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Application of Expectation-Maximization Algorithm to the Detection of a Direct-Sequence Signal in Pulsed Noise Jamming

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***Abstract*— We consider the detection of a direct-sequence spread-spectrum signal received in a pulsed noise jamming environment. The expectation-maximization algorithm is used to estimate the unknown jammer parameters and hence obtain a decision on the binary signal based on the estimated likelihood functions. The probability of error performance of the algorithm is simulated for a repeat code and a (7,4) block code. Simulation results show that at low signal-to-thermal noise ratio and high jammer power, the EM detector performs significantly better than the hard limiter and somewhat better than the soft limiter. Also, at low SNR, there is little degradation as compared to the maximum-likelihood detector with true jammer parameters. At high SNR, the soft limiter outperforms the EM detector.**

I. Introduction

SPREAD-spectrum communication systems offer an inherent advantage of reducing interference. The reduction achieved depends on the processing gain. Pulsed, but broadband, noise jamming may cause considerable degradation in performance of a direct-sequence spread-spectrum system [1]. The performance of the system may be further improved by using additional techniques [2]–[5].

We consider here the performance of a maximum-likelihood detector for the following detection problem [1]. Let the r_i's represent the outputs of the direct-sequence correlator, corresponding to different symbols transmitted as DS-BPSK signals, and let $\{\theta_i^j, i = 1, \cdots, m, \theta_i^j \varepsilon (\pm 1)\}$, $j \varepsilon (1, \cdots, 2^k = M)$ be one of the code vectors of a given (m, k) block code.

Choose one of the following M hypotheses, $\{H_j, j = 1, 2, \cdots, M\}$:

$$H_j : \theta_i = \theta_i^j, \qquad i = 1, 2, \cdots, m \tag{1}$$

given by observations

$$H_j : r_i = \theta_i^j s + n_i + J_i Z_i \,. \tag{2}$$

The significance of various variables appearing in (2) are explained below. For a given (m, k) block code, $\{\theta_i^j, i = 1, 2, \cdots, m\}$ are known sequences for every j. In the case of repeat code, the same bit of information is transmitted m times, i.e., $\theta_i = \theta, i = 1, 2, \cdots, m$. The detection problem (1) reduces to

$$H_1 : \theta = -1$$

versus

$$H_2 : \theta = +1 \,. \tag{3}$$

Perfect interleaving is assumed so that the probability that a symbol is jammed is independent of any other symbol being jammed or not. Let ρ be the duty cycle of the pulse jammer $n_J(t)$ with two-sided power spectral density $N_J/(2\rho)$, and let $n(t)$ be the thermal noise with two-sided power spectral density $N_0/2$. Both the noises are assumed to be independent, zero mean Gaussian. With an equivalent baseband representation for direct-sequence correlator, n_i is a zero-mean white Gaussian noise with known variance $\sigma^2 = N_o/2, J_i$ is zero-mean Gaussian jamming noise with variance $N_J/(2\rho)$. Z_i's $\varepsilon(0,1)$ denote whether the ith symbol is jammed or not. They are independent random variables with $P(Z_i = 1) = \rho$. n_i, J_i, and Z_i are all mutually independent. Z_i, ρ, and $N_J/2$ are typically unknown. Assuming these parameters are known, we construct an optimal (but practically unrealizable) detector in Section III. When Z_i's are known, the r_i's are Gaussian; and when Z_i's are unknown, the r_i's are samples from a *mixture density* as shown below. The signal level s is assumed known. There may be situations where s cannot be determined easily, and the discussions in this paper do not apply to those situations [2]. The expectation-maximization (EM) algorithm is used in order to obtain the estimates of ρ and $\sigma_J^2 = \sigma^2 + N_J/(2\rho)$. The likelihood function with estimated jammer parameters is maximized to obtain a decision on the hypotheses for the testing problem in (1). A complete discussion of the EM algorithm can be found in [6]. Recently, the EM algorithm has been applied to other types of detection and estimation problems [7], [8].

In Section II, we discuss how the EM algorithm can be applied to the testing problem outlined above. In Section III, simulation results are presented for the repeat and block coding cases. The performances of the EM detector are compared to those of hard and soft limiters, an optimal detector, maximum likelihood and linear detectors. In Section IV, we discuss the results.

II. Direct-Sequence Detection and EM Algorithm

Consider the detection problem stated in (1) and (2) with the observations being the sum of the data signal, the channel noise, and the jammer noise. When Z_i's are unknown, the sum of the channel noise and the jammer component may be viewed as a variate from a mixture of two normal distributions with zero means, variances σ^2 and σ_J^2, and mixing ratios $1 - \rho$ and ρ, respectively. In other words, the interference is from channel noise alone with probability $1 - \rho$ and from

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channel plus jammer with probability ρ. The observations (2) are distributed as

$$H_j : r_i \sim f(r_i) = (1-\rho) f_1(r_i) + \rho f_2(r_i) \quad (4)$$

where

$$f_1(r_i) = \left(1/\sqrt{2\pi}\sigma\right) \exp\left(-\left(r_i - \theta_i^j s\right)^2 / 2\sigma^2\right) \quad (5)$$

and

$$f_2(r_i) = \left(1/\sqrt{2\pi}\sigma_J\right) \exp\left(-\left(r_i - \theta_i^j s\right)^2 / 2\sigma_J^2\right). \quad (6)$$

Define the parameter vector $\mathbf{\Phi} = (\mathbf{\Theta}^j, \sigma_J^2, \rho)$, where $\mathbf{\Theta}^j = (\theta_1^j, \theta_2^j, \cdots, \theta_m^j)$. The log-likelihood function is given by $L(\mathbf{\Phi}|\mathbf{r}) = \sum_{i=1}^m \ln f(r_i)$. Then the proposed detector for $\mathbf{\Theta}$, which we shall call the EM detector, maximizes $L(\mathbf{\Phi}|\mathbf{r})$ using the estimates of σ_J^2 and ρ obtained via the EM algorithm.

A. Repeat Codes

Using the procedure in [11], the maximum-likelihood estimates of θ, σ_J^2, and ρ can be obtained as the simultaneous solution to the set of following equations:

$$\hat{\theta} = \arg\max_\theta (L(\mathbf{\Phi}|\mathbf{r})), \quad \theta \epsilon (+1, -1) \quad (7)$$

$$\hat{\sigma}_J^2 = \frac{\sum_{i=1}^m \rho (r_i - \theta s)^2 \cdot f_2(r_i)/f(r_i)}{\sum_{i=1}^m \rho \cdot f_2(r_i)/f(r_i)} \quad (8)$$

$$\hat{\rho} = \frac{\sum_{i=1}^m \hat{\rho} \cdot f_1(r_i)/f(r_i)}{m} = \frac{\sum_{i=1}^m \hat{\rho} \cdot f_2(r_i)/f(r_i)}{m}. \quad (9)$$

There may be several solutions to (7), (8), and (9), and the one which maximizes $L(\mathbf{\Phi}|\mathbf{r})$ has to be picked. Equations (7), (8), and (9) are used to provide the following iteration scheme. However, as explained later, the solution obtained via the iterations does not necessarily correspond to the global maximum of $L(\mathbf{\Phi}|\mathbf{r})$.

Let $\mathbf{\Phi}^{(p)}$ denote the estimate of $\mathbf{\Phi}$ at the pth iteration, $p \geq 1$.

$$\theta^{(p+1)} = +1 \text{ or } -1 \text{ whichever maximizes } L(\mathbf{\Phi}^{(p)}|\mathbf{r}) \quad (10)$$

$$\sigma_J^{2(p+1)} = \frac{\sum_{i=1}^m \left[r_i - \theta^{(p)} s\right]^2 \cdot f_2^{(p)}(r_i)/f^{(p)}(r_i)}{\sum_{i=1}^m f_2^{(p)}(r_i)/f^{(p)}(r_i)} \quad (11)$$

$$\rho^{(p+1)} = \frac{\sum_{i=1}^m \rho^{(p)} \cdot f_2^{(p)}(r_i)/f^{(p)}(r_i)}{m} \quad (12)$$

where $f_\ell^{(p)}(r_i)$, $\ell = 1, 2$, and $f^{(p)}(r_i)$ are the density functions evaluated at r_i and $\mathbf{\Phi}^{(p)}$. A starting value, $\mathbf{\Phi}^{(1)}$, is required. The iteration scheme is insensitive to these initial values, and any reasonable set can be assumed [10]. For example, we assume $\rho^{(1)} = 0.5$, $\sigma_J^{2(1)} = 1.0$, and $\theta^{(1)} = 0$, in all the simulations. Although 0 is not an allowed value for θ, it is used as an unbiased starting value for the EM algorithm. The decision on θ given by the EM detector will always be $+1$ or -1 since these are the only allowed values in subsequent iterations. The fact that (10)–(12) form the EM algorithm for mixture density (4) is shown in [6].

B. Block Codes

Let the code vector be $\mathbf{\Theta}^j$. Then $\hat{\mathbf{\Theta}}^j = \left(\hat{\theta}_1, \hat{\theta}_2, \cdots, \hat{\theta}_m\right)$ is the maximum-likelihood estimate of $\mathbf{\Theta}^j$ given by

$$\hat{\mathbf{\Theta}}^j = \arg\max_{\mathbf{\Theta}} L(\mathbf{\Phi}|\mathbf{r}). \quad (13)$$

The maximum of $L(\mathbf{\Phi}|\mathbf{r})$ is to be searched over the M valid codes. The maximum-likelihood estimate of the jammer variance has to satisfy

$$\hat{\sigma}_J^2 = \frac{\sum_{i=1}^m \rho \cdot \left(r_i - \theta_i^j s\right)^2 \cdot f_2(r_i)/f(r_i)}{\sum_{i=1}^m \rho f_2(r_i)/f(r_i)}. \quad (14)$$

The only difference between (8) and (14) is the index i on θ, as they are no longer the same for each i. The equations for the maximum-likelihood estimate of ρ for the block codes remain the same as for the repeat code, although $f_1(r_i)$ and $f_2(r_i)$, as in (5) and (6), will have the appropriate θ_i^j for each i.

The EM algorithm has been shown to result in a nondecreasing likelihood at each successive step and, under some conditions, to converge to a maximum-likelihood estimator [6], [9]. However, in general, the alorgithm will converge to a compact set of stationary point(s).

III. Simulation Performance

In this section, simulated performances of the EM detector, maximum-likelihood detector with known jammer parameters, the linear, hard-limiter, and soft limiter [1], [4], [5] are studied. The clipping level of the soft-limiter is set at s. If s is also unknown, the resulting EM detector would be the linear detector, which would also be the maximum-likelihood detector because the maximum-likelihood estimate of the common mean of the mixture of two normal distributions is the sample mean [12].

A. Repeat Coding Performance

The bit energy for a repeat code is given by $E_b = m \cdot s^2$, where $m = 7$ is the code length assumed. In the case of repeat code, we look at an optimal, but unrealizeable, detector for performance comparison purposes.

Optimal Detector: With Z_i known, the likelihood ratio for the testing problem (3) is given by

$$\Lambda(r) = \prod_i \frac{\left(1/\sqrt{2\pi}\sigma\right) \exp\left(-(x_i - s)^2/2\sigma^2\right)}{\left(1/\sqrt{2\pi}\sigma\right) \exp\left(-(x_i + s)^2/2\sigma^2\right)}$$

$$\cdot \prod_i \frac{\left(1/\sqrt{2\pi}\sigma_J\right)\exp\left(-(y_i - s)^2/2\sigma_J^2\right)}{\left(1/\sqrt{2\pi}\sigma_J\right)\exp\left(-(y_i + s)^2/2\sigma_J^2\right)} \quad (15)$$

where $r_i = y_i$ if the symbol is jammed, and $r_i = x_i$ if it is not. Equivalently, a test based on the likelihood ratio is given by

$$T(r) = \frac{\Sigma x_i}{\sigma^2} + \frac{\Sigma y_i}{\sigma_J^2} \underset{\hat{\theta}=-1}{\overset{\hat{\theta}=+1}{\gtrless}} 0. \quad (16)$$

In order to implement this detector, value of σ_J^2, and whether each sample is jammed or not, are needed. In this sense, it is an ideal detector and the required information is usually not available. Let k be the number of jammed samples. The error probability of the optimal detector is given by

$$P_{\text{opt}}(e) = \sum_{k=0}^{m} \binom{m}{k} \rho^k (1-\rho)^{m-k} P_{\text{opt}}(e|k) \quad (17)$$

where $P_{\text{opt}}(e \mid k) = Q(s \cdot \sqrt{(m-k)/\sigma^2 + k/(\sigma^2 + N_J/2\rho)})$, and $Q(\cdot)$ is one minus the standard normal cdf.

The EM detector described in Section II-A is simulated for at least 10^5 and up to 10^6 trials for each probability of error estimation. Each trial creates a realization of $r = (r_1, r_2, \cdots, r_m)$ as in (2). The stopping criterion used for the EM algorithm iterations is the following rule of convergence of the likelihood functions:

Stop iterations and obtain the current decision on $\underline{\Theta}$ if

$$\text{abs}((L(\Phi^{(p)} \mid r) - L(\Phi^{(p-1)} \mid r))/L(\Phi^{(p-1)} \mid r)) \leq 0.01 \quad (18)$$

or if the number of iterations exceeded 30.

A benchmark for the performance of the algorithm is the simulated performance of the maximum-likelihood detector with known σ_J^2 and ρ, but unknown jammer state, that is, the maximum-likelihood detector based on the mixture density (4).

B. (m, k) Block Coding Performance

The energy per information bit for an (m, k) block code is given by $E_b = m \cdot s^2/k$. A (7,4) block code is assumed, and hence a single error correcting capability is available. The hard limiter detector makes a decision on each bit of the coded word, and a word decision error is made if the hard limiter makes an error in more than one bit. The soft limiter detector computes

$$\underset{j\varepsilon(1,\cdots,M)}{\arg\max} \left\{ \beta_j = \sum_{i=1}^{m} c(r_i)\, \theta_i^j \right\},$$

$c(\cdot)$ being the output of the soft limiter. The EM detector for the block coding case as described in Section II-B is simulated for 100 000 trials for each E_b/N_J.

The error probabilities of these detectors are shown in Figs. 1 and 2 against ρ for various s, σ^2, and E_b/N_J values, and in Figs. 3–6 against E_b/N_J for various s, σ^2, and ρ values.

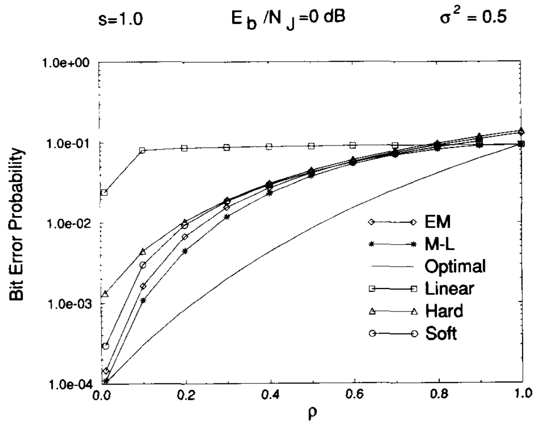

Fig. 1. Bit error probability for $m = 7$ repeat code.

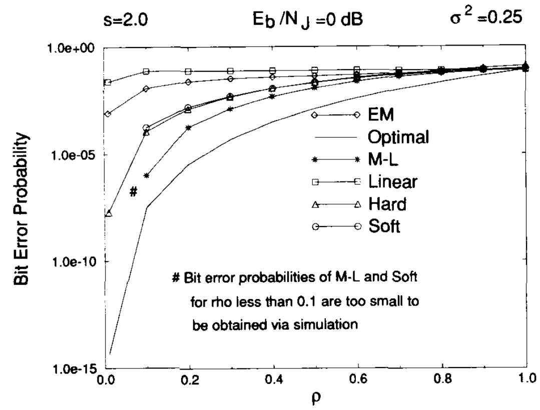

Fig. 2. Bit error probability for $m = 7$ repeat code.

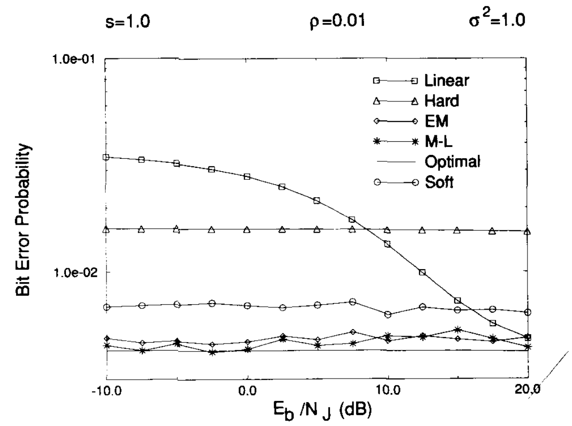

Fig. 3. Bit error probability for $m = 7$ repeat code.

IV. Discussion and Conclusions

Comparing the proposed EM detector to other schemes in terms of the probability of error performance as a function of ρ for different s, σ^2, and E_b/N_J values, it is observed that at low signal-to-thermal noise ratio $\left(\text{SNR} \triangleq s^2/\sigma^2\right)$, there is little

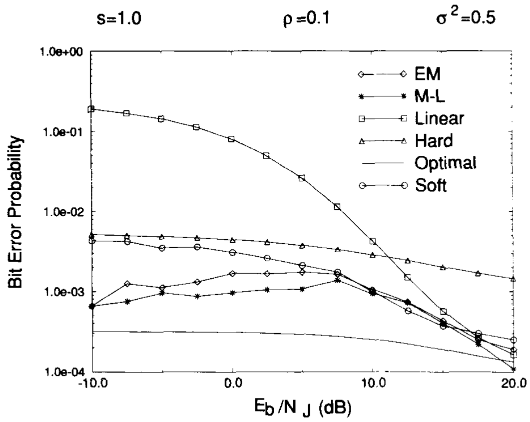

Fig. 4. Bit error probability for $m = 7$ repeat code.

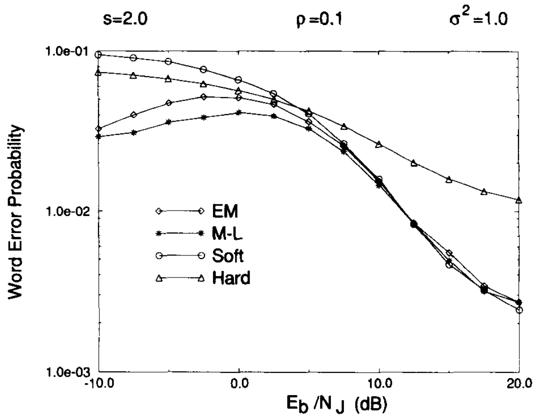

Fig. 5. Word error probability for (7,4) block code.

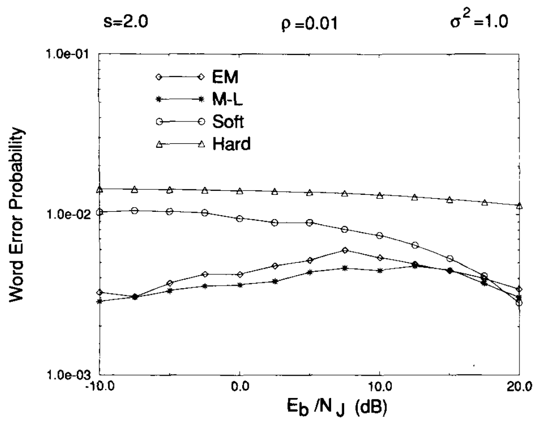

Fig. 6. Word error probability for (7,4) block code.

degradation in performance as compared to the maximum-likelihood detector with known jammer parameters (Fig. 1). At high SNR, the EM detector performance is considerably poorer than the maximum-likelihood detector (Fig. 2), especially at low ρ values. The gap between the performances of the optimal (unrealizable) and the EM detector is considerable for large SNR values (Figs. 1 and 2). The same relative performances of the EM, the maximum-likelihood, and the optimal detectors are also observed when the probability of error is plotted as a function of E_b/N_J for different s, σ^2, and ρ values (Figs. 3 and 4). For (7,4) block code also, the EM and maximum likelihood detectors exhibit close probability of error performances at low SNR (Figs. 5 and 6).

When the EM detector performance is close to that of the maximum-likelihood detector, the estimate of the likelihood function does not necessarily correspond to the true likelihood function. It was observed that, after the EM algorithm had converged according to (18), the estimated jammer parameters did not converge to the true jammer parameters at all even when the probability of error curves for the EM and the maximum-likelihood detectors were close. With such a small sample size as 7, parameter convergence is not expected. The convergence of the EM algorithm is observed to be quite rapid. Very few times (ranging from single digits to a maximum of 50 out of 100 000 for all simulations) did the algorithm fail to converge according to (18) and had to exit after 30 iterations.

Comparing the performance of the EM detector to the other detectors, it is seen that it performs consistently better than the hard limiter detector at low SNR (Figs. 1, 3, and 4). Compared to the soft limiter, the EM detector performs better at low SNR and high jammer power levels. For high SNR conditions, the soft limiter outperforms the EM detector (Fig. 2). In general, the (7,4) block code performs better than the length 7 repeat code at equivalent signal and noise conditions.

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