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On SNR as a Measure of Performance for Narrowband Interference Rejection in Direct Sequence Spread Spectrum Systems

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***Abstract* - The usefulness of *SNR* as a figure of merit to quantify the narrowband interference rejection capability of a DS receiver is examined. The interference considered is a peaked autoregressive Gaussian process. The probability of error and *SNR* estimates of a Kalman, a modified Kalman, and a nonlinear filter proposed in [2] are obtained by simulation. Based on this simulation study and the available theoretical error rate analysis of transversal filters, we can conclude that *SNR* is a useful measure if the processing gain, PG, of the DS system is moderately large. When the PG is small, such as 7, and if thermal noise is negligible compared to the signal, the *SNR* is not a reliable measure of performance.**

I. Introduction

Narrowband interference in a received direct sequence (DS) signal can be reduced by filtering the received signal prior to despreading and demodulation (see [1] for review). The enhanced rejection of narrowband interference by the inclusion of a filter becomes more significant when the processing gain of the DS system is small. In several previous studies, the signal-to-noise ratio (*SNR*) obtained at the filter output has been used as a figure of performance of the interference rejection capability of a DS receiver [1], [2]. In general, *SNR* is used as a measure of performance when an analytical expression for the bit error rate (*BER*) is not available, and error rate evaluation by simulation is time consuming. However, performance comparison of two receivers based on *SNR* could be troublesome. Even though the error probability for each receiver is a monotonic function of its *SNR*, the monotonic curves of the two receivers may be different. Therefore, it is possible that the receiver showing a larger output *SNR* may have a larger error rate. In this letter, we study the suitability of *SNR* as a measure of performance of a DS-BPSK system employing one of the following for interference rejection: Kalman filter, a linear modification, the non-linear modification of [2] or a transversal filter. Simulations with a Kalman filter and its modifications are carried out to estimate the *SNR*'s at the filters' output, which are sequences at the chip rate, *SNR*'s of the test statistics, and *BER*'s of the receivers. To evaluate the transversal filter, the theoretical error rate and *SNR* expressions obtained in [3] are used.

The received DS-BPSK signal, after chip matched filtering and sampling at the chip rate of the PN sequence, gives the following samples [2] for $k \in \{0, \pm 1, \pm 2, \ldots\}$,

$$z_k = s_k + n_k + i_k \tag{1}$$

Here $s_k = d_{[\frac{k}{L}]} c_k$, $\{d_{[\frac{k}{L}]}\}$ denotes the data bit sequence, L is the PG, $[\frac{k}{L}]$ denotes the integer portion of $\frac{k}{L}$ for $k \geq 0$ and integer portion of $\frac{k}{L}$ minus 1 for $k < 0$,and $c_k \in \{-1,1\}$ is the kth chip of the PN sequence. For later convenience, bit d_0 is denoted as d. Each bit is *i.i.d* with value +1 or -1. $\{n_k\}$ is an i.i.d. Gaussian sequence with zero mean and variance σ_n^2 . The narrowband interference, $\{i_k\}$, is a Gaussian autoregressive process of order p and variance σ_i^2.

$$i_k = \sum_{i=1}^{p} \phi_i \, i_{k-i} + e_k \tag{2}$$

where $\{e_k\}$ is zero mean white Gaussian noise and ϕ_i's are known to the receiver. The sequences $\{s_k\}$, $\{n_k\}$ and $\{i_k\}$ are mutually independent. In section II, the interference filters that are simulated, are introduced along with a discussion of the key simulation variables. Section III provides a discussion and conclusions derived from this study.

II. Filtering for Narrowband Interference Rejection

The Kalman filter and its nonlinear modification are described in [2]. By putting the autoregressive interference in a state-space representation, the Kalman filter algorithm is obtained. Asymptotically, as the observation interval becomes large, a Kalman filter is equivalent to a Wiener filter. However, consider the situation where a decision on a given bit is made using the observations corresponding to all the past and present bits. Whereas an observation interval longer than the bit interval is useful because of the correlated interference, by extending the observation interval beyond the current bit, we also inherit the uncertainty associated with the previous bits [3]. Hence, for certain ranges of parameters, the Kalman filter may perform poorly as compared to a transversal filter that has only a finite memory. Also, if the PN sequence is nearly *i.i.d*, then the

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minimum mean square error filter that estimates the interference from the observations $\{z_k\}$ is not linear [2].

A different linear filter is obtained by modifying the state space model. The received signal (1) with i_k as in (2) can be represented as

$$
\begin{aligned}
x_k &= \Phi_k x_{k-1} + w_k \\
z_k &= H_k x_k + n_k
\end{aligned}
\tag{3}
$$

where $x_k = \left[i_k \quad i_{k-1} \quad \cdots \quad i_{k-p+1} \quad d_{[\frac{k}{L}]} \right]^T$,

$H_k = [1 \quad 0 \quad \cdots \quad 0 \quad c_k]$, $w_k = [e_k \quad 0 \quad \cdots \quad 0 \quad u_k]^T$,

$$
\Phi_k = \begin{bmatrix}
\phi_1 & \phi_2 & \cdots & \phi_{p-1} & \phi_p & 0 \\
1 & 0 & \cdots & 0 & 0 & 0 \\
0 & 1 & \cdots & 0 & 0 & 0 \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
0 & 0 & \cdots & 1 & 0 & 0 \\
0 & 0 & \cdots & 0 & 0 & j_k
\end{bmatrix}.
$$

For $k = 0, \pm L, \pm 2L, \ldots,$ $j_k = 0, u_k = \pm 1$, with equal probability, otherwise, $j_k = 1, u_k = 0$.

The rationale for the above modified linear filter is as follows. If (3) is compared with the state model of a Kalman filter in [2], it can be seen that the x_k vector has an added component $d_{[\frac{k}{L}]}$ as shown in (3). This reflects the knowledge that the data bit does not change over the L chip intervals within a data bit. This knowledge is obtained at the expense of the unknown created in the very first chip of a bit. That is, for d_0, $j_0 = 0$ is assumed, since the true bit (which is either +1 or -1) is unknown, and $j_k = 1$ represents the fact that the bit contribution is the same for $1 \le k \le L-1$. The filtering and update equations in [2], with H, Q and Φ replaced by H_k, Q_k and Φ_k respectively, are the filtering equations for this modified filter:

$$
Q_k = \begin{bmatrix}
E\{e_k^2\} & 0 \cdots 0 \\
0 & 0 \cdots 0 \\
\vdots & \vdots \quad \vdots \\
0 & 0 \cdots u_k^2
\end{bmatrix}, \quad
u_k^2 = \begin{cases} 1, & k = 0, \pm L, \pm 2L, \cdots \\ 0, & \text{otherwise} \end{cases}
$$

The bit decision procedure for the DS receiver is shown in Fig. 1. The input to the PN correlator is denoted as ε_k. The output of the correlator is the test statistic TS:

$$
TS = \sum_{k=0}^{L-1} \varepsilon_k c_k \underset{d=-1}{\overset{d=+1}{\gtrless}} 0 \tag{4}
$$

The per chip SNR at the output of the filter is defined as [2]:

$$
SNR_c = \frac{E(s_k^2)}{E(|\varepsilon_k - s_k|^2)} \tag{5}
$$

The test statistic SNR is defined as

$$
SNR_{TS} = \frac{E^2(TS)}{Var(TS)} \tag{6}
$$

Since an interference rejection filter cannot eliminate the interference completely, ε_k has some residual correlation from chip to chip, especially when the interference is strong. Hence SNR_{TS} cannot be estimated from SNR_c unless the residual correlations and any signal distortions are accounted for.

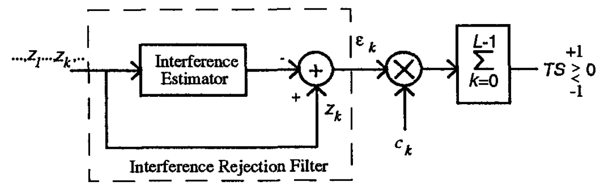

Fig. 1. Filter Structure for Narrowband Interference Rejection and Bit decision of the Spread Spectrum receiver

In a receiver with a Kalman filter, the test statistic for the current bit d is affected by a number of previous bits due to the filter memory. When the filter is designed for strong, highly correlated interference and thermal noise with a low variance, the intersymbol interference (ISI) introduced will be significant. Since the filter is linear, this effect may be studied by applying superposition. Let $f_K(\cdot)$ be the function of the present and all past observations defining the Kalman filter operation. The test statistic is given by

$$
\begin{aligned}
TS &= \sum_{k=0}^{L-1} [z_k - f_K(z_k)] c_k \\
&= \sum_{k=0}^{L-1} \left[\{dc_k - f_K(d_{[\frac{k}{L}]} c_k)\} + \{n_k - f_K(n_k)\} + \{i_k - f_K(i_k)\} \right] c_k
\end{aligned}
\tag{7}
$$

Define $t_s = \sum_{k=0}^{L-1} \left[dc_k - f_K(d_{[\frac{k}{L}]} c_k) \right] c_k$ as the contribution of the signal component to the receiver test statistic. The density of t_s provides a measure of the ISI effect on the test statistic . For example, if no filter is used, the density of t_s will be two impulses at $+L$ and $-L$, indicating that the signal component of the test statistic is the true bit, i.e. no signal distortion or ISI exists in this case. With the Kalman filter, a large variance of t_s indicates that the interference from previous bits significantly affects the current test statistic. If t_s is Gaussian, then so is TS. As seen from the simulation results below, for certain ranges of parameters, the approximate Gaussianity of TS holds, whereas in certain other ranges, TS is either non-Gaussian or approximately Gaussian conditioned on the previous bit.

The test statistic corresponding to a nonlinear filter is, in general, non-Gaussian and its distribution is required in order to find the BER. Any inference on the comparative performances of the linear and non-linear filters based on a test statistic SNR could be misleading, since the nature of the tail of the test statistics' densities of linear and nonlinear filters may be different.

A. Simulation

Simulation of the receiver in Fig. 1 is done using samples generated according to (1). Denoting N as the length of the

PN sequence, the three cases considered are (i) L=N=7, (ii) L=7, N=1023 and (iii) L=N=63. Case (i) is an example of small PG situations where an entire PN sequence is embedded in each bit. In case (ii), L is still small, but a large N is used so that the PN sequence is approximately *iid*. This is the situation where the nonlinear Kalman filter is expected to be better than the Kalman filter[2]. Our *BER* simulation results given below verify this. Case (iii) is for a moderate PG situation. L exceeding 63 was not considered because of excessive simulation time requirements. Values of σ_n^2 and σ_i^2 used are those typically expected. For example, with L=7, σ_n^2=1, no filter and no interference, the *SNR* is $10\log\left(\frac{7^2}{7}\right) = 8.45dB$ (without a filter and interference, the test statistic is Gaussian and $P_b = Q(\sqrt{SNR})$, where $Q(\cdot)$ is one minus the standard normal cdf). For σ_n^2 = 0.01, *SNR* = 28.45*dB*-a situation expected when the TX-RX pair are close. σ_i^2=10,000 for L=7 and σ_i^2=100,000 for L=63 correspond to a strong jammer.

The interference obeys (2) with ϕ_1=1.98, ϕ_2=-0.9801. The spectrum of this interfering signal is sharply peaked. Sufficient number of realizations of *TS* are used to ensure that the variance in the *BER* estimate is small. We have typically used a number of samples exceeding 80/*BER*. More details on the simulation procedure can be found in [4].

The simulation results and some related calculations are shown in Table I (L=N=7), Table II (L=7,N=1023) and Table III (L=N=63) for various values of σ_n^2 and σ_i^2. The labeling of the estimates in the tables are explained below. SNR^* is the estimate of the test statistic *SNR*, SNR_{TS} of (6). P_b^* is the *BER* estimate obtained directly from simulations. P_{SNR}^* is an estimate of the *BER* obtained as $Q(\sqrt{SNR^*})$, i.e. under the assumption that the test statistic is Gaussian. SNR_2, appearing in Table I only, is an estimate of the test statistic *SNR* obtained as L times the estimate of SNR_c (5), i.e. an estimate obtained by ignoring the residual chip correlation and signal distortion. P_2^* is an estimate of the *BER* obtained via $Q(\sqrt{SNR_2})$. In the case of Kalman-based filters, the two columns correspond to filter estimates and predictor estimates respectively. In the case of transversal filters, SNR^*, P_{SNR}^*, and P_b^* shown in Table I are obtained analytically [3]. For the case of Kalman filter, the estimated conditional densities of t_s given present bit equal to +1 and -1 are shown in Figs. 2-4.

III. Discussion and Conclusions

A. Small Processing Gain and Short PN Sequence (L=N=7, Table I)

For the two linear Kalman-based filters, the *BER* estimate from the test statistic *SNR*, P_{SNR}^*, agrees with the *BER* estimate P_b^* for weak interference and relatively large thermal noise variance (σ_n^2 =1 and σ_i^2=1000). In this situation, the ISI effect is small and the Gaussian assumption for the test statistic is valid. This is somewhat justified by the density estimate of t_s for the Kalman filter(Fig. 2). When the interference is strong and the thermal noise variance is small ($\sigma_n^2 = 0.01$ and $\sigma_i^2 = 10{,}000$), the variance of t_s is large and the two peaks for each case of the true bit in the density estimate clearly show the ISI effect (Fig. 3). As observed from the simulation data, the two peaks are due to the previous bit,l, being 1 or -1. The error estimate obtained from the conditional Gaussian approximation, $\sum_{l\in\{-1,1\}} \frac{1}{2} Q\left(\sqrt{SNR_{TS}} \,|\, l, d=+1\right)$, agrees with the *BER* estimate P_b^*. In the case of a transversal filter , the correct *BER* is in fact the average of the conditional errors[3]. As shown in Table I for a one-sided, 4-tap transversal filter, weak thermal noise (σ_n^2 =0.01 and 0.1), and strong interference (σ_i^2=10,000), the *BER* (P_b^*) is larger than the estimate $P_{SNR}^* = Q(\sqrt{SNR^*})$, where SNR^* is the average of the two conditional *SNR*s. This indicates significant ISI for these parameters. For the two linear Kalman filters, the effect of ignoring residual correlation and signal distortion can be studied by comparing the two estimates SNR_2 and SNR^*. Irrespective of the strengths of the interference and thermal noise, the two estimates differ. Therefore, in general $SNR^* \neq L\, SNR_C$.

When the filter is nonlinear, the test statistic is in general non-Gaussian. However, when the interference is not strong and the thermal noise variance is relatively high, (σ_n^2 =1 and σ_i^2=1000), the Gaussian approximation to the test statistic of the nonlinear filter also yields the correct *BER* (compare P_b^* and P_{SNR}^*). Although SNR^* of the nonlinear filter is much larger than that of the modified linear filter or transversal filter, when $\sigma_n^2 = 0.01$ and σ_i^2 = 10000, P_b^* of the nonlinear

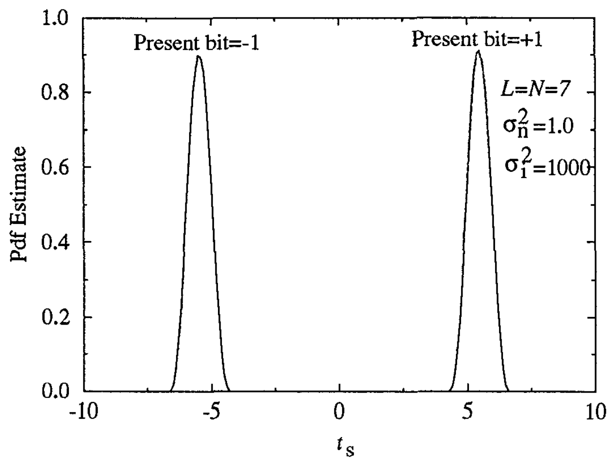

Fig. 2. Estimate of the conditional densities of t_S for Kalman Filter

Table I
SNR and *BER* Estimates

L=*N*=7			KALMAN		MODIFIED		NONLINEAR		Trans. Filter
σ_n^2	σ_i^2		FILTER	PRED.	FILTER	PRED.	FILTER	PRED.	1-sided,4 taps
1.0	1000	SNR^*	5.4290	5.4290	5.1442	5.1442	5.3215	5.4254	4.7891
		P^*_{SNR}	9.9E-3	9.9E-3	1.2E-2	1.2E-2	1.0E-2	9.9E-3	1.432E-02
		SNR_2	7.3093	4.3601	7.7073	4.23	7.7429	4.411	–
		P^*_2	3.4E-3	1.8E-2	2.7E-3	1.9E-2	3.2E-3	1.8E-2	–
		P^*_b	9.8E-3	9.8E-3	1.2E-2	1.2E-2	9.5E-3	9.5E-3	2.386E-02
0.01	10000	SNR^*	8.6684	8.6684	41.725	41.725	843.36	87.309	89.235
		P^*_{SNR}	1.6E-3	1.6E-3	5.E-11	5.E-11	1E-185	4.E-21	1.753E-21
		SNR_2	13.614	7.2408	8.6836	2.7871	2073.4	85.0	–
		P^*_2	1.1E-4	3.5E-3	1.6E-3	4.7E-2	0**	1.E-20	–
		P^*_b	2.7E-4	2.7E-4	#	#	4.0E-5	4.0E-5	2.273E-17
0.1	10000	SNR^*	6.8427	6.8427	6.6322	6.6322	7.9695	8.9847	20.1482
		P^*_{SNR}	4.4E-3	4.4E-3	5.0E-3	5.0E-3	2.3E-3	1.3E-3	3.583E-06
		SNR_2	12.68	6.4178	10.51	4.503	23.627	9.91	–
		P^*_2	1.8E-4	5.6E-3	5.9E-4	1.7E-2	5.8E-7	8.2E-4	–
		P^*_b	1.7E-3	1.7E-3	8.2E-4	8.2E-4	8.9E-3	6.4E-3	3.791E-04
1.0	10000	SNR^*	3.3297	3.3297	2.8303	2.8303	3.2994	3.4016	3.0504
		P^*_{SNR}	3.4E-2	3.4E-2	4.6E-2	4.6E-2	3.4E-2	3.2E-2	4.0375E-02
		SNR_2	7.007	2.954	7.481	2.831	7.1305	2.9890	–
		P^*_2	4.0E-3	4.3E-2	3.1E-3	4.6E-2	3.8E-3	4.2E-2	–
		P^*_b	3.9E-2	3.9E-2	5.0E-2	5.0E-2	3.8E-2	3.8E-2	6.583E-02

#1 error observed in 10^6 trials **Zero up to machine precision

filter is also higher. The nonlinear predictor and the transversal filter have the same SNR^* but the latter has a P_b^* several decades below that of the former. Even a comparison of the two nonlinear filter estimates based on SNR^* could be misleading. The nonlinear predictor and filter show almost the same *BER*s (P_b^*s) but differ in *SNR*s by almost 10*dB*. For another set of noise parameters , σ_n^2=0.1 and σ_i^2=10000, the SNR^*s of all three Kalman-based filters are comparable (SNR^* of the nonlinear is slightly higher) but P_b^* for the nonlinear filter is higher than P_b^* of the modified linear filter.

B. Small Processing Gain and Long PN Sequence (L=7, N=1023, Table II)

For the Kalman filter, the BER estimates P^*_{SNR} and P_b^*

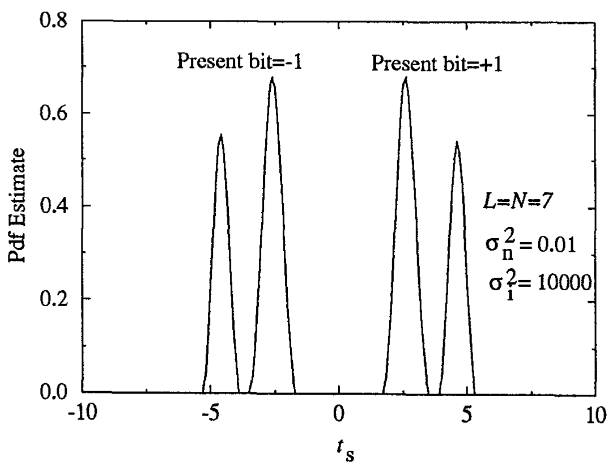

Fig. 3. Estimate of the conditional densities of t_s for Kalman Filter

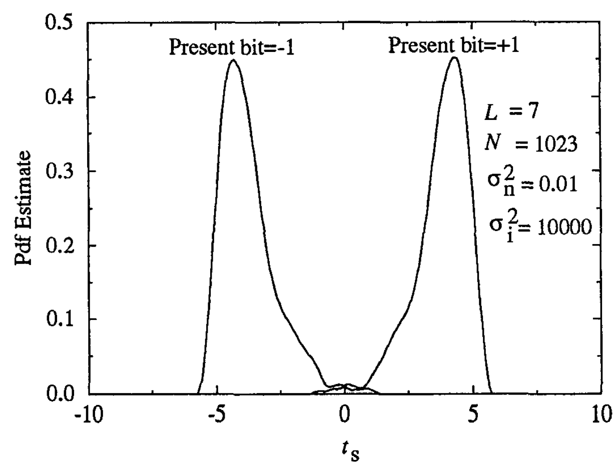

Fig. 4. Estimate of the conditional densities of t_s for Kalman Filter

Table II
SNR and *BER* Estimates

L=7, N=1023			KALMAN		MODIFIED		NONLINEAR	
σ_n^2	σ_i^2		FILTER	PRED.	FILTER	PRED.	FILTER	PRED.
1.0	1000	SNR^*	4.2585	4.2585	4.18026	4.18026	4.2388	4.3052
		P_{SNR}^*	1.95E-2	1.95E-2	2.04E-2	2.04E-2	1.97E-2	1.89E-2
		P_b^*	2.32E-2	2.32E-2	2.33E-2	2.33E-2	2.33E-2	2.30E-2
0.01	10000	SNR^*	8.083	8.083	2.699	2.699	708.548	78.598
		P_{SNR}^*	2.23E-3	2.23E-3	5.02E-2	5.02E-2	0^{**}	3.8E-19
		P_b^*	1.63E-2	1.63E-2	4.82E-2	4.82E-2	2.2E-4	1.9E-4
0.1	10000	SNR^*	7.0672	7.0672	4.7866	4.7866	8.951	10.692
		P_{SNR}^*	3.92E-3	3.92E-3	1.43E-2	1.43E-2	1.38E-2	5.38E-4
		P_b^*	1.80E-2	1.80E-2	2.45E-2	2.45E-2	1.35E-2	1.24E-2
1.0	10000	SNR^*	2.9706	2.9706	2.8912	2.8912	2.909	2.988
		P_{SNR}^*	4.24E-2	4.24E-2	4.45E-2	4.45E-2	4.40E-2	4.19E-2
		P_b^*	4.72E-2	4.72E-2	4.82E-2	4.82E-2	4.79E-2	4.74E-2

Table III
BER Estimates

L=N=63			KALMAN		MODIFIED		NONLINEAR	
σ_n^2	σ_i^2		FILTER	PRED.	FILTER	PRED.	FILTER	PRED.
10.0	10,000	P_{SNR}^*	1.84E-2	1.84E-2	1.83E-2	1.83E-2	1.85E-2	1.85E-2
		P_b^*	1.98E-2	1.98E-2	1.97E-2	1.97E-2	1.98E-2	1.98E-2
10.0	100,000	P_{SNR}^*	3.79E-2	3.79E-2	3.75E-2	3.75E-2	3.79E-2	3.79E-2
		P_b^*	3.38E-2	3.38E-2	3.35E-2	3.33E-2	3.38E-2	3.38E-2
3.0	100,000	P_{SNR}^*	1.2E-3	1.11E-3	1.09E-3	1.09E-3	1.20E-3	1.11E-3
		P_b^*	1.4E-3	1.4E-3	1.43E-3	1.43E-3	1.38E-3	1.4E-3
5.0	100,000	P_{SNR}^*	7.13E-3	7.13E-3	6.90E-3	6.90E-3	7.16E-3	7.14E-3
		P_b^*	7.57E-3	7.57E-3	7.22E-3	7.22E-3	7.59E-3	7.58E-3

**Zero up to machine precision

disagree for weak thermal noise and strong interference (σ_n^2=0.01 and σ_i^2=10,000). This is due to the non-Gaussian nature of t_s as shown by its density estimate (Fig. 4). The nonlinear filter has P_b^* two decades below the others. However, based on *SNR* alone, one would have anticipated a much lower error rate for the nonlinear filter (the nonlinear filter(predictor) has a *SNR** 100(10) times the *SNR** of the Kalman). For the modified linear filter, the estimates P_{SNR}^* and P_b^* agree for all parameters considered.

C. Moderate Processing Gain and PN Sequence (L=N=63, Table III*)*

For moderate processing gain, the simulations had to be restricted to not too small thermal noise variances in order to observe enough errors and obtain an estimate of the *BER*. For all parameters considered , P_{SNR}^* and P_b^* estimates agree.

In conclusion, the *SNR* can be used as a measure of performance of a DS system employing narrowband interference rejection filters if the PG is moderately large. When the PG is small and the thermal noise is negligible as compared to the signal, *SNR* is not a reliable measure.

REFERENCES

[1] L.B. Milstein, "Interference rejection techniques in spread spectrum communications," *Proc. IEEE*, vol. 76, June 1988, pp 657-671.
[2] R. Vijayan and H. V. Poor, "Nonlinear techniques for interference suppression in spread spectrum systems," *IEEE Trans. Commun.*, Vol. 38, July 1990, pp 1060-1065.
[3] A. Ansari and R. Viswanathan, "Performance study of maximum-likelihood receivers and transversal filters for the detection of direct-sequence spread spectrum signal in narrowband interference," *IEEE Trans. Commun.*, Feb./Mar./April 1994, pp 1939-1946.
[4] A. Ansari, "Performance of direct sequence spread spectrum receivers in pulsed noise jamming and in narrowband interference," Ph.D. Dissertation, Southern Illinois University, Carbondale, IL 62901, 1993.