal-to-jammer ratio increases. Also, the peaks are relatively broad suggesting that in practice the jammer could attain the optimum.

B. Error Rate Analysis of PCR When L Equals 4

By proceeding as in Section II-A, it is possible to evaluate the performance of PCR for fourth-order diversity. In this case, numerical integration is required. The details are lengthy but straightforward [16].

In Fig. 6, the worst case error rate is plotted as a function of (E_b/N_J) for fixed (E_b/N_O) of 13.35 dB. The error rate curve of the clipper receiver is also shown for comparison purposes [4]. We also carried out a limited simulation study for (E_b/N_J) values of 15, 20, and 25 dB. The IMSL routines GGEXP and GGNML were used to generate the exponential and Gaussian samples and hence simulate the receiver performance. All the simulations were carried out with the number of simulation trials exceeding $10/\hat{Pe}$ where $\hat{Pe}$ is the estimate of the error probability. This assures that the normalized standard deviation of the estimation error would be less than about 0.25 [11]. From Fig. 6, we observe the close agreement between simulation and the theoretical results. Fig. 7 shows the error rate for PCR for $L = 1$, 2, and 4. For moderate (E_b/N_J), the improvement due to moderate diversity is clearly seen.

Figs. 4 and 6 show the curves corresponding to $(E_b/N_O) = \infty$. Comparing to $(E_b/N_O) = 13.35$ dB curve, it is seen that the thermal noise causes significant additional degradation, for large (E_b/N_J) values.

C. Error Rate Analysis of PCR with Coding and Viterbi's Ratio Threshold Technique

In this subsection, the effect of coding and diversity on the performance of FH/BFSK system is analyzed. Consider the limiting case of a long convolutional code and a sequential decoder operating at its cutoff rate [3]. We neglect the thermal noise but consider a two level partial band jammer [3]. Also, it is possible to improve the diversity performance by using hard decision with a quality bit as proposed by Viterbi in his ratio threshold mitigation technique. Recently, the ratio threshold technique in conjunction with the clipper receiver combiner has been analyzed [12]. The aim is to examine the performance of the PCR with the ratio threshold and compare it to the ratio threshold technique alone (without diversity). We analyze second-order diversity and comment on the higher order diversity case.

Details of the ratio threshold technique can be found in [3]. When diversity is employed, each binary symbol is transmitted in L different hops. After the combiner (in this case PCR), the sample values corresponding to the mark and the space frequency channels will be used to perform the ratio threshold test. This test leads to an equivalent binary input quartenary output channel (see Fig. 8). By using these quartenary outputs with a sequential decoder, decisions could be made regarding

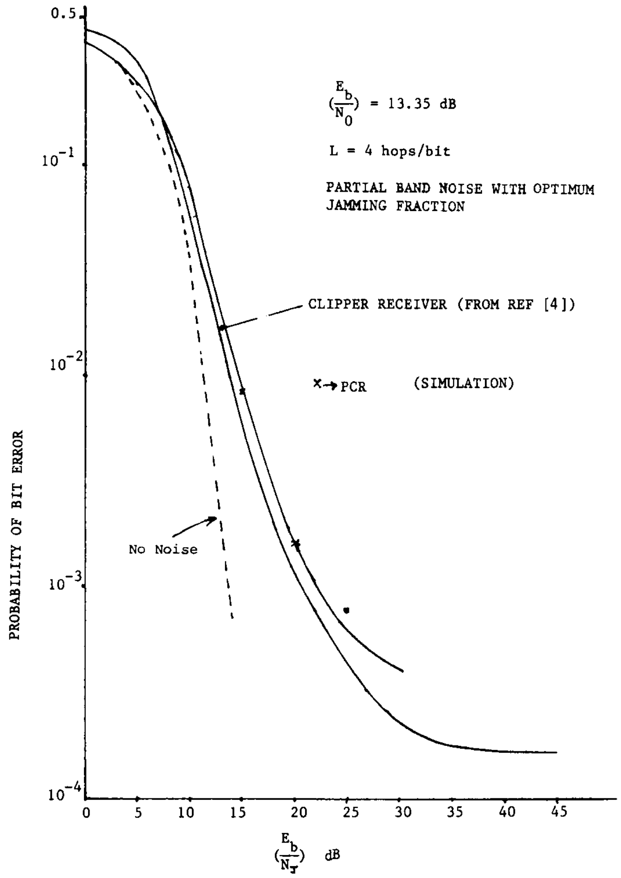

Fig. 6. FH/BFSK receiver performance comparison.

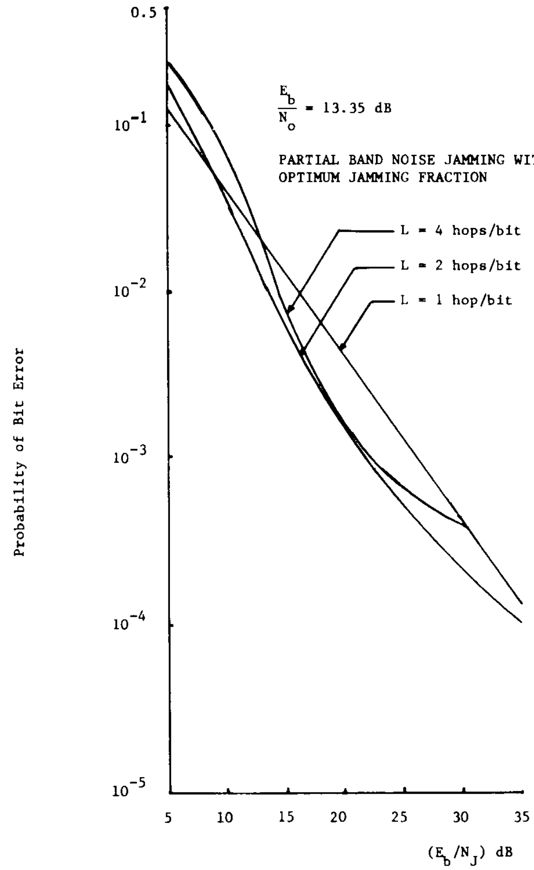

Fig. 7. Probability of bit error versus (E_b/N_j) for PCR.

the binary digit transmitted. The cutoff rate r_o of the sequential decoder is related to the transitional probabilities by

$$r_o = 1 - \log_2 (1 + 2\sqrt{P_E P_C} + 2\sqrt{P_{EX} P_{CX}}). \tag{22}$$

The worst case situation occurs when the jammer forces the user to employ a maximum (E_b/N_J) value at a certain r_o. The jammer employs noise density $\tilde{N}_1$ over a fraction ρ of the spread bandwidth and noise density $\tilde{N}_2$ over the remaining fraction. The relation between these parameters is given by

$$N_J = \rho \tilde{N}_1 + (1-\rho)\tilde{N}_2. \tag{23}$$

The user mitigates the worst situation to some extent, by using the ratio threshold parameter θ [3]. θ equals 1 corresponds to no ratio test situation or PCR with convolutional coding alone. Let

$$\left(\frac{E_b}{N_J}\right) = \frac{1}{\rho \Big/ \left(\frac{E_b}{\tilde{N}_1}\right) + (1-\rho) \Big/ \left(\frac{E_b}{\tilde{N}_2}\right)}. \tag{24}$$

It only remains to compute the transitional probabilities in (22) in terms of θ, the signal-to-jammer noise ratios and ρ.

If l denotes the number of slots jammed with noise density $\tilde{N}_1$ we have three distinct events E_0, E_1, and E_2, corresponding to $l = 0$, $l = 1$, and $l = 2$. The transitional probabilities can be computed conditioned on these events, and then averaged. For example,

$$P_C = P(C|E_0)\rho^2 + 2P(C|E_1)\rho(1-\rho) + P(C|E_2)(1-\rho)^2. \tag{25}$$

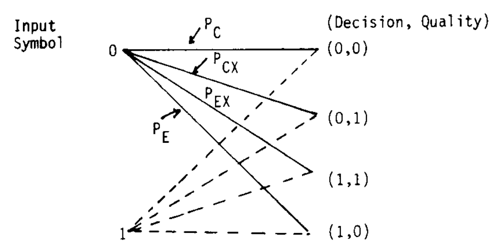

Fig. 8. Channel model for ratio threshold.

TABLE I
WORST CASE (E_b/N_j) IN dB (CORRESPONDING ρ SHOWN IN PARENTHESIS) VITERBI'S SCHEME

r_o/θ	2	3.7
1/1.2	18.21 (0.97)	16.38 (0.96)
1/2	10.61 (0.68)	9.9 (0.58)
1/3	9.58 (0.36)	9.29 (0.21)
1/8	10.5 (0)	10.41 (0)

Similar expressions can be written for P_E, P_{EX}, and P_{CX}. Derivation of the expressions for the conditional probabilities is given in [16].

For different values of θ and r_o, worst case (E_b/N_J)'s are obtained. The results are shown in Tables I and II, and Fig. 9. With $L = 2$, the worst case (E_b/N_J) occurs when $\tilde{N}_1 = 0$ and ρ is appropriately chosen. That is, the optimum two level

TABLE II
WORST CASE (E_b/N_J) IN dB (CORRESPONDING ρ SHOWN IN PARENTHESIS) RATIO THRESHOLD TECHNIQUE WITH PCR

r_o/θ	1	2	5	8	15
1/1.2	14.28 (0.78)	13.23 (0.74)	12.16 (0.68)	11.77 (0.64)	11.48 (0.56)
1/1.5	14.03 (0.71)	11.1 (0.5)	10.45 (0.32)	10.3 (0.28)	10.28 (0.17)
1/2	11.21 (0.244)	10.47 (0.16)	10.12 (0.02)	10.11 (0)	10.18 (0)
1/3	11.22 (0)	10.7 (0)	10.44 (0)	10.43 (0)	10.49 (0)
1/4	11.52 (0)	11.01 (0)	10.79 (0)	10.78 (0)	10.83 (0)
1/5	11.83 (0)	11.31 (0)	11.1 (0)	11.09 (0)	11.13 (0)
1/6	12.09 (0)	11.56 (0)	11.37 (0)	11.36 (0)	11.41 (0)
1/7	12.32 (0)	11.79 (0)	11.61 (0)	11.6 (0)	11.64 (0)
1/8	12.52 (0)	12.0 (0)	11.83 (0)	11.82 (0)	11.85 (0)

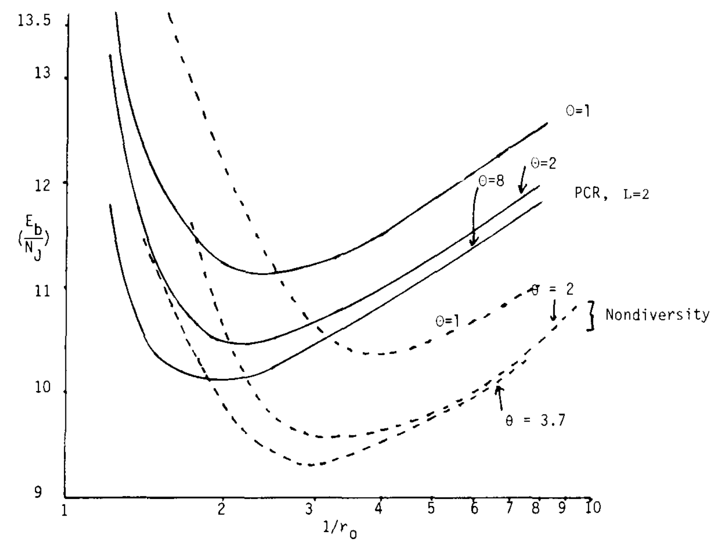

Fig. 9. Cutoff rate versus (E_b/N_J).

jammer is of the on/off type. For high rate codes, the PCR with ratio threshold is better than a simple ratio threshold scheme. $\theta = 8$ gives the best result for most r_o of interest. For example, with $r_o = 1/2$, $\theta = 8$, an (E_b/N_J) of 10.11 dB is required. In Fig. 9 and Table I, we also show Viterbi's result (i.e., without diversity). Without diversity, a best value of $\theta = 3.7$, and $r_o = 1/3$, an (E_b/N_J) of 9.29 dB is required. Hence, with diversity, a penalty of about 0.82 dB exists when compared to the nondiversity case. However, diversity with the ratio threshold is useful in the sense that the worst case ρ for this scheme is different from the worst case ρ in Viterbi's scheme. For example, in Viterbi's scheme with $r_o = 1/2$ and $\theta = 3.7$, the worst case ρ equals 0.58, whereas for $r_o = 1/2$ and $\theta = 8$, the worst case ρ in PCR with the ratio threshold technique equals 0. That is, the jammer is forced to employ wide-band jamming. Also for $\theta = 8$ and $r_o = 1/2$, and PCR with the ratio threshold, the (E_b/N_J) requirement is only 2.92 dB when the jammer employs $\rho = 0.58$. Similar reduction is also obtained by changing to a different coding rate rather than employing diversity. For example, (E_b/N_J) required is only 4.41 dB with $r_o = 1/4$ and $\theta = 3.7$.

Though not shown here, we have evaluated PCR with ratio threshold technique for $L = 4$ and $\theta = 1$, assuming an on/off type of jammer (a two-level jammer of the type (23) with $\tilde{N}_1 = 0$). The worst case (E_b/N_J)'s are considerably larger than the corresponding values for $L = 2$ case for all $1/r_o > 1.2$. Therefore, we conjecture that larger values of L may not lead to useful performance. Finally, it must be mentioned that with Viterbi's scheme, the (E_b/N_J) requirement can be reduced below 9.29 dB by moving to higher M-ary alphabets [3].

III. Performance Under Tone Jamming

As explained in the Introduction, the conditional probabilities $P(e|E_i)$ are needed for evaluating $P(e)$. Evaluation of (9) for the clipper and PCR are lengthy but straightforward. Details can be found in [16].

For a given (E_b/N_J) ratio and N, the $P(e)$ can be calculated for different receivers as a function of K. We assume that the jammer optimizes K to cause the largest error rate. The worst case error rates are shown in Fig. 10 as a function of (E_b/N_J), assuming N equals 1000. From the figure it is seen that the PCR is competitive to the clipper receiver. Both receivers show an order of magnitude improvement in the error rates over the nondiversity receiver. Even in the presence of thermal noise, the diversity improvement with these receivers should be possible.

IV. Conclusion

In this paper, a new scheme of diversity combining for FH/BFSK system is proposed to combat partial band jamming.

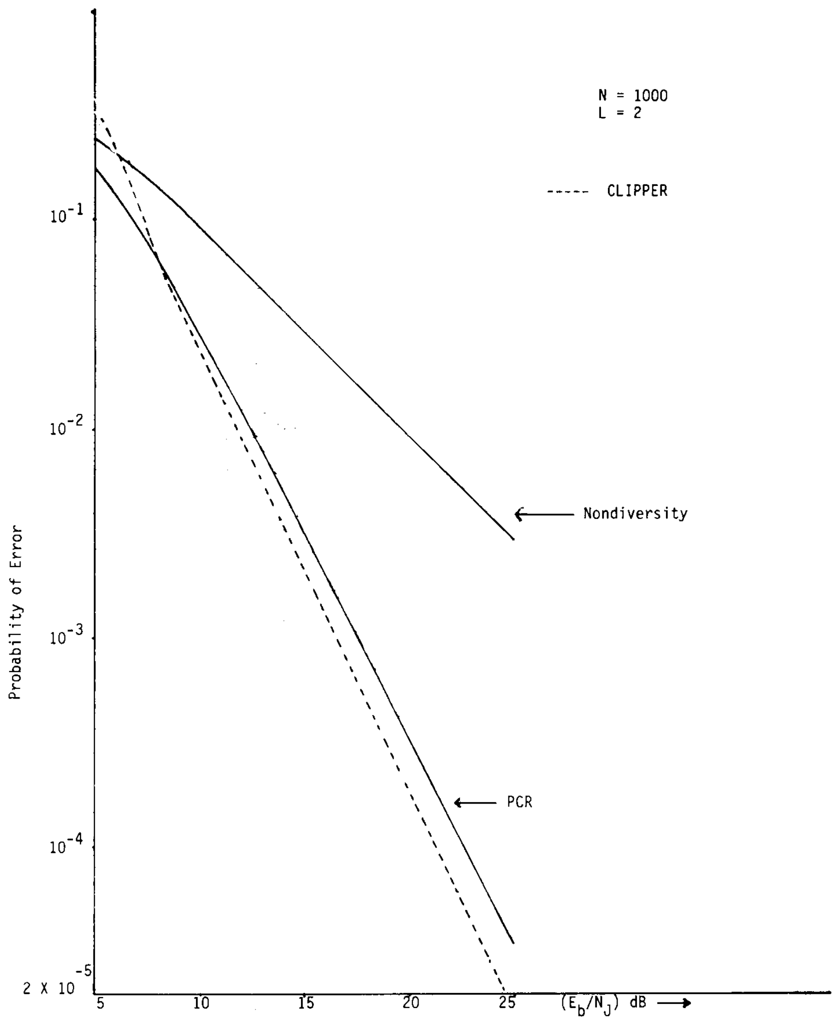

Fig. 10. Worst case probability of error under tone jamming.

Under partial band noise jamming, when compared only on the basis of diversity, the PCR is comparable in performance to that of the clipper receiver considered in [4]. Whereas the clipper requires the signal-to-noise ratio for threshold adjustments, the PCR does not require this knowledge for its operation. The PCR shows improvement over the nondiversity receiver for moderate (E_b/N_J) values.

We also evaluated the performance of PCR of diversity two with convolutional coding and Viterbi's ratio threshold technique. Against the best jammer, the best performance of this receiver occurs with an (E_b/N_J) value which is about 0.82 dB higher than the value required by a simple ratio threshold scheme (without diversity). However, diversity with the ratio threshold is useful in the sense that the worst case jamming fractions with and without diversity are different. Also, with high rate codes PCR with the ratio threshold performs better than a simple ratio threshold scheme.

Finally, the second-order diversity performances of PCR and the clipper under tone jamming, and no thermal noise are analyzed. These combiners exhibit some diversity gain over the nondiversity receiver. The presence of thermal noise is expected to affect the relative performances of the receivers to some extent.

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R. Viswanathan (S'81–M'83) received the B.E. (Hons.) degree in electronics and communication engineering from the University of Madras, India, in 1975, the M.E. degree with distinction in electrical communication engineering from the Indian Institute of Science, Bangalore, India, in 1977, and the Ph.D. degree in electrical engineering from Southern Methodist University, Dallas, TX, in 1983.

From 1977 to 1980, he worked as a Deputy Engineer in the Digital Communication Department of the Research and Development Division of Bharat Electronics Limited, Bangalore, India. Since 1983 he has been an Assistant Professor in the Electrical Engineering Department at Southern Illinois University, Carbondale, IL. His research interests include spread-spectrum systems, statistical theory of communication, and distributed detection and estimation theory.

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Kashfieh Taghizadeh received the M.S.E.E. degree in 1985 from Southern Illinois University at Carbondale.

For the past few years she has been working as a Technical Support Engineer for 32-bit microprocesser-based product line at National Semiconductor Corporation, CA.