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A Selection and Estimation Test for Multiple Target Detection

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We apply a selection and estimation procedure for the detection of multiple targets in clutter. The selection and estimation (SE) test performs better than a fixed order statistic (OS) detector. The SE test has some similarity to a variable trimmed mean (VTM) test and has a slight performance advantage. Unlike the VTM, whose performance is evaluated only by simulation studies, the performance of the SE test, for Rayleigh fluctuating target model, can be evaluated analytically. Further improvement in the performance is possible if a better selection procedure can be found.

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I. INTRODUCTION

The detection of multiple targets in clutter has been a research topic for the past twenty years [1-17]. Here we are concerned with search type radars, looking for targets at certain range, elevation, and azimuth. Closely spaced multiple targets in search volume arises when 1) some of the three parameters are nearly the same for the targets and 2) the other parameters are in the resolution cells not too far from each other. The return from a target is present in a noise-plus-clutter background whose power level and the amplitude distribution may be unknown and possibly changing. For convenience, the clutter-plus-noise is referred sometimes as clutter or noise. For the solution proposed here, it is assumed that the amplitude distribution but not the power level is known. We assume that the radar return is processed in a conventional way, using noncoherent envelope detection followed by a square law nonlinearity and comparison to a threshold. Earlier researchers realized the fact that if a constant threshold value is used, the probability of false alarm can increase dramatically, even when the power level of the clutter changes by a small amount. In target detection, it is important to keep the probability of false alarm below a specified value. Therefore, it becomes necessary to take samples from adjacent resolution cells in order to have an estimate of the clutter-plus-noise level. If the resolution cells are purely from a clutter and noise background, one can have a consistent estimator whose value will approach asymptotically to the clutter-plus-noise power level, as the number of samples increases. Realistically, when the number of adjacent cells is increased, it is likely that 1) some of these samples are from other interfering targets and 2) two groups of samples may be from differing clutter backgrounds when a clutter transition occurs within the range of resolution cells. When two different clutter backgrounds exist, we call the low power clutter as noise. When the clutter transition occurs, ideally we want the estimate of the power level of the clutter background that is present in the intended target cell. Therefore, given a number N of samples from the adjacent resolution cells, the problem is to estimate the power level of the clutter-plus-noise background that is present in the test cell under investigation. Since estimates are based on a finite number of samples, the ideal value cannot be realized. The tradeoff parameters are 1) the performance in the homogeneous background, that is, no interfering targets or clutter power variations, 2) a good resolution of closely spaced multiple targets, and 3) low false alarm rate swings during clutter transitions.

A number of schemes proposed in the literature are termed variously as 1) CA-CFAR, cell averaging constant false alarm rate detector, 2) GO-CFAR, greatest of cell averaging CFAR, 3) SO-CFAR,

smallest of cell averaging CFAR, 4) OS, order statistic CFAR 5) TM, trimmed mean CFAR [10, 18], 6) CMLD, censored mean level detector, and 7) weighted cell averaging CFAR [10, 14]. Among these schemes, only the OS and the TM provide a reasonable tradeoff between the above various factors [10]. Also, it is of interest to look at the multiple decision procedure described in [9]. The scheme tests the reference samples for homogeneity, for possible clutter transition and position of clutter transition if a transition is suspected, and for the samples from possible interferers. The author has quoted encouraging results based on simulation studies. The drawback of such a multistage procedure is that it introduces statistical dependencies and it cannot be analytically evaluated. Simulation studies will have to be restricted to fewer situations. Other recent procedures are a variable trimmed mean (VTM) test suggested in [16] and an adaptive order statistic (AOS) test in [25]. Because of analytical difficulty one resorts to simulation for performance studies. The VTM performs slightly better than the OS detector. For few example cases, we compare the performances of the OS and the VTM to the selection and estimation (SE) test. The idea of selection and ranking used in statistics for reliability and other applications is applied to this estimation problem. Given N adjacent cells samples, a selection procedure determines samples from possible interfering targets or clutter samples from differing background. Based on the selection, an estimate of the power level is derived. The scheme is discussed in detail in Section II and the design and performance issues are studied in Section III.

II. A NEW PROCEDURE BASED ON SELECTION AND ESTIMATION

Let us denote the N samples from the adjacent resolution cells as $X_1, X_2, \dots, X_n, X_{n+1}, \dots, X_N$ and the test sample as X_0 . For convenience, the samples X_1, \dots, X_n are assumed to be from a lagging window and the remaining are from the leading window. Typically, $n = N/2$. The samples are assumed independent, though in practice this may be violated. Correlation between samples usually leads to worse than the performance achieved with independence. If we denote the estimate derived from these samples as λ^* , then the test for the target detection is based on the following:

$$\text{Declare target present if } X_0 > t\lambda^* \quad (1)$$

where t is some positive number. For any of the schemes, assuming a homogeneous background, t will be found to achieve a specified probability of false alarm P_f . Since the unknown clutter or noise power level appears as a scale parameter in the distributions of X_0 and the $\{X_i\}$; under homogeneous background,

a constant P_f will be achieved for any of the above schemes and the proposed scheme. Hence, all these procedures are termed CFAR, though interfering targets and/or clutter transitions can change the false alarm rate.

Although the proposed SE method can be designed for any target/noise models, the design and the performance analysis of the SE test using an analytical method is possible for a Rayleigh fluctuating target model. For other target/noise models, it may be necessary to rely on simulation studies. In the rest of the discussions we restrict our attention to the Rayleigh fluctuating target.

A. Target Model

In the sequel, we assume that $\{X_i\}$ and X_0 are exponentially distributed (Rayleigh target). The probability density functions of the samples are assumed as follows.

Homogeneous Background: Let

$$f(x) = \begin{cases} 1/\lambda e^{-x/\lambda}, & x \geq 0 \\ 0, & \text{elsewhere} \end{cases} \quad (2)$$

For convenience, denote the above density function as $\exp(\lambda)$. Then

$$X_i \sim \exp(1) \quad i = 1, 2, \dots, N. \quad (3)$$

Without any loss of generality, the mean of the exponential is taken to be one when the samples are from the homogeneous background. For the test cell,

$$\begin{aligned} X_0 &\sim \exp(1) && \text{for no target} \\ &\sim \exp(1 + \text{SNR}) && \text{for target.} \end{aligned} \quad (4)$$

where SNR denotes the signal-to-noise power ratio.

Clutter Transition: Assuming the clutter transition occurs at the $(\ell + 1)$ th sample,

$$\begin{aligned} X_1, \dots, X_\ell &\sim \exp(1) \\ X_{\ell+1}, \dots, X_N &\sim \exp(1 + \text{CNR}) \end{aligned} \quad (5)$$

where CNR denotes the clutter power-to-noise (or low clutter) power ratio. For the test cell,

$$X_0 \sim \exp(1) \quad \text{or} \quad \exp(1 + \text{CNR}) \quad (6)$$

for no target, according to whether or not the test sample is from noise background or clutter background, respectively,

$$X_0 \sim \exp(1 + \text{SNR}) \quad \text{or} \quad \exp(1 + \text{CNR} + \text{SNR}) \quad (7)$$

for target, according to whether or not the test sample is from noise background or clutter background respectively.

Interferers: When some of the samples are from interfering target returns, these samples are assumed distributed exponentially with parameter $1 + \text{INR}$, where INR denotes the interfering target power-to-noise power ratio.

B. Selection and Estimation

Given the reference samples X_1, X_2, \dots, X_N , a subset of samples with power levels equal to that of the noise plus clutter present in the test cell is required. In other words, the subset should exclude the samples from the possible interferers and clutter cells of different power level. Obviously if the number of interferers is too large or if the number of clutter cells with different levels is large, then the selected subset size will be small and the estimate λ^* will not be good. If this happens, nothing can be done to improve the estimate, but this is not the problem of the selection scheme. In reliability and other applications, selection and ranking schemes are widely used.

A complete treatment of different ranking and selection procedures can be found in [19–21]; we discuss only a subset selection procedure that is relevant to the present problem. Assume that there are n_1 samples from a population characterized by a parameter Θ_1 and n_2 samples from a population characterized by a parameter Θ_2 . For example, the Θ s could correspond to the means of the respective populations. Let $\Theta_2 > \Theta_1$, without any loss of generality, and $n_1 + n_2 = N$ be a fixed number. The value of n_1 (and hence n_2) is not known a priori and the problem is to identify all the samples from one group (say one with the parameter Θ_1). Let the observations be denoted as X_1, X_2, \dots, X_N and the rank ordered values be denoted as $X_{(1)}, X_{(2)}, \dots, X_{(N)}$. Assume X_i s to be positive random variables. Since large values of X_i s are more likely from the populations with a larger mean Θ_2 , a subset selection scheme that identifies the population Θ_1 is of the following form: include all those samples in the subset for which $X_i \leq dX_{(1)}$, where d is greater than or equal to one.

Selection and Estimation Method: The subset selection approach is applied to select a subset of the reference samples X_1, X_2, \dots, X_N . Ideally, the following aspects of selection are required. 1) All X_1, \dots, X_N should be selected, if they are identically distributed (i.e., homogeneous background). 2) If there are multiple interfering targets, the samples due to these should not be selected but the rest should be included. 3) If clutter background with differing power levels exist, the samples whose power levels are the same as the one present in the test cell must be selected and the rest should not be selected.

In practice, all these requirements cannot be met. Let us consider the following procedure:

include in the subset all those X_i satisfying $X_i \leq dX_{(b)}$ (8)

where $d \geq 1$, and $X_{(b)}$ is the b th smallest order statistic of the samples $\{X_i\}$. Let r denote the size of the selected subset and $P_s(r)$ be the corresponding probability. Notice that $b \leq r \leq N$. The probability is given by

$$P_s(r) = \begin{cases} \Pr(X_{(r)} \leq dX_{(b)} < X_{(r+1)}), & r = b, b+1, \dots, N-1 \\ \Pr(X_{(N)} \leq dX_{(b)}), & r = N \end{cases} \quad (9)$$

The joint distribution of $X_{(b)}$, $X_{(r)}$, and $X_{(r+1)}$ is known in terms of the distribution of $\{X_i\}$, and hence (9) can be evaluated [22, 23]. As discussed below, knowing $P_s(r)$, a reasonable choice of d for a given b can be made.

From the subsequent discussions, it is seen that the proposed SE test can be considered as a two-step procedure. In the first step, as mentioned above, an application of the selection procedure to the adjacent cell resolution samples, yields the parameter r . In the second step, based on r and a predetermined look-up table, a parameter β is obtained and the test for the presence of a target in the test cell is carried out by using (1) with $\lambda^* = X_{(\beta)}$. The design of the SE test involves the choice of the parameters d , b , and the assignment of β as a function of r (the look-up table).

Choice of d and b : By explicitly evaluating (9), the selection probability for homogeneous background is found to be as follows:

$$P_s(r) = \binom{N}{r} \binom{r}{b} b/(d-1) \sum_{i=0}^{b-1} \binom{b-1}{i} (-1)^i \times \text{beta}(r-b+1, (r-b+1+i+d(N-r))/(d-1)) \quad (10)$$

Above $\text{beta}(\cdot, \cdot)$ denotes the beta function.

The Tables I through V show the selection probabilities for different values of d and b and for homogeneous, five interferers, and the clutter transition in the middle situations. (The selection probabilities for the interferer and the clutter transition situations are obtained from $P(f, r)$ expression in (11) with t replaced by zero.)

Tables I–III show the selection probabilities for $N = 24$, $b = 8$, and $d = 3, 5$, and 10 , respectively. Ideally, in the homogeneous background, nearly all the samples should be selected and in the case of five interferers, all the samples except those from the interferers should be selected. In general, identifying the interfering targets samples become easier when the INR are large, say greater than 20 dB. Consider the specific value of $b = 8$. The d value of 10 provides a good selection probability in the homogeneous background (probability of selecting a subset of size

TABLE I
Selection Probability

r	Selection Probability (N=24)				
	Homog.	#interf.=5	#interf.=5	Clutt.in middle	Clutt. in middle
	Back.	INR =10dB	INR = 20dB	CNR= 10dB	CNR= 20dB
8	5.e-4	7.e-4	8.e-4	0.0017	0.0035
9	0.003	0.0043	0.0047	0.0115	0.0258
10	0.0097	0.0142	0.0158	0.04	0.1
11	0.0228	0.0337	0.0378	0.0953	0.263
12	0.043	0.0638	0.0718	0.1665	0.436
13	0.0689	0.101	0.1138	0.2128	0.143
14	0.0965	0.1371	0.1535	0.2	0.024
15	0.12	0.161	0.177	0.1429	0.0025
16	0.1347	0.163	0.1725	0.0786	2.e-4
17	0.1359	0.1402	0.1371	0.0339	1.e-5
18	0.1232	0.099	0.082	0.0116	5.e-7
19	0.0994	0.0542	0.0301	0.0031	1.e-8
20	0.0703	0.021	0.0025	7.e-4	4.e-10
21	0.0422	0.0054	1.e-4	1.e-4	8.e-12
22	0.0204	9.e-4	2.e-6	1.e-5	1.e-13
23	0.0072	8.e-5	2.e-8	9.e-7	0
24	0.0014	3.e-6	1.e-10	3.e-8	0

Note: $b = 8, d = 3$.

TABLE II
Selection Probability

r	Selection Probability (N=24)				
	Homog.	#interf.=5	#interf.=5	Clutt.in middle	Clutt. in middle
	Back.	INR =10dB	INR = 20dB	CNR= 10dB	CNR= 20dB
8	1.e-5	1.e-5	2.e-5	6.e-5	1.e-4
9	8.e-5	1.e-4	2.e-4	6.e-4	0.0017
10	3.e-4	6.e-4	7.e-4	0.003	0.0128
11	0.0011	0.002	0.0025	0.0125	0.081
12	0.0029	0.0056	0.0068	0.0405	0.5078
13	0.0065	0.0132	0.0164	0.0947	0.2934
14	0.0129	0.0275	0.0351	0.159	0.085
15	0.0232	0.0519	0.0682	0.1984	0.0158
16	0.0387	0.0891	0.1204	0.1908	0.0021
17	0.0598	0.1382	0.1909	0.1459	2.e-4
18	0.0861	0.1889	0.26	0.0883	1.e-5
19	0.1154	0.2126	0.2585	0.0433	1.e-6
20	0.1429	0.1628	0.0376	0.0169	5.e-8
21	0.1605	0.079	0.0024	0.0051	2.e-9
22	0.1581	0.0235	8.e-5	0.0011	6.e-11
23	0.1261	0.004	1.e-6	2.e-4	1.e-12
24	0.0646	3.e-4	1.e-8	1.e-5	0

Note: $b = 8, d = 5$.

TABLE III
Selection Probability

r	Selection Probability (N=24)				
	Homog.	#interf.=5	#interf.=5	Clutt.in middle	Clutt. in middle
	Back.	INR =10dB	INR = 20dB	CNR= 10dB	CNR= 20dB
8	5.e-8	9.e-8	1.e-7	4.e-7	1.e-6
9	5.e-7	9.e-7	1.e-6	5.e-6	2.e-5
10	2.e-6	5.e-6	6.e-6	4.e-5	3.e-4
11	1.e-5	2.e-5	2.e-5	3.e-4	0.0052
12	3.e-5	8.e-5	1.e-4	0.002	0.3236
13	9.e-5	2.e-4	3.e-4	0.009	0.3621
14	2.e-4	7.e-4	0.001	0.0278	0.2046
15	5.e-4	0.002	0.0031	0.0622	0.077
16	0.0012	0.0057	0.01	0.108	0.0214
17	0.0027	0.016	0.0324	0.1516	0.0046
18	0.0057	0.0466	0.1221	0.1758	8.e-4
19	0.0119	0.146	0.6353	0.1702	1.e-4
20	0.0247	0.2641	0.1732	0.137	1.e-5
21	0.0516	0.2749	0.0208	0.0899	1.e-6
22	0.1098	0.1716	0.0013	0.046	6.e-8
23	0.2422	0.0616	5.e-5	0.0166	2.e-9
24	0.5489	0.01	7.e-7	0.0032	5.e-11

Note: $b = 8, d = 10$.

22 or larger is more than 0.9, as seen from Table III). However, $d = 3$ is better when there exists interfering targets. This follows from the following observation. With INR of 20 dB or larger, the probability that all of the five interferers samples occupy the five highest order statistics in the combined ranking of 24 samples is large (using the theory of order statistics [22], this probability exceeds 0.99). With $d = 10$, the probability that the 20th order statistic is included in the selected subset is 0.19 ($\approx 0.1732 + 0.0208 + 0.0013$, from Table III), whereas with $d = 3$, the corresponding probability is only 0.0026. From Tables I and II, with the clutter transition in the middle and CNR of 20 dB, the probability of selecting $r = 12$ or 13 is 0.8 for $d = 5$ whereas the corresponding probability is only 0.578 for $d = 3$. Also, for $d = 3, r = 10$ or 11 is selected with higher probability than when $d = 5$. Hence, in this situation, $d = 5$ is preferred over $d = 3$. For $N = 24$ and $b = 4$, a similar effect of different d values on the selection probabilities is seen from Tables IV and V. For a given b , the choice of d to be a large or a small value only trades off the performances under homogeneous and nonhomogeneous situations. Therefore, the behavior of the selection scheme is far from ideal. A judicious choice of an estimator, based on the subset selection is needed in order to achieve an overall best performance. We digress briefly to discuss the choice of an estimator.

In the search of an estimator we restrict ourself to one of the order statistics. This is due to the following reasons. 1) A fixed OS detector provides nearly as good a performance as a trimmed mean detector [10]. The additional complexity of trimmed averaging does not yield very significant improvement in performance. 2) An analytical evaluation of the proposed SE scheme, for the Rayleigh target model, is possible when the estimator is chosen as one of the order statistics.

Let us now get back to the choice of d values. With the choice of the estimation procedure discussed in the next section, and from the numerical study, it is found that the performance in the homogeneous background is not changed significantly for a wide range of d values except for very small d values. Hence, it would be productive to choose a small d value that is consistent with the following. The d value should not be so small that 1) it gives very low selection probability for r values in the upper half, for the homogeneous case, 2) it identifies rather poorly the clutter transition occurring in the middle, and 3) it gives a large probability of selection for r values close to $N/2$, when there exists an expected maximum number of interfering targets of reasonable strength, thereby making the selection scheme confused between the clutter transition in the middle and the interfering target situation. Based on the above discussion, and the

TABLE IV
Selection Probability

r	Selection Probability (N=24)				
	Homog.	#interf.=5	#interf.=5	Clutt.in middle	Clutt. in middle
	Back.	INR =10dB	INR = 20dB	CNR= 10dB	CNR= 20dB
4	1.e-4	1.e-4	2.e-4	2.e-4	2.e-4
5	6.e-4	7.e-4	7.e-4	9.e-4	0.0012
6	0.0015	0.0018	0.0018	0.0026	0.0034
7	0.003	0.0036	0.0038	0.0059	0.0084
8	0.0052	0.0065	0.0068	0.0116	0.0182
9	0.0083	0.0106	0.0113	0.0211	0.0376
10	0.0123	0.0163	0.0176	0.0362	0.0771
11	0.0174	0.0239	0.0262	0.0594	0.1661
12	0.0237	0.0336	0.0374	0.093	0.4242
13	0.0311	0.0459	0.0519	0.13	0.2
14	0.0397	0.0607	0.0701	0.1553	0.0521
15	0.0494	0.0785	0.0927	0.1555	0.0093
16	0.0601	0.0986	0.1198	0.1309	0.0013
17	0.0715	0.1202	0.1508	0.0933	1.e-4
18	0.0831	0.14	0.1818	0.0565	1.e-5
19	0.0944	0.1491	0.1935	0.029	9.e-7
20	0.1041	0.1188	0.0308	0.0125	5.e-8
21	0.1107	0.0638	0.0025	0.0043	2.e-9
22	0.1113	0.022	9.e-5	0.0012	8.e-11
23	0.1011	0.0045	2.e-6	2.e-4	2.e-12
24	0.0709	4.e-4	2.e-8	2.e-5	2.e-14

Note: $b = 4$, $d = 10$.

tables, a value of 5 for d is reasonable for $b = 8$ and a value between 10 and 15 is reasonable for $b = 4$.

By knowing the selected subset size, a decision regarding the clutter transition in the middle is desirable. Hence we choose b so that $b < N/2$. For a given design problem (an example is considered in the next section) few values of b in the range $(1, N/2)$ are chosen, and the performance is assessed. The value of b that yields the best performance is then finally chosen. In the next section we show by means of a design example how one can design an estimator based on the selection and achieve some definite improvement in the performance over the OS detector.

III. ESTIMATOR ASSIGNMENT AND PERFORMANCE OF SELECTION AND ESTIMATION

As pointed out in the previous section, a reasonable value for d can be chosen for a given value of b . The choice of b can then be decided by evaluating the performances for different b s. In order to show that a definite improvement in performance over the fixed OS detector is possible, we consider the following case. Consider $N = 24$, false alarm

probability of 10^{-6} , and the 20th OS detector [10]. With $INR/SNR = 1$, the OS detector can tolerate up to 4 interfering targets with the detection probability of the test target approaching 1 as a function of increasing signal power and can provide a detection probability of only 0.27 with infinite test signal power, when there are 5 interferers. Let the estimator λ^* be $X_{(\beta)}$. By a judicious choice of β as a function of the selected subset size r , we show that a performance better than the 20th OS detector can be obtained. In order to evaluate the performance, we derive the expressions for the false alarm probability. A given design parameter is the false alarm probability in the homogeneous background. Given a value of P_f , the threshold t can be solved using the expressions for the joint probability of a false alarm and the selected subset size r , $P(f, r)$, and the following:

$$P_f = \sum_{r=b}^N P(f, r) \quad (11)$$

where $P(f, r) = \Pr(X_0 > tX_{(\beta)}, X_{(r)} < dX_{(b)} < X_{(r+1)} \mid \text{no target})$. The detection probability P_d is computed using the same expression for P_f but with t replaced

TABLE V
Selection Probability

r	Selection Probability (N=24)				
	Homog.	#interf.=5	#interf.=5	Clutt.in middle	Clutt. in middle
	Back.	INR =10dB	INR = 20dB	CNR= 10dB	CNR= 20dB
4	3.e-5	3.e-5	3.e-5	4.e-5	5.e-5
5	1.e-4	1.e-4	1.e-4	2.e-4	2.e-4
6	3.e-4	4.e-4	4.e-4	6.e-4	8.e-4
7	7.e-4	8.e-4	9.e-4	0.0014	0.0021
8	0.0013	0.0016	0.0017	0.0031	0.0051
9	0.0021	0.0028	0.003	0.0062	0.012
10	0.0034	0.0046	0.0051	0.012	0.0297
11	0.0051	0.0074	0.0082	0.0227	0.0858
12	0.0074	0.0113	0.0129	0.043	0.4401
13	0.0105	0.017	0.0198	0.075	0.2872
14	0.0146	0.0251	0.0302	0.1127	0.1037
15	0.0198	0.0369	0.0462	0.143	0.0265
16	0.0267	0.0542	0.0716	0.1543	0.0053
17	0.0355	0.08	0.1147	0.1432	8.e-4
18	0.0468	0.1196	0.1965	0.1153	1.e-4
19	0.0615	0.181	0.3955	0.08	1.e-5
20	0.0808	0.2043	0.084	0.0484	1.e-6
21	0.1062	0.1532	0.0082	0.0245	8.e-8
22	0.1405	0.0743	4.e-4	0.0099	4.e-9
23	0.1877	0.0218	1.e-5	0.0028	1.e-10
24	0.2485	0.003	2.e-7	4.e-4	2.e-12

Note: $b = 4, d = 15$.

TABLE VI
Possible Estimator Assignment

r	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
β	21	21	22	24	24	21	21	20	19	19	19	19	19	18	18	18	18

Note: $d = 5, b = 8, N = 24$.

by $t/(1 + \text{SNR})$. Hence,

$$P_d = \sum_{r=b}^N P(d, r) \quad (12)$$

where

$$P(d, r) = \Pr(X_0 > tX_{(\beta)}, X_{(r)} < dX_{(b)} < X_{(r+1)} \mid \text{target}) \\ = \Pr\left(X_0 > \frac{tX_{(\beta)}}{(1 + \text{SNR})}, \right. \\ \left. x_{(r)} < dX_{(b)} < X_{(r+1)} \mid \text{no target}\right).$$

The derivation of the expressions for $P(f, r)$ is straightforward, but tedious. In the Appendix, a sketch

of the method employed in obtaining these expressions is provided. Proceeding along the indicated lines, the derivations can be completed or the readers can refer to the technical report [24]. In order to find the false alarm swing with the clutter of different backgrounds, the same probability expressions derived for the interferers are used with INR replaced by CNR.

A. Estimator Assignment

One possible choice of β , as a function of r , for $b = 8$ and $d = 5$ is shown in Table VI. The entries in the table are made keeping in mind that the scheme needs to exhibit performance better than an OS-20 detector. We show that the SE detector, as designed

using Table VI, can tolerate an additional interfering target as compared with an OS-20 and that the false alarm performance of the SE detector is better than that of the OS-20 detector.

Consider the assignment of β for $21 \leq r \leq 24$. In homogeneous background all the 24 samples are independent identically distributed. From Table II, the probability of selecting $r \geq 21$, for $d = 5$ and $b = 8$ is about 0.51. A suitable choice would be the sample mean of the selected samples. However we have restricted our choice to one of the OSs. Results in [10] show that the median ($X_{(N/2)}$) has a detection loss as compared with the cell averaging, and that the OS $X_{(3N/4)}$ gives nearly as good a detection probability as the cell-averaging detector. The choice of median as an estimator would show a considerable false alarm increase when the clutter transition in the middle of the cells exists. This is because, for a CNR of 10 dB, the following is true. 1) The selection probability of $r = 21$, for the clutter transition in the middle situation is not very low (from Table II, it is 0.0051). 2) For the OS detector $X_{(N/2)}$, the probability of false alarm under clutter transition in the middle is very large (0.167 at CNR of 10 dB, [10]). 3) From the statements 1), 2), and (11), the P_f is seen large as a result of a large value of the joint probability $P(f, 21)$, when $\beta = N/2$. Also, the false alarm increase for the OS detector $X_{(3N/4)}$ is very much less as compared with the OS detector $X_{(N/2)}$. Therefore a value of β near 18 is preferred over 12. Values of β close to 24 would not be suitable, because 1) up to 4 targets need be tolerated and 2) from Table II, the selection probability of $r \geq 21$, for INR = 10 dB, is not very small. Hence, for $21 \leq r \leq 24$, choice of β near 24 would lead to the intended target masking when the interferers are present. Hence, we have

$$\beta = 18; \quad 21 \leq r \leq 24.$$

Let us consider $16 \leq r \leq 20$. As the interfering targets or clutter transition of higher power level appears in the reference cells, these samples occupy higher order places and, therefore, the selected subset size is more likely to be smaller. Assume a maximum number of five interferers. Table II shows that the selected subset size is more likely to be in the range of 16 to 20. Also, the probability of selecting $r = 11$ or $r = 12$ is extremely low. Hence any particular assignment of β values for $r = 11$ or 12 will have an insignificant influence on the detection probability when five or less interfering targets are present. We show below that the choice of $\beta = 24$ is appropriate for $r = 11$ and 12, in order to keep the false alarm increase to a low value, during the presence of a clutter transition in the middle of the reference cells. Since we do not want an interfering target sample to be our estimate, we assign

$$\beta = 19; \quad 16 \leq r \leq 20.$$

As said, the choice of $\beta = 24$ for $r = 12$ or 11 cannot mask the detection of the intended target when less than or equal to five interferers are present, because, under this situation 1) the probability of the event ($r = 11$ or $r = 12$) is extremely low, and 2) (12) shows that although $P(d, 12)$ and $P(d, 11)$ will be very small, P_d will be high due to the contribution of $P(d, r)$ for r values over (16, 20).

The performance in the homogeneous background is essentially determined by the choice of β for $16 \leq r \leq 24$. (This is because $P_s(r)$ for $r \geq 16$ is ≥ 0.988 ; from Fig. 1, the detection probability of the SE detector with the estimator assignment Table VI is nearly that of the OS-20 detector). Selection of $r = 14$ or 15 is more likely due to the clutter transition in the middle, rather than due to a homogeneous background or the interferers situation. In order to keep the false alarm increase to a low value, we assign

$$\beta = \begin{cases} 20, & r = 15 \\ 21, & r = 14 \end{cases}$$

A value such as 23 or 24 for $r = 15$ and 14 is not used because, though the probability of selecting $r = 15$ or 14 when a maximum number of interferers are present is low, it is not sufficiently small. Hence, such a high value for β would decrease the detection probability when the interferers are present.

The choice of β for $10 \leq r \leq 13$ is again dictated by the false alarm increase requirement. We choose

$$\beta = \begin{cases} 21, & r = 13 \\ 24, & r = 11, 12 \\ 22, & r = 10 \end{cases}$$

For $r = 11$ and 12, the maximum value of 24 is assigned to β , in order to keep the false alarm increase to a low value. As explained, this does not lead to target masking when the interferers are present. We observe that the β value is increased from 20 to 24 as r decreases from 15 to 12.

When the number of clutter cells is greater than $N/2$, the test cell sample has the higher clutter power. Hence, the selected subset (according to (8)) no longer contains the required samples. In fact, the samples not in the selected subset will be of interest. Therefore, a choice of $\beta > N/2$ is to be used when $r < N/2$. Consider $8 \leq r \leq 9$. As the number of clutter cells exceeds $N/2$, the probability of selecting $8 \leq r \leq 12$ increases. Even though a higher value of $\beta = 24$ for these r values would decrease the false alarm increase above the designed value (when the test cell is from the high clutter), the probability of selection of ($8 \leq r \leq 10$) is not small, when the number of clutter cells is less than $N/2$ (number of clutter cells $< N/2$ implies that the test cell is from low clutter). Hence, when the number of clutter cells is less than but close to $N/2$, choice of $\beta = 24$ leads to an overestimation of threshold and leads to a false alarm decrease very

much below the desired value. Hence we assign

$$\beta = 21; \quad 8 \leq r \leq 9.$$

For a similar reason, $\beta = 22$ is used for $r = 10$, as shown above.

It should be mentioned that the OS-19 detector also tolerates up to 5 targets. However, its false alarm during clutter transition in the middle increases to about 8.6×10^{-4} and 1.4×10^{-3} for CNR of 10 dB and 20 dB, respectively [10]. In contrast, the SE test (Table VI) exhibits a considerably better performance (see Figs. 3–5). Since $r \geq b = 8$, the choice of β needs to be considered only for $r \geq 8$.

In the discussions above, for the purpose of estimator assignment, the range of r is divided into several segments such as (24, 21), (20, 16) and so on. The demarcation was heuristically determined in order to achieve the stated objective. Slight variations in the segmentation of the range of r and in the assignment of β values are possible but would not lead to a significant change in performance.

Table VI shows that for $r \leq 18$, the estimator $X_{(\beta)}$ falls outside the selected subset. A natural question is the following: Why find a subset and then choose an estimate which is outside the subset? The discussions above contain the answer. Let us reiterate the pertinent points that are dispersed in various paragraphs. 1) The selection scheme is not good enough. Of course there does exist an inherent discrimination limitation in identifying the samples from two groups, when the power levels of the two groups do not differ a lot, such as when $\text{CNR} \leq 10$ dB or $\text{INR} \leq 10$ dB. 2) When $r = 12$ or 11 is selected, this event is most likely when the clutter transition is in the middle and $\text{CNR} > 20$ dB. Hence the choice of $\beta = 24$ limits the false alarm to a reasonable value. Also, the above event is less likely when there are only a few interfering targets present and hence the assignment of $\beta = 24$ does not mask the detection of the intended target. 3) When r is a little below $N/2$, this event is more likely to happen when the number of clutter cells is greater than $N/2$. In this case, the samples of interest are outside the subset selected.

B. Performance of Selection and Estimation

The performance of the selection and estimation test, Table VI, is shown in Figs. 1–6. Fig. 2 shows that the SE test can tolerate even a 5th target to a good degree and provide a detection probability of 0.85 for a large test signal strength. The upper false alarm swing for the SE test is better than the OS detector when the CNR is 20 dB or higher (compare Figs. 3, 4, and 5). This is to be expected since a reasonable discrimination between clutters, based on a finite number of samples, is possible when the power levels are considerably different. Moreover, Figs. 4 and 5

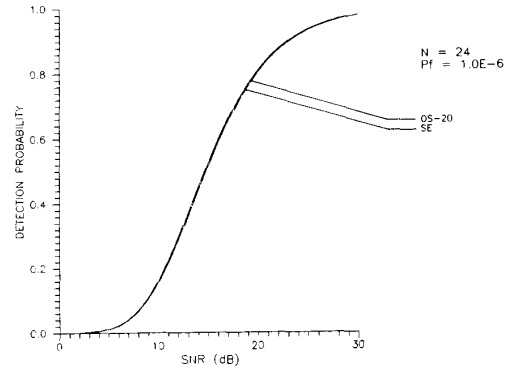


Fig. 1. Probability of detection of SE and OS-20 in homogeneous background.

show that the false alarm swing curves of the SE test take a dip when the number of clutter cells is between 12 and 24. For CNR of 30 dB (Fig. 5), the false alarm, with the number of clutter cells equal to 13, even decreases slightly below the designed value of 10^{-6} . Such a behavior is not seen with the fixed OS detectors. We provide a reason for the above behavior. The SE test can be described as a varying OS detector or as a randomized OS detector, with the randomization dependent on the observations. This is in contrast to what can be termed as a fixed randomized OS detector (FROS). Consider the following FROS. At random, with probability 1/2, either an OS 20 or an OS 19 detector is chosen and the test conducted with the chosen detector, for any given scenario. Clearly, any performance measure of such a FROS is just a mixture (mixture coefficient 0.5) of the performance measures of OS 20 and OS 19. OS 20 has a better false alarm increase performance whereas OS 19 can tolerate an additional interfering target as compared with OS 20. By taking a mixture, a FROS compromises the performance criterias under the two situations, and achieves the average performance measures. For this reason, the false alarm curve of FROS, corresponding to Figs. 4 and 5, will be monotonic decreasing with the number of clutter cells. In contrast, the SE test chooses one of the OSs as the estimate, based on the observations themselves. Whereas the constant t is fixed, once the SE test is designed, the choice of β in $X_{(\beta)}$ changes with the observations. This somewhat explains the nonmonotonic behavior of the SE curves in Figs. 4 and 5. In homogeneous background, Fig. 1 shows that the performances of both the SE test and OS 20 are nearly the same.

The estimator assignment, Table VI, is tuned to the need that a scheme better than the 20th OS detector is desired. Therefore, the estimator assignment has judiciously placed the OSs 18 through 24 to achieve the purpose. This shows that some knowledge of expected maximum number of interferers is required and that

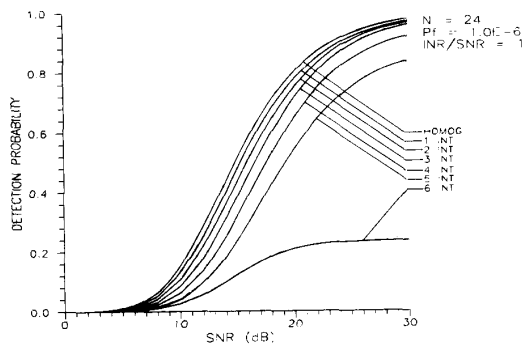


Fig. 2. Probability of detection of SE in multiple targets environment.

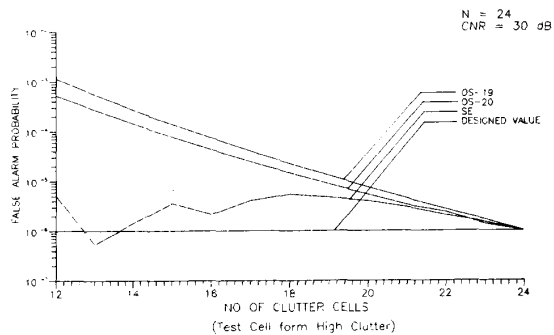


Fig. 5. False alarm rate performance of SE, OS-20, and OS-19 processor in clutter transition (upper swing).

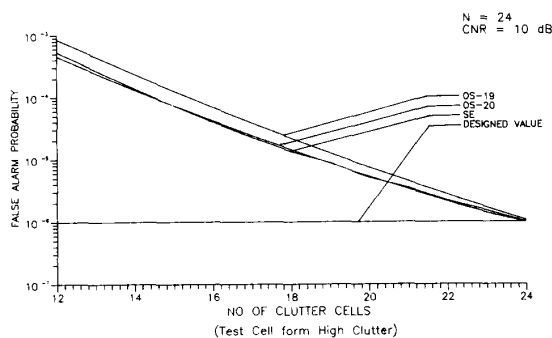


Fig. 3. False alarm rate performance of SE, OS-20, and OS-19 processor in clutter transition (upper swing).

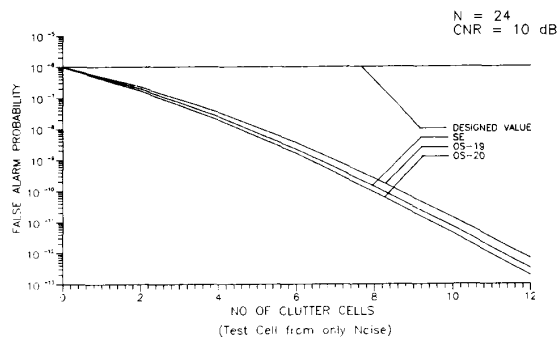


Fig. 6. False alarm rate performance of SE, OS-19, and OS-20 processor in clutter transition (lower swing).

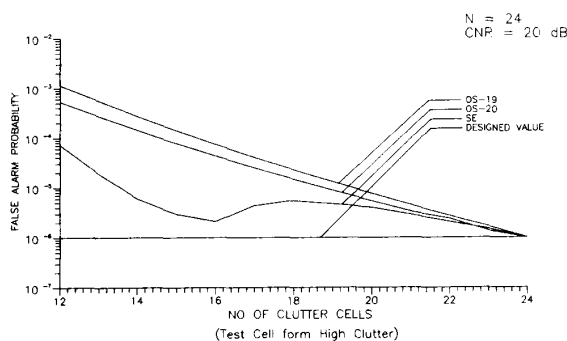


Fig. 4. False alarm rate performance of SE, OS-20, and OS-19 processor in clutter transition (upper swing).

the SE test can provide an improved performance over the fixed OS test to a certain degree only. We also designed the SE tests with $b = 4$ and $d = 10$, $b = 4$ and $d = 15$ and compared their performances with the OS test and the SE test with $b = 8$ and $d = 5$. It is observed that the SE ($b = 8$) performs slightly better than the SE ($b = 4$). Also, the choice of $b = 1$ gives a performance well below that of $b = 8$, $d = 5$. Therefore it is clear that for a specific situation, the parameters of the SE test can be fine

tuned for the best performance. In order to evaluate the performance of the SE test for target models other than the Rayleigh target, one has to resort to simulation studies. At the present time we have not done any such simulation studies.

In Table VII we compare the performances of the VTM test and the SE test for the situation considered in [16]. The designed false alarm probability is 10^{-3} , $b = 8$, $d = 5$, and $N = 24$. The estimator assignment chosen for this problem is also shown in Table VII. This choice is motivated by the requirement that up to 4 interfering targets need to be tolerated. Hence only 20 is assigned to β when the selected subset size is 16 or larger. The general philosophy that guides the choice of β values for $b \leq r \leq N$ is the same as outlined earlier. For large CNR of the order of 20 dB, the SE test exhibits much smaller false alarm increase in clutter transition. Performances of the two schemes ($k = 20$, $\gamma = 0.75$ for VTM) with regard to other parameters are comparable. A higher value of $\gamma = 3$ or 4 for VTM reduces slightly the probability of false alarm increase, when the clutter transition exists in the middle of reference cells, but also decreases the probability of detection when 4 interfering targets are present.

One reason for the slight difference in the performance is that the SE test is able to test for the

TABLE VII
Performances of SE and VTM and Estimator Assignment II

CNR dB	False Alarm Probability			
	SE	VTM (k=20)		
	(ES.ASS. II)	($\gamma = 0.75$)	($\gamma = 3$)	($\gamma = 4$)
10	1.51e-2	1.55e-2	1.15e-2	1.13e-2
20	2.13e-3	1.57e-2	1.20e-2	9.84e-3
30	4.55e-4	1.58e-2	1.27e-2	9.92e-3

No. of Interferers	Detection Probability (SNR=15dB)			
	SE	VTM (k=20)		
	(ES.ASS. II)	($\gamma = 0.75$)	($\gamma = 3$)	($\gamma = 4$)
3	0.661	0.66	0.635	0.608
4	0.585	0.59	0.524	0.497

r	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
β	21	21	22	24	24	22	22	21	20	20	20	20	20	20	20	20	20

Note: $d = 5, b = 8, N = 24$.

clutter transition in the middle, and then apply the largest OS as the estimate. In the VTM, an averaged value of a few of the higher OSs is used as the estimate and hence the false alarm during the clutter transition is larger than the false alarm obtained with the SE test. The principle of selection used in both the schemes is the same. Both the tests use a selection scheme whose performance is far from ideal. This is a contributory reason for the limited success of the SE test. It is not clear whether a selection scheme better than the one employed is possible.

IV. CONCLUSION

A selection scheme widely used in selection and ranking applications is applied for the estimation of noise power level in multiple target constant false alarm rate detection. With a proper choice of the design parameters, the SE test performs better than the OS detector. Further improvement is possible if a better selection procedure can be found.

ACKNOWLEDGMENT

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APPENDIX

The following is the derivation of the probability expressions, $P(f, r)$. Let us define the following:

$$F_1(u) = \begin{cases} 1 - e^{-u}, & u \geq 0 \\ 0, & u < 0 \end{cases} \quad (A1)$$

$$f_1(u) = \begin{cases} e^{-u}, & u \geq 0 \\ 0, & u < 0 \end{cases} \quad (A2)$$

$$F_2(u) = \begin{cases} 1 - e^{-u/p}, & u \geq 0 \\ 0, & u < 0 \end{cases} \quad (A3)$$

$$f_2(u) = \begin{cases} 1/p e^{-u/p}, & u \geq 0 \\ 0, & u < 0 \end{cases} \quad (A4)$$

$f(x)$ is the density of the sample from the test cell where $p = 1 + \text{INR}$.

Joint probability of a false alarm and the selected subset size being r is given by

$$P(f, r) = \begin{cases} \int_0^\infty P(x > tX_{(b)}, X_{(r)} < dX_{(b)} < X_{(r+1)} | X = x) f(x) dx & \text{for } r = b, \dots, N-1 \\ \int_0^\infty P(x > tX_{(b)}, X_{(r)} < dX_{(b)} | X = x) f(x) dx & \text{for } r = N \end{cases} \quad (A5)$$

Let $u, y, z,$ and v denote $X_{(b)}, X_{(r)}, X_{(r+1)},$ and $X_{(\beta)}$, respectively. Define the following: ℓ is the total number of interfering target returns, λ is the total number of interferer samples assuming one or many or all of the OSs $u, y, z,$ and v , as illustrated in the appropriate figures shown below, corresponding to the interferer case, and $k, k_k,$ and j are the number of interferer samples falling in each region of the OSs, as illustrated in the appropriate figures corresponding to the interferer case.

A. Homogeneous Background

Depending on the value of r , six different cases are identified for evaluating the $P(f, r)$ expressions. In all the cases, the $P(f, r)$ expression is of the form

$$P(f, r) = \int_0^\infty G(x) e^{-x} dx. \quad (\text{A6})$$

Different cases have different G functions. The G functions can easily be derived by looking at diagrams A-1 to A-6. These illustrate the number of noise samples falling in different regions, as alienated by $r, r+1, b,$ and β . The expressions written below the upward pointing arrows denote the number of samples falling in the corresponding regions. Below we show the six cases, the corresponding figures, and the G expressions. In the expressions for G below, we have used $[du]$ to denote the differential du . This is to distinguish it from the product of d and $u, du,$ that appears in these integrals as limits. The final, simplified expressions for $P(f, r)$ can be obtained from the technical report [24].

Case I. $r = b$: In this case, a reasonable assumption is $\beta \geq b + 2$.

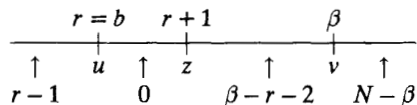


Diagram A-1

$$G = \int_0^{x/t} \int_0^{v/d} \int_{du}^v f_{zuv}(\cdot) dz [du] dv \quad (\text{A7})$$

and

$$f_{zuv}(\cdot) = \frac{N!}{(r-1)!(\beta-r-2)!(N-\beta)!} [F_1(u)]^{r-1} \times [F_1(v) - F_1(z)]^{\beta-r-2} [1 - F_1(v)]^{N-\beta} \times f_1(z) f_1(u) f_1(v). \quad (\text{A8})$$

Case II. $b < r < \beta - 1$:

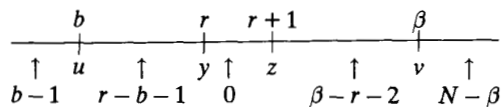


Diagram A-2

$$G = \int_0^{x/t} \int_0^{v/d} \int_{du}^v \int_u^{du} f_{yzuv}(\cdot) dy dz [du] dv \quad (\text{A9})$$

and

$$f_{yzuv}(\cdot) = \frac{N!}{(b-1)!(r-b-1)!(\beta-r-2)!(N-\beta)!} \times [F_1(u)]^{b-1} [F_1(y) - F_1(u)]^{r-b-1} \times [F_1(v) - F_1(z)]^{\beta-r-2} \times [1 - F_1(v)]^{N-\beta} f_1(y) f_1(z) f_1(u) f_1(v). \quad (\text{A10})$$

Case III. $r = \beta - 1$:

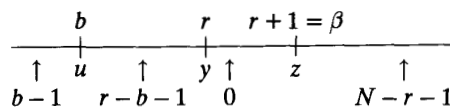


Diagram A-3

$$G = \int_0^{x/t} \int_0^{z/d} \int_u^{du} f_{yuz}(\cdot) dy [du] dz \quad (\text{A11})$$

and

$$f_{yuz}(\cdot) = \frac{N!}{(b-1)!(r-b-1)!(N-r-1)!} [F_1(u)]^{b-1} \times [F_1(y) - F_1(u)]^{r-b-1} [1 - F_1(z)]^{N-r-1} \times f_1(y) f_1(u) f_1(z). \quad (\text{A12})$$

Case IV. $r = \beta$:

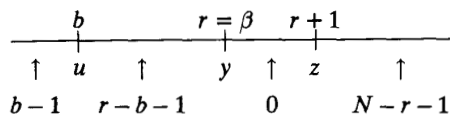


Diagram A-4

$$G = \int_0^{x/dt} \int_{du}^\infty \int_u^{du} f_{yzu}(\cdot) dy dz [du] + \int_{x/dt}^{x/t} \int_{du}^\infty \int_u^{x/t} f_{yzu}(\cdot) dy dz [du] \quad (\text{A13})$$

and

$$f_{yzu} = \frac{N!}{(b-1)!(r-b-1)!(N-r-1)!} [F_1(u)]^{b-1} \times [F_1(y) - F_1(u)]^{r-b-1} [1 - F_1(z)]^{N-r-1} \times f_1(y) f_1(z) f_1(u). \quad (\text{A14})$$

Case V. $\beta < r < N$:

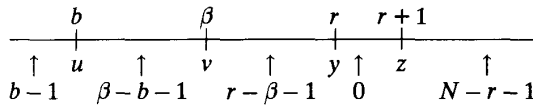


Diagram A-5

$$G = \int_0^{x/t} \int_{v/d}^v \int_{du}^{\infty} \int_v^{du} f_{yzuv}(\cdot) dy dz [du] dv \quad (\text{A15})$$

and

$$f_{yzuv}(\cdot) = \frac{N!}{(b-1)!(\beta-b-1)!(r-\beta-1)!(N-r-1)!} \times [F_1(u)]^{b-1} [F_1(v) - F_1(u)]^{\beta-b-1} \times [F_1(y) - F_1(v)]^{r-\beta-1} \times [1 - F_1(z)]^{N-r-1} f_1(u) f_1(v) f_1(y) f_1(z). \quad (\text{A16})$$

Case VI. $r = N$:

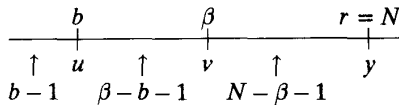


Diagram A-6

$$G = \int_0^{x/t} \int_{v/d}^v \int_{du}^{du} f_{yuv}(\cdot) dy [du] dv \quad (\text{A17})$$

and

$$f_{yuz}(\cdot) = \frac{N!}{(b-1)!(\beta-b-1)!(N-\beta-1)!} [F_1(u)]^{b-1} \times [F_1(v) - F_1(u)]^{\beta-b-1} [F_1(y) - F_1(v)]^{N-\beta-1} \times f_1(y) f_1(u) f_1(v). \quad (\text{A18})$$

B. Interfering Targets

As before, we have to examine the six different cases. For the sake of brevity we consider only the $r = b$ case here. The results of the rest of the cases can be obtained by proceeding along the same lines. A complete treatment, along with the final expressions, is available in [24]. Diagram A-7 illustrates the $r = b$ case. The expressions written below the upward pointing arrows denote the number of samples, either the noise or the interferer, falling in the corresponding regions. The expressions written above the downward pointing arrows denote the number of interferer-only samples falling in the corresponding regions. Using the definition of λ given earlier, we observe that $0 \leq \lambda \leq 3$ (maximum limit is only 3, as there are only three OSs of interest in this case). Each of the possibilities $\lambda = 0$,

$\lambda = 1$, $\lambda = 2$, $\lambda = 3$ are mutually exclusive and $P(f, r)$ is computed as

$$\sum_{\lambda=0}^3 P(f, r, \lambda).$$

Also for $\lambda = 1$, there are three possibilities corresponding to an interferer sample assuming either u or z or v . Similarly, $\lambda = 2$ corresponds to three possibilities.

Case I. $r = b$:

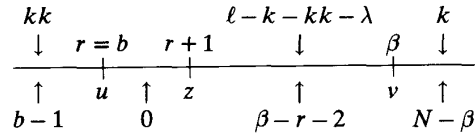


Diagram A-7

Let

$$I(k, k, k, \lambda, \theta, \alpha, \eta) = \int_0^{\infty} G(x) e^{-x} dx \quad (\text{A19})$$

where

$$G(x) = \int_0^{x/t} \int_0^{v/d} \int_{du}^v f_{zuv}(\cdot) dz [du] dv \quad (\text{A20})$$

$$f_{zuv}(\cdot) = C_{\lambda+1} [F_2(u)]^{kk} [F_2(v) - F_2(z)]^{\ell-k-kk-\lambda} \times [1 - F_2(v)]^k [F_1(u)]^{b-1-kk} \times [F_1(v) - F_1(z)]^{\beta-r-2-\ell+k+kk+\lambda} \times [1 - F_1(v)]^{N-\beta-k} \times f_1^\theta(z) f_1^\alpha(u) f_1^\eta(v) f_2^\delta(z) f_2^\phi(u) f_2^\psi(v) \quad (\text{A21})$$

where $\delta = 1 - \theta$, $\phi = 1 - \alpha$, $\psi = 1 - \eta$, and θ, α, η are 1 or 0 variables, and

$$C_{\lambda+1} = \frac{\ell!}{k!kk!(\ell-k-kk-\lambda)!} \times \frac{(N-\ell)!}{(\beta-r-2-\ell+k+kk+\lambda)!(b-1-kk)!(N-\beta-k)!} \quad (\text{A22})$$

By looking at Diagram A-7, and from the definition of λ given earlier, it can be seen that $P(f, r)$ is given by

$$P(f, r) = S_1 \{ C_1 [I(k, k, k, 0, 1, 1, 1)] \} + S_2 \{ C_2 [I(k, k, k, 1, 0, 1, 1) + I(k, k, k, 1, 1, 0, 1) + I(k, k, k, 1, 1, 1, 0)] \} + S_3 \{ C_3 [I(k, k, k, 2, 0, 0, 1) + I(k, k, k, 2, 0, 1, 0) + I(k, k, k, 2, 1, 0, 0)] \} + S_4 \{ C_4 [I(k, k, k, 3, 0, 0, 0)] \} \quad (\text{A23})$$

where the summations operator $S_{\lambda+1}$ is defined as follows

$$S_{\lambda+1} = \sum_{kk=0}^{\min(\ell-\lambda, b-1)} \sum_{k=\max(0, \ell-kk-\lambda-(\beta-r-2))}^{\min(\ell-kk-\lambda, N-\beta)} \quad (\text{A24})$$

REFERENCES

- [1] Hansen, V. G. (1973)
Constant false alarm rate processing in search radars.
In *Proceedings of the IEEE 1973 International Radar Conference*, London, 1973, 325-332.
- [2] Steenson, B. O. (1968)
Detection performance of a mean-level threshold.
IEEE Transactions on Aerospace and Electronic Systems, AES-4 (July 1968), 529-534.
- [3] Hansen, V. G., and Sawyers, J. H. (1980)
Detectability loss due to greatest of selection in a cell averaging CFAR.
IEEE Transactions on Aerospace and Electronic Systems, AES-16 (Jan. 1980), 115-118.
- [4] Trunk, G. V. (1978)
Range resolution of targets using automatic detectors.
IEEE Transactions on Aerospace and Electronic Systems, AES-14 (Sept. 1978), 750-755.
- [5] Dillard, G. M. (1974)
Mean-level detection of nonfluctuating signals.
IEEE Transactions on Aerospace and Electronic Systems, AES-10 (Nov. 1974), 795-799.
- [6] Rickard, J. T., and Dillard, G. M. (1977)
Adaptive detection algorithm for multiple target situations.
IEEE Transactions on Aerospace and Electronic Systems, AES-13 (July 1977), 338-343.
- [7] Rohling, H. (1983)
Radar CFAR thresholding in clutter and multiple target situations.
IEEE Transactions on Aerospace and Electronic Systems, AES-19 (July 1983), 608-621.
- [8] Rohling, H. (1984)
New CFAR processor based on order statistic.
In *Proceedings of the IEEE International Radar Conference*, Paris, 1984, 38-42.
- [9] Finn, H. M. (1986)
A CFAR design for a window spanning two clutter fields.
IEEE Transactions on Aerospace and Electronic Systems, AES-22 (Mar. 1986), 155-169.
- [10] Gandhi, P., and Kassam, S. A. (1988)
Analysis of CFAR processors in nonhomogeneous background.
IEEE Transactions on Aerospace and Electronic Systems, 24 (July 1988), 427-445.
- [11] Ritcey, J. A. (1986)
Censored mean level detector analysis.
IEEE Transactions on Aerospace and Electronic Systems, AES-22 (July 1986), 443-454.
- [12] Goldstein, G. B. (1973)
False alarm regulation in log-normal and Weibull clutter.
IEEE Transactions on Aerospace and Electronic Systems, AES-9 (Jan. 1973), 84-92.
- [13] Weber, P., and Haykin, S. (1985)
Ordered statistic CFAR processing for two-parameter distributions with variable skewness.
IEEE Transactions on Aerospace and Electronic Systems, AES-21 (Nov. 1985), 819-821.
- [14] Barkat, M., and Varshney, P. K. (1987)
A weighted adaptive cell averaging CFAR detector for multiple target situations.
In *Proceedings of the 21st Annual Conference on Information Sciences and Systems*, The Johns Hopkins University, Baltimore, 1987, 118-123.
- [15] Holm, J. R., and Ritcey, J. A. (1989)
On the Neyman-Pearson optimality of the censored mean level detector.
In *Proceedings of the Conference on Information Sciences and Systems*, The Johns Hopkins University, Baltimore, 1989, 346.
- [16] Ozgunes, I., Gandhi, P., and Kassam, S. A. (1989)
A CFAR detection scheme based on modified trimmed mean thresholding.
In *Proceedings of the Conference on Information Sciences and Systems*, The Johns Hopkins University, Baltimore, 1989, 356-361.
- [17] Weiss, M. (1982)
Analysis of some modified cell-averaging CFAR processors in multiple target situations.
IEEE Transactions on Aerospace and Electronic Systems, AES-18 (Jan. 1982), 102-113.
- [18] Hawkins, D. M. (1980)
Identification of Outliers.
London: Chapman and Hall, 1980.
- [19] Gupta, S. S., and Panchapakesan, S. (1979)
Multiple decision procedures.
New York: Wiley, 1979.
- [20] Gibbons, J. D., Olkin, I., and Sobel, M. (1977)
Selecting and Ordering Populations: A New Statistical Methodology.
New York: Wiley, 1977.
- [21] Gupta, S. S., and Huang, D. Y. (1981)
Multiple Statistical Decision Theory: Recent Developments.
New York: Springer-Verlag, 1981.
- [22] Mood, A. M., Graybill, F. A., and Boes, D. C. (1974)
Introduction to the Theory of Statistics.
New York: McGraw-Hill, 1974.
- [23] David, H. A. (1981)
Order Statistics.
New York: Wiley, 1981.
- [24] Viswanathan, R., and Eftekhari, A. (1991)
Application of selection procedure to multiple targets in clutter detection.
Technical report, SIUC/DEE/TR 90-8, Southern Illinois University at Carbondale, Mar. 1991.
- [25] Gandhi, P. P., and Kassam, S. A. (1989)
An adaptive order statistic constant false alarm rate detector.
In *Proceedings of the International Conference on Systems Engineering*, Wright State University, 1989.



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