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Performance Study of Maximum Likelihood Receivers and Transversal Filters for the Detection of Direct Sequence Spread Spectrum Signal in Narrowband Interference

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Abstract **–Linear least squares estimation techniques can be used to enhance suppression of narrowband interference in direct sequence spread spectrum systems. Nonlinear techniques for this purpose have also been investigated recently. In this paper, we derive maximum likelihood receivers for direct sequence signal in Gaussian interference with known second order characteristics. It is shown that if the receiver uses samples from outside the bit interval, then the receiver structure is nonlinear. The bit error rate performances of these receivers are compared to those of linear receivers employing one-sided and two-sided least squares estimation filters, for the case of Gaussian autoregressive interference. The results in this paper show that intersymbol interference due to filter taps extending beyond the bit interval cannot be ignored for small processing gains. In some cases, not accounting for intersymbol interference yields too optimistic error estimates, very much away from the true error rates.**

I. INTRODUCTION

Direct sequence spread spectrum systems offer an inherent capability of rejecting narrowband interference. This is achieved by modulating the bit waveform with a pseudonoise (PN) signal before transmission and correlating the received signal with a replica of the PN signal. In this way, interfering signals, whose bandwidths are narrow compared to the spread signal, are attenuated by the receiver. Processing the received signal prior to correlating with the PN sequence has been employed to improve the suppression of narrowband interference. Linear least squares estimation techniques to estimate and subtract the narrowband interference have been studied in [1]-[5]. Nonlinear techniques for interference suppression in spread spectrum and other communication systems have been investigated in [6],[7]. All of these techniques are based on the idea that the spread signal and the white noise, having a flat spectrum, cannot be predicted from their past values, while the narrowband interfering signal can be predicted. Therefore, an attempt to predict the received signal will, in effect, produce an estimate of the interfering signal. This estimate is then subtracted from the received signal, yielding an error signal as the input to the correlator. An overview of signal processing techniques for interference rejection in spread spectrum communications is provided in [8]. Transform domain processing structures for this purpose are proposed in [9],[10]. A detailed discussion on least squares estimation and transform domain techniques for interference rejection can be found in [11]. As shown in [4] and [14], there is a close correspondence between linear prediction filters for suppressing interference and prewhitening filters in the solutions of maximum likelihood receivers for the detection of signals in colored Gaussian noise.

In this paper, we study the performance of maximum likelihood receivers for direct sequence spread spectrum signals received in Gaussian interference with known second order statistics. When the receiver operates on the observations in the bit duration only, the receiver is the linear detector (matched filter). When the observation interval extends outside the bit interval, the receiver structure is shown to be nonlinear. The nonlinearity arises not due to the modeling of the PN sequence as random, as in [6], but due to the uncertainty on the bits adjacent to the bit being tested. In the case of transversal filters, when the current chip sample from which the interferer estimate is to be subtracted, lies in the beginning of the bit interval, the filter taps will extend to the previous bit. Also, in the case of two sided transversal filter, when the current chip sample lies at the end of the bit interval, the filter taps will extend into the next bit. This extension of the filter taps into neighboring bits introduces a signal distortion that can be termed intersymbol interference (ISI). The analytical results in [4] are intended for application in systems with large processing gain as compared to the filter length, a situation where this ISI is negligible. The numerical results presented there for small processing gains do not account for ISI, but the results obtained here show that ISI cannot be ignored for small processing gains.

In section II, the spread spectrum signal model is introduced and maximum likelihood receivers are developed for this model. In section III, for Gaussian AR interference, the performance of the receivers obtained in section II are

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compared to those of one-sided and two-sided transversal filters. Results and conclusions from this study are presented in section IV.

II. Maximum Likelihood Receivers

We consider here the performance of maximum likelihood receivers for the following problem (see Fig. 1.(a)). We shall restrict to the case where an entire maximal length PN code sequence is embedded in each bit (so called short PN sequences [15]). A similar analysis can be easily done for the case of long PN sequences. Let the received signal be processed by a chip-matched filter and sampled at the chip rate of the PN sequence to yield [6]:

$$z_k = s_k + n_k + j_k \tag{1}$$

The symbols in (1) are explained below.

$s_k = S\, b_k\, c_k$. S is the signal strength. Without loss of generality, $S = 1.0$ is assumed.

$c_k \varepsilon \{+1,-1\}$ is the k^{th} chip of the PN sequence with chip interval τ_c.

c_k for $k<0$ and $k>L-1$ is taken k modulo L.

$b_k \varepsilon \{+1,-1\}$ is the binary information with bit duration $T_b = L\tau_c$, L is the processing gain given as the number of PN chips per message bit. Note that $b_k = b \varepsilon \{+1,-1\}$ for all k in the same bit interval.

n_k is a sequence of zero mean i.i.d. Gaussian noise with known variance σ_n^2.

j_k is a sequence of narrowband interference modeled as a zero mean Gaussian process with autocovariance $R_j(k)$.

The detection problem is:

$$\text{all } b_k \text{ over the current bit (i.e. } b) = \begin{cases} -1: & H_0 \\ +1: & H_1 \end{cases} \tag{2}$$

Equivalently,

$$H_0 : z_k = -c_k + n_k + j_k \quad \text{vs.} \quad H_1 : z_k = c_k + n_k + j_k, \tag{3}$$

$k = 0, 1, \ldots, L-1$

Let $v_k = n_k + j_k$ be the white noise plus the interference with autocovariance

$$R_v(m) = \sigma_n^2\, \delta(m) + R_j(m).$$

Let Λ be the $L\times L$ covariance matrix of $\{v_k\}$. The maximum likelihood detector for the detection problem in (3) is given by [12]:

$$\mathbf{z}^T \Lambda^{-1} \mathbf{c} \underset{-1}{\overset{+1}{\gtrless}} 0 \tag{4}$$

where

$$\mathbf{z}^T = [z_0, z_1, \ldots, z_{L-1}], \quad \mathbf{c}^T = [c_0, c_1, \ldots, c_{L-1}],$$

and T denotes the transpose of the vector. Let us call this the ML I receiver. The bit error probability of the ML I receiver is given by

$$Q(\sqrt{\mathbf{c}^T \Lambda^{-1} \mathbf{c}}),$$

where $Q(\cdot)$ is one minus the standard normal cdf.

A. ML II Receiver and its Bit Error Rate

Now consider the observation vector to consist of the chips corresponding to the bit under test appended with some chips from the previous bit, i.e., the receiver has to test the present bit but uses observation samples from the present bit interval and a part of the previous bit interval.

Let

$$\mathbf{r}^T = [\bar{\mathbf{z}}^T : \mathbf{z}^T]$$

where

$$\bar{\mathbf{z}}^T = [\bar{z}_{L-i}, \bar{z}_{L-i+1}, \ldots, \bar{z}_{L-1}]$$

is the vector of the last i chip samples from the previous bit, $i \le L$. The likelihood ratio, $\lambda(\mathbf{r})$, and the corresponding maximum likelihood detector for the detection problem in (3) are then given by:

$$\lambda(\mathbf{r}) = \frac{\sum_{d\varepsilon\{+1,-1\}} \exp\{\mathbf{s}_{d,+1}^T \Lambda^{-1}(\mathbf{r} - \frac{1}{2}\mathbf{s}_{d,+1})\}}{\sum_{d\varepsilon\{+1,-1\}} \exp\{\mathbf{s}_{d,-1}^T \Lambda^{-1}(\mathbf{r} - \frac{1}{2}\mathbf{s}_{d,-1})\}} \underset{-1}{\overset{+1}{\gtrless}} 1 \tag{5}$$

where Λ is the $(L+i) \times (L+i)$ covariance matrix of the sequence $\{v_k\}$, the sequence $\mathbf{s}_{d,b}$ is defined as $\mathbf{s}_{d,b}^T = [dc_{L-i}, dc_{L-i+1}, \ldots, dc_{L-1}, bc_0, bc_1, \ldots, bc_{L-1}]$, and d indicates the previous bit, $d \varepsilon \{\pm 1\}$. Using straightforward calculations involving partitioned vectors and matrices, it is shown in the Appendix that the bit error probability for the detector in (5) is given by:

$$P_e = \frac{1}{2}\left(\mathrm{P}(error | H_0, d = +1) + \mathrm{P}(error | H_0, d = -1)\right) \tag{6}$$

where

$$\mathrm{P}(error | H_0, d) = \mathrm{P}\left(\sinh(\theta_1) > \gamma \sinh(\theta_2) | H_0, d\right)$$

$$\theta_1 = \mathbf{s}_{+1,+1}^T \Lambda^{-1} \mathbf{r}$$

$$\theta_2 = \mathbf{s}_{-1,+1}^T \Lambda^{-1} \mathbf{r}$$

γ is a negative constant obtained from the entries of Λ^{-1} matrix and $s_{+1,+1}$ vector.

The numerical evaluation of P_e is addressed in the appendix. The test statistic given by (5) is nonlinear in observations. An equivalent, simplified test is given by (A3). The receiver based on (5) (or (A3)) will be called ML II.

B. Generalization of ML II Receiver

Apart from appending chips from only the previous bit, we may also let the receiver use samples from the next bit interval. The receiver observation samples are

$$\mathbf{r}^T = \left[\bar{\mathbf{z}}^T : \mathbf{z}^T : \overset{+}{\mathbf{z}}^T\right],$$

where

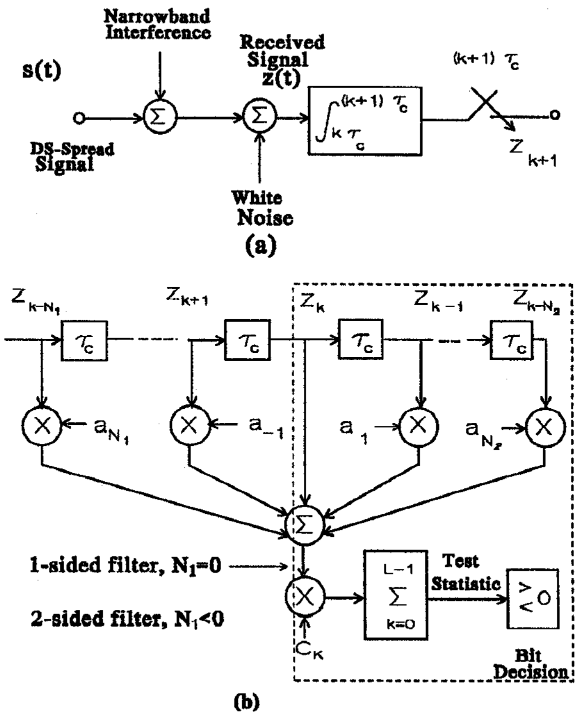

Fig. 1. (a) Model for received DS Signal. (b) LLSE Filter.

$$\overset{+}{\mathbf{z}}^T = \left[\overset{+}{z}_0, \overset{+}{z}_1, \ldots, \overset{+}{z}_{i-1}\right]$$

are the first i chip samples from the next bit, $i \leq L$. The maximum likelihood receiver for this case is given by:

$$\lambda(\mathbf{r}) = \frac{\sum_{e\varepsilon\{+1,-1\}} \sum_{d\varepsilon\{+1,-1\}} \exp\{\mathbf{s}_{d,+1,e}^T \Lambda^{-1}(\mathbf{r} - \frac{1}{2}\mathbf{s}_{d,+1,e})\}}{\sum_{e\varepsilon\{+1,-1\}} \sum_{d\varepsilon\{+1,-1\}} \exp\{\mathbf{s}_{d,-1,e}^T \Lambda^{-1}(\mathbf{r} - \frac{1}{2}\mathbf{s}_{d,-1,e})\}} \underset{-1}{\overset{+1}{\gtrless}} 1 \quad (7)$$

where Λ is the $(L+2i) \times (L+2i)$ covariance matrix of $\{v_k\}$,

$\mathbf{s}_{d,b,e}^T =$

$[dc_{L-i}, dc_{L-i+1}, \ldots, dc_{L-1}, bc_0, bc_1, \ldots, bc_{L-1}, ec_0, ec_1, \ldots, ec_{i-1}]$,

$d, b, e\,\varepsilon\{\pm 1\}$ are the previous bit, the hypothesis on the present bit and the next bit respectively. As before, the test statistic derived from (7) is nonlinear in observations.

C. The Asymptotic Case

In order to study the asymptotics (using infinite past bits) of the ML II receiver, we shall consider the following general form:

$$\lambda(\mathbf{r}) = \frac{E_{\mathbf{d}}\left[\exp\left\{\mathbf{s}_{\mathbf{d},+1}^T \Lambda^{-1}\left(\mathbf{r} - \frac{1}{2}\mathbf{s}_{\mathbf{d},+1}\right)\right\}\right]}{E_{\mathbf{d}}\left[\exp\left\{\mathbf{s}_{\mathbf{d},-1}^T \Lambda^{-1}\left(\mathbf{r} - \frac{1}{2}\mathbf{s}_{\mathbf{d},-1}\right)\right\}\right]} \quad (8)$$

where the observation vector $\mathbf{r}$ consists of observation samples from the bit being tested and from M number of previous bits. The observation interval consists of $(M+1)L$ chips and Λ is a $(M+1)L \times (M+1)L$ covariance matrix of $\{v_k\}$. The $M \times 1$ vector $\mathbf{d}$ denotes the M previous bits outside the test bit interval,

$\mathbf{d}^T = [d^{(-M)}, d^{(-M+1)}, \ldots, d^{(-1)}]$, $\quad d^{(j)}\varepsilon\{\pm 1\}, j = -M, \ldots, -1$,

the s vectors are given by

$\mathbf{s}_{\mathbf{d},b}^T = [d^{(-M)}\mathbf{c}^T \vdots \cdots \vdots d^{(-1)}\mathbf{c}^T \vdots b\mathbf{c}^T]$, $\quad b\varepsilon\{\pm 1\}$ is the test bit,

and $E_{\mathbf{d}}(.)$ denotes expectation with respect to the $\mathbf{d}$ vector. Equation (8) may be simplified as follows. Upon partitioning the matrices and vectors in the following way,

$\mathbf{s}_{\mathbf{d},b}^T = [\mathbf{s}_{\mathbf{d}}^T \vdots \mathbf{s}_b^T]$, where $\mathbf{s}_{\mathbf{d}}^T = [d^{(-M)}\mathbf{c}^T \vdots \cdots \vdots d^{(-1)}\mathbf{c}^T]_{1\times ML}$,

$\mathbf{s}_b = b\mathbf{c}$ is a $L \times 1$ vector,

$$\Lambda^{-1}\mathbf{r} = \begin{bmatrix} G_1 \\ \cdots\cdots \\ G_0 \end{bmatrix}_{(M+1)L\times 1}$$

where G_1 is $ML \times 1$, G_0 is $L \times 1$,

$$\Lambda^{-1} = \begin{bmatrix} Q_1 \\ \cdots\cdots \\ Q_0 \end{bmatrix}_{(M+1)L\times(M+1)L}$$

where Q_1 is $ML \times (M+1)L$, Q_0 is $L \times (M+1)L$,

the likelihood ratio in (8) becomes:

$$\lambda(\mathbf{r}) = \frac{E_{\mathbf{d}}\left[\exp\left\{\mathbf{s}_{\mathbf{d}}^T G_1 + \mathbf{s}_1^T G_0 - \frac{1}{2}\left(\mathbf{s}_{\mathbf{d}}^T Q_1 + \mathbf{s}_1^T Q_0\right)\begin{bmatrix}\mathbf{s}_{\mathbf{d}} \\ \cdots \\ \mathbf{s}_1\end{bmatrix}\right\}\right]}{E_{\mathbf{d}}\left[\exp\left\{\mathbf{s}_{\mathbf{d}}^T G_1 - \mathbf{s}_1^T G_0 - \frac{1}{2}\left(\mathbf{s}_{\mathbf{d}}^T Q_1 - \mathbf{s}_1^T Q_0\right)\begin{bmatrix}\mathbf{s}_{\mathbf{d}} \\ \cdots \\ -\mathbf{s}_1\end{bmatrix}\right\}\right]} \quad (9)$$

where $\mathbf{s}_{-1} = -\mathbf{s}_1$ is used. Further partitioning gives the following:

$\frac{1}{2}\mathbf{s}_{\mathbf{d}}^T Q_1 = [P_1^T \vdots P_2^T]$, P_1 is $ML \times 1$ and P_2 is $L \times 1$.

$\frac{1}{2}\mathbf{s}_1^T Q_0 = [R_1^T \vdots R_2^T]$, R_1 is $ML \times 1$ and R_2 is $L \times 1$.

Let

$\phi_1 = \mathbf{s}_{\mathbf{d}}^T G_1$, $\psi_1 = \mathbf{s}_1^T G_0$, $\phi_2 = P_1^T \mathbf{s}_{\mathbf{d}}$, $\phi_3 = P_2^T \mathbf{s}_1$, $\phi_4 = R_1^T \mathbf{s}_{\mathbf{d}}$,

and $\psi_2 = R_2^T \mathbf{s}_1$.

The scalars ϕ_j's $(j = 1, 2, .., 4)$ depend on the vector $\mathbf{d}$ while ψ_1 and ψ_2 do not. Equation (9) becomes:

$$\lambda(\mathbf{r}) = \frac{E_{\mathbf{d}}[\exp\{\phi_1 + \psi_1 - \phi_2 - \phi_3 - \phi_4 - \psi_2\}]}{E_{\mathbf{d}}[\exp\{\phi_1 - \psi_1 - \phi_2 + \phi_3 + \phi_4 - \psi_2\}]} = \exp\{2\psi_1\}\frac{E_{\mathbf{d}}[\exp\{\phi_1 - \phi_2 - \phi_3 - \phi_4\}]}{E_{\mathbf{d}}[\exp\{\phi_1 - \phi_2 + \phi_3 + \phi_4\}]} \quad (10)$$

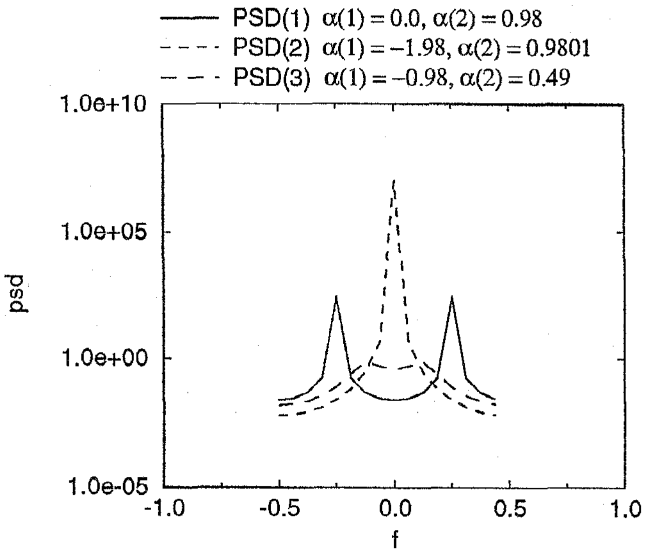

Fig. 2. Power spectral densities of autoregressive Gaussian interference.

If the ratio of expectations on the right hand side of equation (10) were to go to unity as M becomes large, then $\ln\lambda$ would asymptotically be linear in **r**. It does not seem that the ratio of expectations in (10) will be one, even when $M \to \infty$, and we conjecture that for an arbitrary correlated interference, any monotonic function of λ is nonlinear in observations. If j_k is white, then of course (10) leads to a test that is linear in observations.

III. Performance Comparison of ML Receivers With LLSE Filters

The bit error rate performances of the ML I and ML II receivers discussed in Section II are evaluated numerically and compared to the performances of the one-sided and two-sided transversal filters designed using linear least squares estimation technique. The bit error rates for different receivers are plotted against the signal energy to noise density ratio, given by $E_b / N_0 = L / 2\sigma_n^2$. The per chip signal-to-interference ratio (*SIR*) is evaluated as $1/R_j(0)$. The narrowband interference is modeled as a second order zero mean Gaussian autoregressive process with known parameters:

$$j_k + \alpha_1 j_{k-1} + \alpha_2 j_{k-2} = e_k \tag{11}$$

where $\{e_k\}$ is zero mean white Gaussian noise. The power spectral densities for the different pairs of parameters considered are shown in Fig. 2. The interfering signal is obtained by passing white noise through a second-order IIR filter, as given by (11). The poles of the filter that give the different power spectral densities are:

PSD 1: $p_1 = -j\,0.99 \quad p_2 = j\,0.99$
PSD 2: $p_1 = 0.99 \quad p_2 = 0.99$
PSD 3: $p_1 = 0.49 + j\,0.5 \quad p_2 = 0.49 - j\,0.5$

One sided and two sided transversal filters for this problem (Fig. 1b) are designed using the following Wiener-Hopf equations [4]:

$$\sum_{\substack{k=N_1 \\ k\neq 0}}^{N_2} a_k R_v(n-k) = -R_j(n),\ n = N_1,\ldots,-1,1,\ldots,N_2 \tag{12}$$

where N_1 is zero for a one sided filter and is $-N_2$ for two sided filter, N_2 is the number of taps on one side. These equations are solved for the tap weights a_k's with $R_j(n)$ being obtained from the known AR parameters of the interference via the Stepdown procedure and Levinson's algorithm [13].

A. Probability of Error Expressions for LLSE Filters

As shown in Fig. 1.(b), the test statistic of a LLSE filter is

$$TS = \sum_{k=0}^{L-1} c_k \sum_{n=N_1}^{N_2} a_n z_{k-n},\ a_0 = 1.0.$$

The mean of the test statistic, given that the present, previous, and next bits (b,d, and e respectively) are +1 is:

$$E(TS|H_1, d=+1, e=+1) = \sum_{k=0}^{L-1} c_k \sum_{n=N_1}^{N_2} a_n c_{k-n} \tag{13}$$

The reason why this mean value is conditioned on the neighboring bit values is that in the inner summation, the index on the chip sequence takes negative values as well as values exceeding L-1. The mean calculation therefore requires the knowledge of the neighboring bits. In general, the conditional mean of the test statistic is

$$E(TS|H_1, d, e) = \sum_{k=0}^{L-1} c_k \sum_{n=N_1}^{N_2} a_n c^*_{k-n} \tag{14}$$

where

$$c^*_{k-n} = \begin{cases} d c_{k-n} & \text{if } k-n<0 \\ c_{k-n} & \text{if } 0 \le k-n \le L-1 \\ e c_{k-n} & \text{if } k-n > L-1 \text{ (two - sided only)} \end{cases} \tag{15}$$

The variance of the test statistic is

$$Var(TS) = \sum_{\ell=0}^{L-1} c_\ell \sum_{k=0}^{L-1} c_k \sum_{n=N_1}^{N_2} a_n \sum_{m=N_1}^{N_2} a_m R_v(k-\ell+m-n) \tag{16}$$

The variance expression is the same as in [4]. Conditioned on hypothesis H_j, d and e, TS is Gaussian. Using (14)-(16), the bit error probability for the filter is

$$P_e = \begin{cases} \dfrac{1}{4} \sum\limits_{d\in\{-1,+1\}} \sum\limits_{e\in\{-1,+1\}} Q\left(\dfrac{E(TS|H_1,d,e)}{\sqrt{Var(TS)}}\right) & \text{for two - sided filter} \\ \dfrac{1}{2} \sum\limits_{d\in\{-1,+1\}} Q\left(\dfrac{E(TS|H_1,d)}{\sqrt{Var(TS)}}\right) & \text{for one - sided filter} \end{cases} \tag{17}$$

The bit error probability for two-sided filter ignoring ISI is given by

$$P_e(\text{ignoring ISI }) = Q\left(\frac{E(TS|H_1, d=+1, e=+1)}{\sqrt{Var(TS)}}\right) \tag{18}$$

When the condition e=+1 is suppressed, (18) is applicable to one-sided filter.

The bit error rates for the ML I, ML II, and the bit error rates for one sided and two sided transversal filters, according to equation (17) (with ISI) and (18) (ignoring ISI), are shown in Figs. 3-8 for various pairs of the AR interference parameters, processing gains and signal-to-interference ratios. The parameters assumed are as follows:

$L = 7, 31$	Processing gain
$N_2 = 4, 2$	For one-sided and two-sided filters, respectively.
$N_1 = 0, -2$	For one-sided and two-sided filters, respectively.
$i = 4$	Number of chips by which the observation interval is extended beyond the bit duration for ML II receiver.

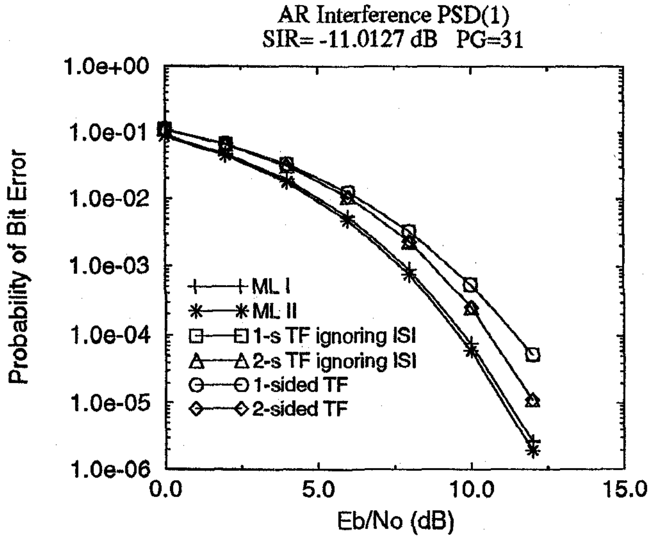

Fig. 3. Performance of ML receivers and transversal filters.

IV. Discussion and Conclusions

The bit error rates for ML I, ML II, one-sided and two-sided transversal filters, accounting for ISI and ignoring ISI, are plotted against E_b / N_0 for autoregressive interference with three specified power spectral densities (see Fig. 2) and various *SIRs* (Figs. 3-8). In Figs. 4 and 5 we also show error rate estimates obtained via simulation. The simulation procedure is explained in the next paragraph. Figs. 3 - 4 correspond to interference with power spectral density PSD 1 illustrated in Fig. 2. In Fig. 3, due to a processing gain of 31, the ISI does not have any significant effect on error estimates. Moreover, extending the observation interval beyond the bit interval under consideration does not offer any improvement for the ML receivers. Reducing the processing gain to 7 (Fig. 4) causes ISI to affect the error estimates, making them slightly lower than when ISI is not accounted for. As pointed out later, accounting for ISI lowers the error estimate only for this type of power spectral density. In all other cases, the true error rate (with ISI accounted for) is larger than the error rate estimate computed with ISI ignored. The gap between ML I and ML II also increases in Fig. 4 as compared to Fig. 3. Figs. 5-6 are for a highly correlated jammer with power spectral density PSD 2 as shown in Fig. 2. As the SIR increases from about -20 dB in Fig. 5 to about 9 dB in Fig. 6, i.e. as the jammer power decreases accordingly, the improvement in the performance of each receiver is marginal. All these curves correspond to a processing gain of 7. Though not shown here, it is found that for this type of power spectral density of the interference, a processing gain of 31 causes all the receivers to exhibit nearly identical performance. In Fig. 5, the error estimate of the transversal filters are a lot lower when ISI is ignored than when it is not. At high E_b / N_0, there is a considerable improvement in the performance of ML II over ML I. Fig. 6 also shows the same performance trend. The performances of the ML receivers are consistently better than the transversal filters. Figs. 7 and 8 are for an interference whose power spectral density is relatively flat. Hence, in Fig. 7, the performances of all the receivers are poor inspite of a processing gain of 31. A processing gain of 7 causes the effect of ISI to be noticeable and ML II offers some improvement over ML I, as seen in Fig. 8.

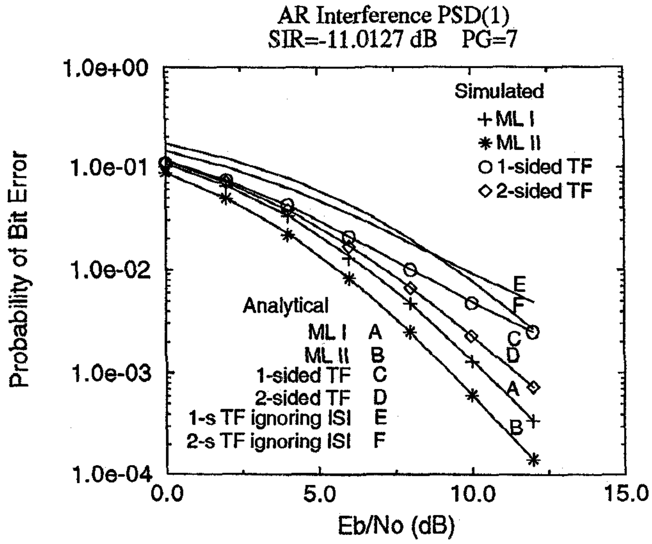

Fig. 4. Performance of ML receivers and transversal filters.

Computer simulations of all the receivers considered here were also carried out. For a given set of E_b / N_0, *SIR*, *PG*, and jammer parameters, 10^6 bits were generated randomly to have values +1 and -1 with probability 1/2 each. The bits were multiplied by the code sequence to give a data vector, of length equal to the processing gain, for each bit. To each of these data samples, interfering signals and thermal noise samples were added (1). The interfering signal was generated according to (11). The white excitation noise and thermal noise samples were obtained using IMSL routine *DRNNOA* [16]. Test statistics for ML I (4), ML II (5), and the transversal filters (Fig. 1b) were computed for each bit and tested for bit decision. The count of the errors as a fraction

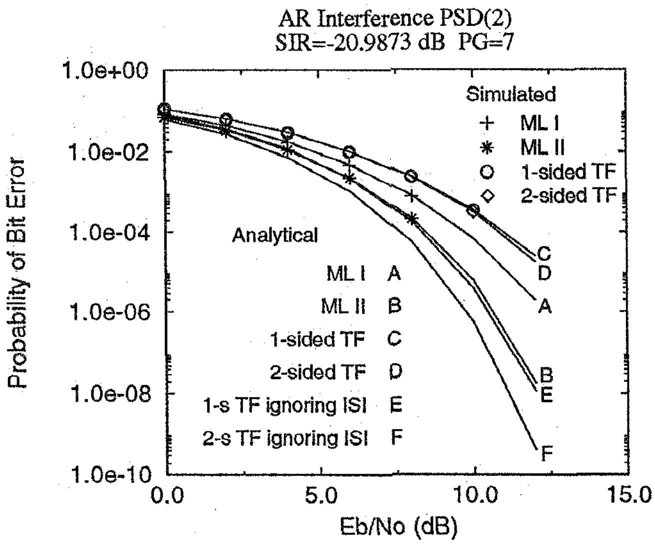

Fig. 5. Performance of ML receivers and transversal filters.

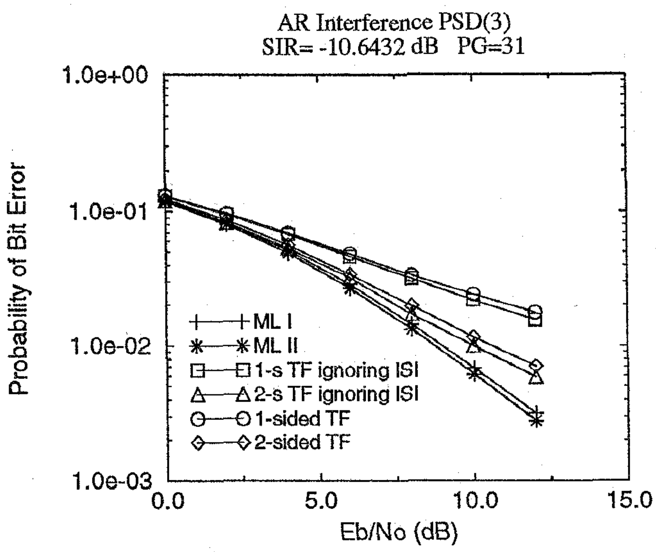

Fig. 7. Performance of ML receivers and transversal filters.

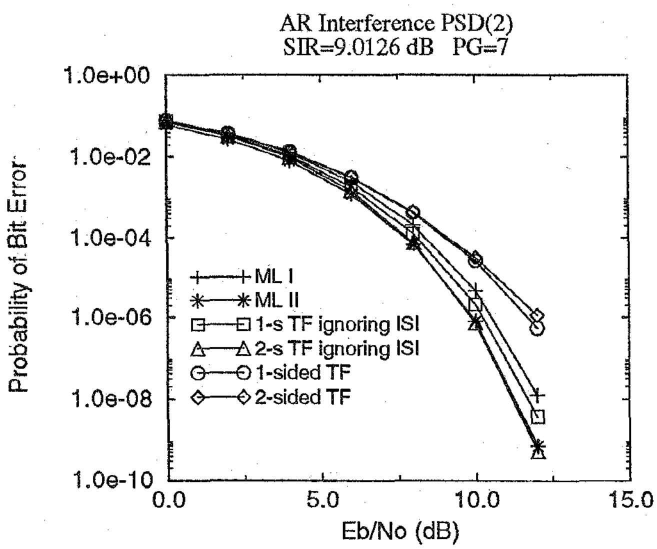

Fig. 6. Performance of ML receivers and transversal filters.

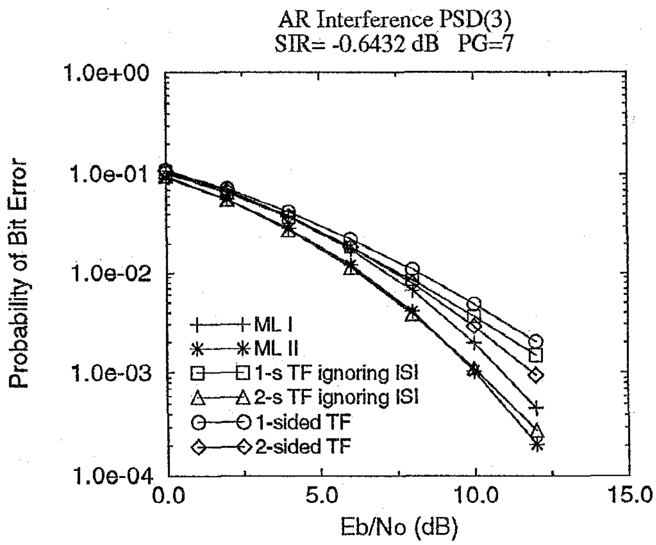

Fig. 8. Performance of ML receivers and transversal filters.

of the total number of bits tested gave the error estimate. All bit error estimates higher than 10^{-4} in Figs. 3-8 were verified via simulations. Figs. 4 and 5 are representations of this verification.

Comparing the bit error rate performances of the ML I and ML II receivers with the one-sided and two-sided transversal filters, it is observed that the maximum likelihood receivers consistently perform better than the transversal filters. The ML II receiver performance is better than that of the ML I receiver, indicating that there is some gain in performing a maximum likelihood test for the bit using observations from outside the bit duration. This gain is more for a processing gain of 7 (Figs. 4 - 6 and 8) than for PG 31 (Figs. 3 and 7). This is because for a high processing gain, the observation interval for the bit is long and additional observation samples do not improve the performance as much as when the processing gain is small. A similar effect of the processing gain is observed in the variation, due to ISI, of the bit error rate estimates for the transversal filters. For higher processing gain, not accounting for ISI does not change the error estimates as much as for lower processing gain with a comparable filter length. For low processing gain, substantial differences between the two error rate estimates can be seen in some cases (Figs. 5 and 6). For power spectral densities of the type PSD(2) and PSD(3), true error rates are the same as or higher than the error rates ignoring ISI, but for PSD(1) it is the other way. Among the transversal filters, the two-sided filters' performance is better than that of one-sided filter (except for the cases of Fig. 6).

As mentioned in [4], any performance comparison of transversal filters and maximum likelihood receivers should be done with both having the same observation interval to work with. Hence, the observation interval of the maximum likelihood receivers should be extended by the number of taps

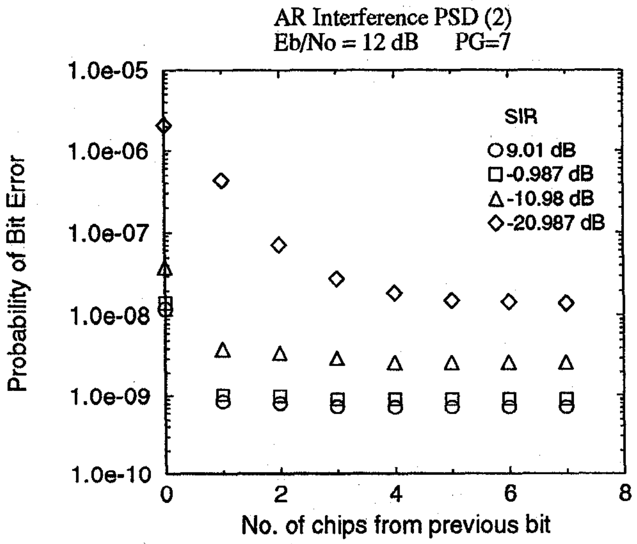

Fig. 9. BER of ML II as a function of the number of chips from the previous bit in observation interval.

of the transversal filter. This results in the ML II receiver, if the observation interval is extended into the previous bit only, and the receiver given in equation (7) if the observation interval is extended to both sides. It is seen that the ML II receiver performs better than both one-sided and two-sided filters for the autoregressive interference considered here.

All the receivers for PG 31 perform better than their counterparts with PG 7. Both the maximum likelihood receivers and transversal filters perform better when the power spectral density is peaky (PSD(1) and PSD(2)). Increasing the filter length of the transversal filters or the observation interval of the ML II receiver beyond the values given here did not improve the performance much. Fig. 9 gives an example of the bit error probability of ML II receiver as a function of the number of chips from the previous bit included in the observation interval. It is seen that even for a strong jammer (corresponding to a SIR of -21 dB), no significant improvement is obtained by extending the observation interval beyond four chips.

In conclusion, for detection of direct sequence spread spectrum signals in Gaussian autoregressive interference, (i) the nonlinear maximum likelihood receiver, which results when the observation interval is extended into the previous bit, outperforms the matched filter receiver and the one-sided and two-sided transversal filters; and (ii) intersymbol interference due to filter taps extending outside the bit interval cannot be ignored for small processing gains. In some cases, the error rates for a small processing gain can be substantially larger than the error rate estimates obtained by ignoring intersymbol interference.

Appendix

In this appendix, the bit error probability of the ML II receiver is derived and its numerical evaluation is discussed.

A. ML II Receiver Error Probability

Let us rewrite the likelihood ratio of (5) as

$$\lambda(\mathbf{r}) = \frac{\exp\{\mathbf{s}^T_{-1,+1}\Lambda^{-1}(\mathbf{r}-\frac{1}{2}\mathbf{s}_{-1,+1})\} + \exp\{\mathbf{s}^T_{+1,+1}\Lambda^{-1}(\mathbf{r}-\frac{1}{2}\mathbf{s}_{+1,+1})\}}{\exp\{\mathbf{s}^T_{-1,-1}\Lambda^{-1}(\mathbf{r}-\frac{1}{2}\mathbf{s}_{-1,-1})\} + \exp\{\mathbf{s}^T_{+1,-1}\Lambda^{-1}(\mathbf{r}-\frac{1}{2}\mathbf{s}_{+1,-1})\}} \tag{A.1}$$

Let us define the following partitions and scalars:

$\mathbf{s}^T_{a,b} = [\hat{\mathbf{s}}^T_a : \mathbf{s}^T_b]$, where $\hat{\mathbf{s}}_a$ is $i \times 1$ and $\mathbf{s}_b$ is $L \times 1$.

$$\Lambda^{-1}\mathbf{r} = \begin{bmatrix} G_1 \\ \cdots\cdots \\ G_0 \end{bmatrix}_{(i+L)\times 1}, \text{ where } G_1 \text{ is } i\times 1,\ \ G_0 \text{ is } L \times 1,$$

$$\Lambda^{-1} = \begin{bmatrix} Q_1 \\ \cdots\cdots \\ Q_0 \end{bmatrix}_{(i+L)\times(i+L)}, \text{ where } Q_1 \text{ is } i\times(i+L),\ \ Q_0 \text{ is } L\times(i+L),$$

$\frac{1}{2}\hat{\mathbf{s}}^T_1 Q_1 = [P^T_1 : P^T_2]$, P_1 is $i \times 1$ and P_2 is $L \times 1$.

$\frac{1}{2}\mathbf{s}^T_1 Q_0 = [R^T_1 : R^T_2]$, R_1 is $i \times 1$ and R_2 is $L \times 1$.

$x_1 = \hat{\mathbf{s}}^T_1 G_1$, $x_2 = \mathbf{s}^T_1 G_0$, $x_3 = P^T_1\hat{\mathbf{s}}_1 + R^T_2\mathbf{s}_1$, $x_4 = P^T_2\mathbf{s}_1 + R^T_1\hat{\mathbf{s}}_1$.

Equation (A1) can be rewritten, using $\hat{\mathbf{s}}_{-1} = -\hat{\mathbf{s}}_1$ and $\mathbf{s}_{-1} = -\mathbf{s}_1$, as

$$\lambda(\mathbf{r}) = \frac{\exp\{x_1 + x_2 - x_3 - x_4\} + \exp\{-x_1 + x_2 - x_3 + x_4\}}{\exp\{x_1 - x_2 - x_3 + x_4\} + \exp\{-x_1 - x_2 - x_3 - x_4\}} \tag{A.2}$$

Let $\theta_1 = x_1 + x_2$ and $\theta_2 = -x_1 + x_2$. Using (A.2) and (5), the likelihood ratio test is simplified as

$$\sinh(\theta_1) - \gamma\sinh(\theta_2) \underset{-1}{\overset{+1}{\gtrless}} 0 \tag{A.3}$$

where $\gamma = -\exp(2x_4)$.

The bit error probability for the ML II receiver (5) is:

$$P_e = \frac{1}{4}\sum_{d\in\{+1,-1\}}\sum_{b\in\{+1,-1\}} P(\text{error}|b,d) \tag{A.4}$$

where b denotes the test bit, b = +1 or -1 corresponding to H_1 or H_0 respectively. Because of the symmetry of the variables in (A.3),

$$\begin{aligned} P_e &= \frac{1}{2}\sum_{d\in\{+1,-1\}} P(\text{error}|H_0,d) \\ &= \frac{1}{2}\sum_{d\in\{+1,-1\}} P(\lambda(\mathbf{r}) > 1|H_0,d) \\ &= \frac{1}{2}\sum_{d\in\{+1,-1\}} P(\sinh(\theta_1) > \gamma\sinh(\theta_2)|H_0,d) \end{aligned} \tag{A.5}$$

B. Numerical Evaluation of P_e of ML II Receiver

From (A.5), the conditional bit error probability is

$$P(\text{error}|H_0,d) = \int_{-\infty}^{\infty}\left[\int_{\sinh^{-1}(\gamma\sinh(\theta_2))}^{\infty} f(\theta_1|\theta_2,H_0,d)\,d\theta_1\right] f(\theta_2|H_0,d)\,d\theta_2 \tag{A.6}$$

where $f(\theta_1|\theta_2,H_0,d)$ is the conditional density of θ_1 given θ_2, H_0, and d, and $f(\theta_2|H_0,d)$ is the conditional density of θ_2 given H_0 and d. Conditioned on H_0 and the previous bit d, θ_1 and θ_2 are bivariate Gaussian with means μ_1, μ_2 and variances σ_1^2,σ_2^2 and covariance c_{12}. These parameters are given by:

$$\mu_1 = \mathbf{s}_{+1,+1}^T\Lambda^{-1}\mathbf{s}_{d,-1} \tag{A.7}$$

$$\mu_2 = \mathbf{s}_{-1,+1}^T\Lambda^{-1}\mathbf{s}_{d,-1} \tag{A.8}$$

$$\sigma_1^2 = \mathbf{s}_{+1,+1}^T\Lambda^{-1}\mathbf{s}_{+1,+1} \tag{A.9}$$

$$\sigma_2^2 = \mathbf{s}_{-1,+1}^T\Lambda^{-1}\mathbf{s}_{-1,+1} \tag{A.10}$$

$$c_{12} = \mathbf{s}_{+1,+1}^T\Lambda^{-1}\mathbf{s}_{-1,+1} \tag{A.11}$$

The conditional density $f(\theta_1|\theta_2,H_0,d)$ is Gaussian with mean μ and variance σ^2, where

$$\mu = E(\theta_1|\theta_2=\theta,d) = \mu_1 + \rho\frac{\sigma_1}{\sigma_2}(\theta-\mu_2) \tag{A.12}$$

$$\rho = \frac{c_{12}}{\sigma_1\,\sigma_2} \tag{A.13}$$

$$\sigma^2 = \sigma_1^2(1-\rho^2) \tag{A.14}$$

Hence the conditional probability of error is

$$\begin{aligned} P(\text{error}|H_0,d) &= \int_{-\infty}^{\infty}\left[1-\Phi\left(\frac{\sinh^{-1}(\gamma\sinh(\theta_2))-\mu}{\sigma}\right)\right] f(\theta_2|H_0,d)\,d\theta_2 \\ &= 1-\int_{-\infty}^{\infty}\Phi\left(\frac{\sinh^{-1}(\gamma\sinh(\theta_2))-\mu}{\sigma}\right) f(\theta_2|H_0,d)\,d\theta_2 \end{aligned} \tag{A.15}$$

where $\Phi(.)$ is the standard normal cdf. The average probability of error is obtained by using (A.5) and (A.15).

The integral in (A.15) is evaluated using the double precision IMSL routine *DQDAGI* [16]. The first term of the integrand was specified using the standard normal distribution function *DNORDF* of IMSL. *DQDAGI* integrates a function over an infinite interval by first transforming an infinite interval into the finite interval [0,1] and then using a 21-point Gauss-Kronrod rule to estimate the integral and the error. Since the function to be integrated is a product of Gaussian pdf and cdf, it is a smooth one. Sufficient numerical accuracy was achieved by specifying a relative error of 0.001.

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