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An Inexact Inferencing Strategy for Spatial Objects with Determined and Indeterminate Boundaries

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Abstract- **For many years, spatial querying has been of interest for the researchers in the GIS community. Any successful implementation and long-term viability of the GIS technology depends on the issue of accuracy of spatial queries. In order to improve the accuracy and quality of spatial querying, the problems associated with the areas of fuzziness and uncertainty need to be addressed. There has been a strong demand to provide approaches that deal with inaccuracy and uncertainty in GIS. In this paper, we develop an approach that can perform fuzzy spatial querying under uncertainty. An inexact inferencing strategy for objects with determined and indeterminate boundaries is investigated, using type-2 fuzzy set theory.**

I. INTRODUCTION

In most spatial queries, verbal language plays an important rule. Words like *some*, *little*, *all*, *directly*, *none*, *slightly*, *mostly*, *somewhat*, and *not*, which happen frequently, give fuzziness to spatial queries. Hence, the ability to perform queries under fuzziness is one of the most important characteristics of any spatial database.

Directional and topological relationships have been shown to be fuzzy concepts [1, 2]. Our earlier works support queries of this nature and provide a basis for fuzzy querying capabilities based on a binary model [3-6]. Our Clips-based implementation for querying binary spatial relationships can distinguish various cases in the same relational classes. As an example, consider the relational description: *"Object A overlaps Object B."* Fuzzy query processing can answer whether *all of Object A overlaps* ***some*** *of Object B,* or *little of Object A overlaps* ***most*** *of Object B?*

However, in these kinds of imprecise queries, the representation of the imprecise variables is based on classical set theory. Although classical sets are suitable for various applications and have proven to be an important tool for mathematics and computer science, they do not reflect the nature of human concepts and thoughts, which tend to be abstract and imprecise. The flaw comes from the sharp transition between inclusion and exclusion in a set. In an earlier paper, we showed a way to use fuzzy sets for dealing with the vague meaning of linguistic terms, in which the smooth transition is characterized by a membership function [10]. However, this approach is applicable only for spatial objects with precisely defined boundaries.

In this paper, we consider spatial objects with indeterminate boundaries, which are found in many applications in geographic analysis and image understanding. For such objects, we introduce type-2 fuzzy membership functions for topological and directional relationships.

The queries expressed by verbal language often involve a mixture of uncertainties in the outcomes that are governed by the meaning of linguistic terms. This uncertainty increases even more for objects with indeterminate boundaries. Therefore, there is an availability-related need for a skilled inexact inferencing approach to handle the uncertain features [7]. Uncertainty occurs when one is not absolutely certain about a piece of information. Although uncertainty is an inevitable problem in spatial queries, there are clear gaps in our understanding of how to incorporate uncertain reasoning into the spatial querying process. This requires performing inexact inferencing. Recently, models of uncertainty have been proposed for spatial information that incorporate ideas from natural language processing, the value of information concept, non-monotonic bgic and fuzzy set, evidential and probability theory. Each model is appropriate for a different type of inexactness in spatial data. By incorporating the type-2 fuzzy set and confirmation theory, we investigate an inexact inferencing approach for fuzzy spatial querying for objects with indeterminate boundaries. The aim is to improve spatial querying accuracy and quality.

The paper is organized as follows. Section 2 briefly overviews our previous works, and shows some basic techniques and strategies to deal with fuzzy multiple relations in spatial querying. Section 3 describes our approaches that can perform fuzzy querying under the uncertainties for objects with both defined and indeterminate boundaries. Finally, a summary is given in section 4.

II. SUMMARY OF THE PREVIOUS WORKS

In this work, minimum bounding rectangles (MBRs) are used to approximate the spatial objects. Figure 1 shows two objects with defined boundaries in two dimensions. Based on the spatial binary model [3-6], some spatial querying techniques and strategies can be briefly overviewed as follows.

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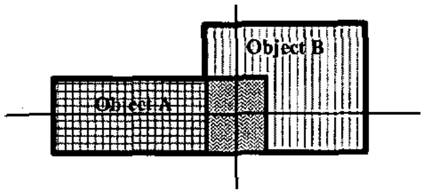

Fig. 1. Two objects with defined boundaries (2-D)

A. Basic Spatial Querying

Topological and directional relationships are critical components in the retrieval of information from spatial databases, including image, map and pictorial databases. Many contributions have been made. The authors in [9] define new families of fuzzy directional relations in terms of the computation of force histograms, which is based on the raster data. In this paper, we will take into account these two major spatial relations based on the vector data.

The topological relationships express the concepts of inclusion and neighborhood. A large body of related work has focused on the intersection mode that describes relations using intersections of objects' interiors and boundaries. By means of geometrical similarity, we defined the topological relationships as a set:

T={disjoint, tangent, surround, overlaps ......}

The paper [3] provides greater details on this.

The directional relationships are commonly utilized in everyday life. The most common directions are the cardinal directions and their refinements. We therefore defined the directional relations as the following set:

D={North, East, South, West, Northeast, Southeast, Southwest, Northwest}

Such relationships provide a significant resource for the basic binary spatial queries. The examples of such queries might look like these:

Does Object A **overlap** Object B?
Is Object A **west** of Object B?

B. Fuzzy Spatial Querying

Although the above querying method can provide topological and directional information, these kinds of information are not associated with any degrees. This means it can only perform a low-level query. A typical example is shown in Figure 2.

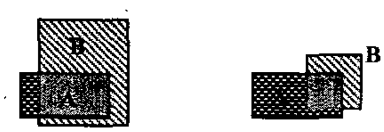

Fig. 2. An example for fuzzy spatial querying

For both cases that belong to the same class (or relational group), the basic spatial querying will provide the same topological and directional relationships, i.e. Object A overlaps Object B and Object A is west of Object B.

The provision of more accurate information, such as most of Object A overlaps some of Object B, or little of Object A overlaps some of Object B and so on, encourages us to make further investigation. Some strategies and techniques can be briefly described as follows (see the details in [6]).

- Partition each object into sub-groups in eight directions based on the reference area (the common part of two objects) shown in Figure 3;
- Map each sub-group to a node, and assign two weights (area and node weights) to each node;
- Calculate two weights to determine the spatial degree

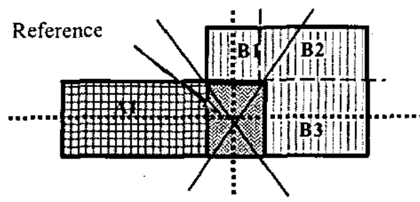

Fig. 3. Partitioning two objects in 2D

where area weight can be calculated by

AW= (area of sub-group) / (area of the entire object)

and node weight can be obtained by

NW=AW · ((axis length) / (longest axis length))

In order to support fuzzy querying, the resulting quantitative figures (*AW, NW*) are mapped to a range that corresponds to a term known as linguistic qualifiers. There is a large body of knowledge and techniques that deal with fuzzy spatial relations in linguistic expression. In this paper, we define the topological qualifier *TQ* and directional qualifier *DQ* as:

TQ={all, most, some, little, none}
DQ={directly, mostly, somewhat, slightly, not}

Relative qualifiers can be defined as subsets of the unit interval and represented as a linguistic term. Based on the classical set, the membership function of qualifiers can be defined as a binary set, that is, complete membership has a value of 1, and no membership has a value of 0. The following tables give the quantifying description.

TABLE I.
TOPOLOGICAL QUALIFIERS

Topological Qualifiers (TQ)	Area Weight (AW)
all	0.96 to 1.00
most	0.60 to 0.95
some	0.30 to 0.59
little	0.06 to 0.29
none	0.00 to 0.05

TABLE 2.
DIRECTIONAL QUALIFIERS

Directional Qualifiers (DQ)	Node Weight (NW)
directly	0.96 to 1.00
mostly	0.60 to 0.95
somewhat	0.30 to 0.59
slightly	0.06 to 0.29
not	0.00 to 0.05

As shown in Figure 1, the Clips-based implementation can provide the following information [6]:

Most of Object A overlaps Object B
Object A overlaps some of Object B

Most of Object A overlaps some of Object B

Most of Object A is west of Object B
Object A is mostly west of Object B

Most of Object A is mostly west of Object B

III. FUZZY QUERYING UNDER UNCERTAINTY

As we mentioned earlier, the spatial relationships depend on human interpretation and therefore fuzzy concepts should be used to improve the accuracy of spatial queries. To support queries of this nature, previous works provided fuzzy queries without uncertainty and limited to objects with defined boundaries. These approaches can handle the fuzziness by defining fuzzy qualifiers. However, in these kinds of fuzzy queries, the particular grades of membership are defined as classical sets. The problem is that there exists a gap between two neighboring members such as '*all*' and '*most*'. Because a jump occurs, no qualifier is defined in some intervals, for instance the interval (0.95, 0.96). This problem is even more visible for objects with indeterminate boundaries [11].

To improve the fuzzy query for objects with defined boundaries, fuzzy set theory and uncertainty concepts were employed in our previous work [10]. Here we apply type-2 fuzzy set theory to expand our previous work to objects with indeterminate boundaries.

In this section, we start with an overview of our approach for objects with defined boundaries and then continue with introduction of our type-2 fuzzy set based method for objects with indeterminate boundaries.

A. *Fuzziness Consideration for Objects with Defined Boundaries*

Fuzziness occurs when the boundary of a piece of information is not clear-cut. Hence, fuzzy querying expands query capabilities by allowing for ambiguity and partial membership. The definition of the grades of membership is subjective and depends on the human interpretation. A way to eliminate subjectivity is another interesting research field. Here, simple membership functions will be considered.

A fuzzy set is a set without a crisp boundary. The smooth transition, for objects with defined boundaries, is characterized by type-1 (classical) membership functions that give fuzzy sets flexibility in linguistic expressions. More formally, a fuzzy set in a universe is characterized by a classical membership function μ: $U \rightarrow [0,1]$. Figure 4 illustrates the primary term of fuzzy variable area weight. Each term represents a specific fuzzy set.

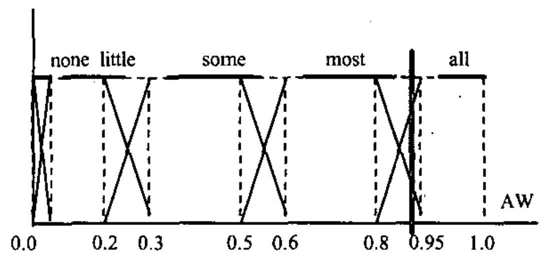

Fig. 4. Membership function for TQ

The fuzzy set functions for topological qualifiers can be described as:

$$\mu_{all}(AW) = \begin{cases} 1.0 & \text{if } 0.95 \le AW \le 1.0 \\ 20\,(AW - 0.80)/3 & \text{if } 0.8 \le AW \le 0.95 \end{cases}$$

$$\mu_{most}(AW) = \begin{cases} 20\,(0.95 - AW)/3 & \text{if } 0.8 \le AW \le 0.95 \\ 1.0 & \text{if } 0.6 \le AW \le 0.80 \\ 10\,(AW - 0.5) & \text{if } 0.5 \le AW \le 0.6 \end{cases}$$

$$\mu_{some}(AW) = \begin{cases} 10\,(0.6 - AW) & \text{if } 0.5 \le AW \le 0.6 \\ 1.0 & \text{if } 0.3 \le AW \le 0.5 \\ 10\,(AW - 0.2) & \text{if } 0.2 \le AW \le 0.3 \end{cases}$$

$$\mu_{little}(AW) = \begin{cases} 10\,(0.3 - AW) & \text{if } 0.2 \le AW \le 0.3 \\ 1.0 & \text{if } 0.02 \le AW \le 0.2 \\ 100(AW - 0.01) & \text{if } 0.01 \le AW \le 0.02 \end{cases}$$

$$\mu_{none}(AW) = \begin{cases} 100\,(0.02 - AW) & \text{if } 0.01 \le AW \le 0.02 \\ 1.0 & \text{if } 0.0 \le AW \le 0.01 \end{cases}$$

In the same way, the fuzzy set functions for directional querying can be described as:

$$\mu_{directly}(NW) = \begin{cases} 1.0 & \text{if } 0.95 \le NW \le 1.0 \\ 20\,(NW - 0.80)/3 & \text{if } 0.8 \le NW \le 0.95 \end{cases}$$

$$\mu_{mostly}(NW) = \begin{cases} 20\,(0.95 - NW)/3 & \text{if } 0.8 \le NW \le 0.95 \\ 1.0 & \text{if } 0.6 \le NW \le 0.80 \\ 10\,(NW - 0.5) & \text{if } 0.5 \le NW \le 0.6 \end{cases}$$

$$\mu_{somewhat}(NW) = \begin{cases} 10\,(0.6 - NW) & \text{if } 0.5 \le NW \le 0.6 \\ 1.0 & \text{if } 0.3 \le NW \le 0.5 \\ 10\,(NW - 0.2) & \text{if } 0.2 \le NW \le 0.3 \end{cases}$$

$$\mu_{slightly}(NW)=\begin{cases}10\,(0.3-NW) & \text{if } 0.2 \le NW \le 0.3\\ 1.0 & \text{if } 0.02 \le NW \le 0.2\\ 100(NW-0.01) & \text{if } 0.01 \le NW \le 0.02\end{cases}$$

$$\mu_{not}(NW)=\begin{cases}100\,(0.02-NW) & \text{if } 0.01 \le NW \le 0.02\\ 1.0 & \text{if } 0.0 \le NW \le 0.01\end{cases}$$

Although fuzzy set theory solves the gap problem in classical set expression, a new problem arises. Because a common feature of the fuzzy sets is overlapping definitions, the qualifiers may be associated with two different terms at the intersecting intervals. For instance, the topological qualifier TQ may take *'all'* and *'most'* at the same time. This reveals uncertainty – the lack of adequate and correct information to make a decision. We deal with this uncertainty in subsection C.

B. Fuzziness Consideration For Objects With Indeterminate Boundaries

In many areas of geographic data handling, particularly in the management of natural resource data, spatial objects tend to have indeterminate boundaries [11]. These objects are also called fuzzy objects [11]. Many examples of fuzzy objects can be found in climatology and soil data. Figure 5 illustrates a simple fuzzy object that includes a core area surrounded by the inside edge and an indeterminate boundary area surrounded by the inside edge and the outside edge. In general and based on [11], a simple fuzzy region A can be decomposed into three major parts: (1) the core area, denoted by A'; (2) the indeterminate boundary, denoted as A^d and (3) the exterior, denoted as A^-. In this paper, we use type-2 membership functions for topological and directional qualifiers of fuzzy objects.

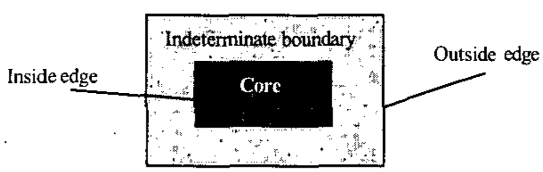

Fig. 5. The representation of a simple fuzzy region

We are now required to define two new concepts: Core Area Weight (CAW)(Figure 6), and Core Node Weight (CNW):

- *CAW=(area of intersection for core regions) / (area of the entire core region)*
- *CNW=CAW · (axis length) / (longest axis length)*

CAW and *CNW* are used with *AW* and *NW* to form the type-2 membership functions for fuzzy objects. To understand how this works, imagine blurring the type-1 membership function

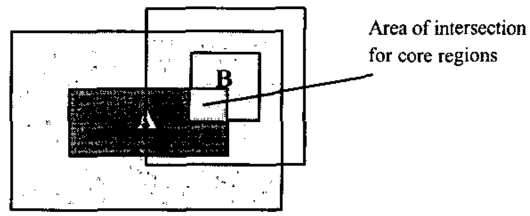

Fig. 6. Area of intersection for core regions

for *'some'* (figure 4) by shifting the points on the graphs either to the left or to the right. Then, at a specific value of *AW*, say 0.45, there is no longer a single value for the *'some'* membership function; instead, the membership function takes on values wherever the vertical line intersects the blur. Those values are not all weighted the same. Hence, we can assign an amplitude distribution to all of those points, which in this case would be *CAWs*. This creates a three dimensional membership function – a type-2 membership function – that characterizes our type two fuzzy sets for topological qualifiers.

More precisely, if the geospatial object is with indeterminate boundaries, to form membership functions for topological qualifiers, we take into account both *AW* and *CAW*. Figure 7 illustrates the membership functions for the topological qualifiers for fuzzy objects. These graphs show the impact of fuzziness on the spatial object queries. They are plotted based on the data we have collected from the experts for a variety of different cases. Table 3 shows the data gathered for topological qualifier *'some'* for objects with indeterminate boundaries.

TABLE 3
TOPOLOGICAL QUALIFIER *'SOME'* FOR FUZZY OBJECTS

AW	CAW	DoM*	AW	CAW	DoM*
0	0	0	0.5	0.5	1
0.1	0	0	0.6	0	0.9
0.1	0.1	0	0.6	0.1	1
0.2	0	0	0.6	0.2	1
0.2	0.1	0	0.6	0.3	1
0.2	0.2	0.2	0.6	0.4	1
0.3	0	0.5	0.6	0.5	1
0.3	0.1	0.6	0.6	0.6	0.9
0.3	0.2	0.7	0.6	0.7	0.8
0.3	0.3	1	0.6	0.8	0.7
0.4	0	0.7	0.6	0.9	0.6
0.4	0.1	0.8	0.6	1	0.5
0.4	0.2	0.9	0.7	0	0
0.4	0.3	1	0.7	0.1	0
0.4	0.4	1	0.7	0.2	0
0.5	0	0.8	0.7	0.3	0
0.5	0.1	0.9	0.7	0.4	0
0.5	0.2	1	0.7	0.5	0
0.5	0.3	1	0.7	0.6	0
0.5	0.4	1	0.7	0.7	0

* Degree of Membership

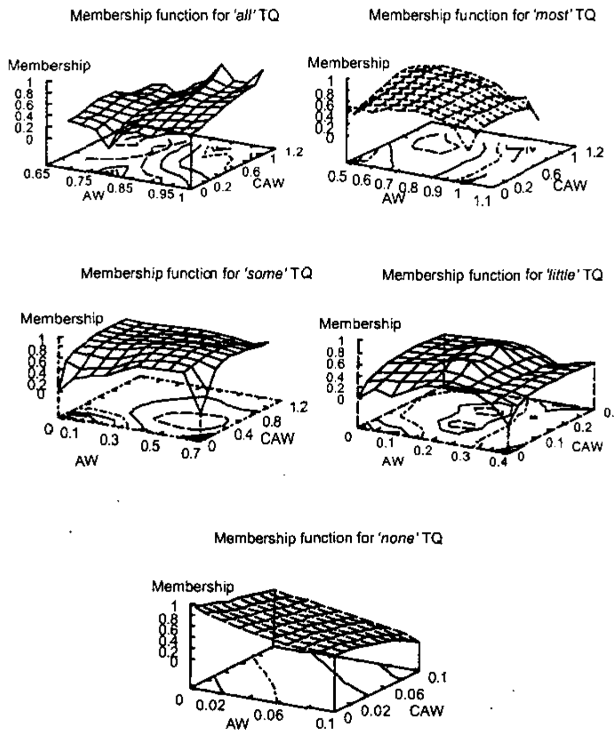

Fig. 7. Membership functions for the topological qualifiers for fuzzy objects.

In the same fashion, we can form the type-2 fuzzy membership functions for the directional qualifiers by using *NW* and *CNW*.

C. *Uncertainty Consideration*

Uncertainty is an inevitable problem in GIS. In our previous work, we explored an approach that could perform the fuzzy querying under uncertainties for objects with determined boundaries [10]. In this work, we use the same approach for fuzzy objects as well.

Uncertainty occurs when one is not certain about a piece of information. This happens more frequently for objects with indeterminate boundaries than objects with defined boundaries. The reason is that for the first case, although we have both *AW* and *CAW* (*NW* and *CNW*) to form the membership functions, we do not have a unique and obvious boundary. That is why we have to come up with the internal and external borders for the fuzzy object.

As an example for uncertainty, given AW=0.90 and CAW=0.3, the fuzzy querying may give the following querying phrase:

<u>All</u> of Object A overlaps Object B.
<u>Most</u> of Object A overlaps Object B.

How do we make the decision according to the information? Which querying information is reliable?

This reveals important deficiencies in areas such as the reliability of query results and the ability to detect inconsistencies in the knowledge. Because we cannot be completely certain if some qualifiers are true or others are false, we construct a certainty factor (CF) to evaluate the degree of certainty. The degree of certainty is usually represented by a crisp numerical value scaled from zero to one. A certainty factor of one indicates that it is very certain that a fact is true, and a certainty factor of zero indicates that it is very uncertain that a fact is true. Some key ideas relevant to the determination of the CF are discussed as following.

<u>Case 1</u>. Consider a single qualifier for each query.

In this case, only one qualifier associated with a single object is involved in each querying result. Here is an example:

<u>All</u> of Object A overlaps Object B.
Object A is <u>directly</u> west of Object B.

Where the fuzzy topological qualifier TQ_A = 'all' and directional qualifier DQ_A='directly' is associated with object A.

- If the qualifier only takes one term at a given interval, the grade of membership $\mu(\ )$ can be used as a CF that represents the degree of belief. The results for fuzzy objects will look like:

  <u>All</u> of Object A overlaps Object B
  with $CF=\mu_{all}(AW_{ai}=0.9, CAW_{ai}=0.6)=1.0$
  Object A is <u>directly</u> west of Object B
  with $CF=\mu_{directly}(NW_{ai}=0.9, CNW_{ai}=0.6)=1.0$

Where AW_{ai} and CAW_{ai} are the area and the core area weights of the sub-groups associated with object *A*. NW_{ai} is the node weight and CNW_{ai} is the core node weight of the subgroups associated with object *A*; and i, j∈I[1, 8], 'I' represents an integer set.

- If the given weight is in the overlapping area, two qualifiers will be related. For example, the fuzzy topological qualifier of the object *A* takes both 'all' and 'most.' The querying results will be:

  <u>All</u> of Object A overlaps Object B.
  <u>Most</u> of Object A overlaps Object B.

It is acceptable if we take the qualifier that has a larger grade of membership. The certainty factor can be determined by the maximum value, that is,

$$CF = \max\{\mu_{all}(AW_{ai}=0.9, CAW_{ai}=0.5), \mu_{most}(AW_{ai}=0.9, CAW_{ai}=0.5)\} = \mu_{all}(AW_{ai}=0.9, CAW_{ai}=0.5)$$

The final querying results should be:

All of Object A overlaps Object B
with CF= $\mu_{all}(AW_{ai}=0.9, CAW_{ai}=0.5)=0.9$

The reason is that μ_{all} = 0.9 is bigger than μ_{most}=0.4 in this example. These numbers can be read from figure 7 or the respective tables which are not included in this paper.

As a result, for fuzzy objects, the CF in case 1 can be obtained by:

CF=max{ $\mu_{TQk}(AW_{\bullet i}$ =const, $CAW_{\bullet i}$ =const), $k\in I[1,5]$, $i\in I[1,8]$ }
CF=max{ $\mu_{DQk}(NW_{\bullet j}$ =const, CNW_{ai}=const), $k\in I[1,5]$, $j\in I[1,8]$ }
where
TQ_k is a topological qualifier such as *all*;
DQ_k is a directional qualifier such as *directly*;
$AW_{\bullet i}$ ($CAW_{\bullet i}$) is an area (core area) weight associated object i-node;
$NW_{\bullet i}$ ($CNW_{\bullet i}$) is a node (core node) weight

For objects with defined boundaries, we drop the CAWs and CNWs to obtain the CF [10].

Case 2. Consider multiple qualifiers for each query

In the querying results, many pieces of fuzzy terms are conjoined, or disjoined. The examples of these types of queries are as follows:

Most of Object A overlaps some of Object B
Some of Object A is slightly south of Object B

Hence, to perform these kinds of queries, we have to handle multiple fuzzy qualifiers. It is easy to understand that the relationship between different object qualifiers is conjunction, and the relationship between the same object qualifiers is disjoined. According to the fuzzy set theory, the conjunction and disjunction of fuzzy terms can be respectively defined as the minimum and maximum of the involved facts. Therefore, the certainty factor containing multiple qualifiers can be determined by the following formulas:

Consider topological relationships:
CF=min{ max{ $\mu_{TQka}(AW_{ai}=\alpha, CAW_{ai}=\beta)$ }, max{ $\mu_{TQkb}(AW_{bj}=\alpha, CAW_{bj}=\beta)$ } },
where ka, kb $\in I[1,5]$ & i,j $\in I[1,8]$
Note: a topological qualifier TQ_1=all if ka=1.

Consider topological/directional relationships:
CF=min{ max{ $\mu_{TQka}(AW_{ai}=\alpha, CAW_{ai}=\kappa)$ }, max{ $\mu_{DQka}(NW_{aj}=\beta, CNW_{ai}=\lambda)$ } },
where ka $\in I[1,5]$ & i,j $\in I[1,8]$
where a_i, b_j represent object node associated with object *A* and *B*, respectively,
α, β, κ, λ are constant.

Again, for objects with defined boundaries, we drop the *CAWs* and *CNWs* to obtain the CF in this case [10].

In this way, an approach in which fuzzy sets and uncertainty can be integrated to perform the fuzzy queries for objects with defined and indeterminate boundaries is developed.

CONCLUSION

Verbal language is a part of most spatial queries and subsequently, fuzziness and uncertainty are frequently involved in such queries. To improve the accuracy of spatial queries, we introduced an inexact inferencing approach that can perform fuzzy querying under uncertainty. In our method, we used type-2 fuzzy membership functions for objects with indeterminate borders. We also judged the reliability of querying information by a certainty factor (CF). This is a flexible method that can return spatial information in a wider variety of forms for both fuzzy and non-fuzzy spatial objects.

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