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A New Distributed Constant False Alarm Rate Detector

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A new constant false alarm rate (CFAR) test termed signal-plus-order statistic CFAR (S + OS) using distributed sensors is developed. The sensor modeling assumes that the returns of the test cells of different sensors are all independent and identically distributed. In the S + OS scheme, each sensor transmits its test sample and a designated order statistic of its surrounding observations to the fusion center. At the fusion center, the sum of the samples of the test cells is compared with a constant multiplied by a function of the order statistics. For a two-sensor network, the functions considered are the minimum of the order statistics (mOS) and the maximum of the order statistics (MOS). For detecting a Rayleigh fluctuating target in Gaussian noise, closed-form expressions for the false alarm and detection probabilities are obtained. The numerical results indicate that the performance of the MOS detector is very close to that of a centralized OS-CFAR, and it performs considerably better than the OS-CFAR detector with the AND or the OR fusion rule. Extension to an N-sensor network is also considered, and general equations for the false alarm probabilities under homogeneous and nonhomogeneous background noise are presented.

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I. INTRODUCTION

For the past several years a considerable amount of work [1–10] on single sensor constant false alarm rate (CFAR) signal detection has been done. The detection of signals becomes complex when radar returns are from nonstationary background noise (or noise plus clutter). The probability of false alarm increases intolerably when a detection scheme employing a fixed threshold is used. Therefore, adaptive threshold techniques are required in order to maintain a nearly constant false alarm rate. Because of the diversity of the radar search environment (multiple target, abrupt changes in clutter, etc.) there exists no universal CFAR scheme. Typically the adaptive threshold of a CFAR scheme is the product of two terms, one is a fixed scaling factor to adjust the probability of false alarm, and the other is an estimate of the total unknown noise power of the test cell. The sample in the test cell is compared with this threshold in order to decide the presence or the absence of a target. A variety of CFAR techniques are developed according to the logic used to estimate the unknown noise power level. Some examples are, cell averaging CFAR (CA-CFAR), order statistics CFAR (OS-CFAR), greatest of CFAR, smallest of CFAR [3], and selection and estimation test [4].

Attraction toward multiple sensor systems with data fusion began to grow in the early 1980s [11]. Distributed signal detection (DSD) schemes are needed when system performance factors such as speed, reliability, and constraint over the communication bandwidth are taken into account. In DSD techniques, each sensor sends either a binary decision or a condensed form of information (statistics) about the observations available at the sensor to the fusion center, where a final decision about the presence of a target is made. DSD with data fusion has been applied to CA-CFAR, adaptive CA-CFAR, and OS-CFAR. Barkat and Varshney [12] considered CA-CFAR detection using multiple sensors and data fusion. In their approach, each CA-CFAR detector transmits a binary decision to the fusion center where a final decision based on the AND or the OR counting rule is obtained. They have also addressed the adaptive CA-CFAR detector problem for parallel and tandem distributed networks [13]. Distributed OS-CFAR detectors with the AND or the OR fusion rule is considered by Uner and Varshney [14]. The problem of distributed CA-CFAR detection of dependent signal returns is studied by Blum and Kassam [15]. The common ground between all of these distributed CFAR detection schemes is that the final decision based on individual decisions of each sensor emerges from a counting rule such as AND or OR.

We propose a new distributed CFAR detection scheme called signal-plus-order statistic CFAR

(S + OS). Instead of a binary decision, each sensor transmits the sample from the test cell and a designated order statistic from the available set of reference observations surrounding the test cell to the fusion center. The selected order statistics among the sensors could have the same or different ranks, and the number of samples in the reference observations for each sensor need not be the same. At the fusion center, the sum of the test samples is compared with an adaptive threshold obtained by the product of a fixed scaling factor and a function of the received order statistics, to decide the presence/absence of a target. The estimate of the noise power level of the test cells is provided by this function. Some examples of this function are: minimum of, maximum of, linear combination of, or in the case of a large number of sensors, an order statistics of the variables. We call the S + OS test that uses the maximum (minimum) order statistic the MOS (mOS) detector. Our problem formulation therefore assumes that the test cells of different sensors all have statistically identical noise (clutter), and that if a target is present in the surveillance regions, all the test cells have statistically identical target returns. What happens if this assumption is violated? The performance of the proposed CFAR test could degrade depending on the statistical dissimilarities between the returns in the test cells of different sensors. In such a case a distributed CFAR test with decisions combining at the fusion center (like the AND or the OR rule) may provide a more robust performance.

The performance of a central order statistic detector (COS-CFAR), whose decision is based on the comparison of the sum of samples of the test cells with an order statistic of the samples from the adjacent cells of all the sensors, is also evaluated. Although the MOS detector requires a little more computation as compared with the existing distributed CFAR techniques, it shows considerable improvements in performance over the AND and the OR schemes. Moreover, its performance is close to that of the COS-CFAR detector, which has all the test and noise data available.

In Section II, for a two-sensor network, we define the problem for detecting a Rayleigh fluctuating target in Gaussian noise. Also, closed-form expressions for the probabilities of false alarm and detection for the MOS and the mOS detectors are derived. Generalization of the S + OS scheme to an N-sensor network is also developed in this section. Section III contains performance comparisons of various schemes based on the numerical study involving a two-sensor network. A summary and the conclusions derived from this study are presented in Section IV. Appendix A provides the performance equations for the COS-CFAR detector and the OS-CFAR detector with the AND and the OR fusion rules.

II. SIGNAL-PLUS-ORDER STATISTIC DISTRIBUTED CFAR

In this section, the S + OS distributed CFAR test for a network of two sensors is defined and appropriate parameters are developed. Extension of the S + OS test to the case of N sensors is also presented. For a two-sensor network, the equations for the probabilities of false alarm and detection for the MOS and the mOS detectors for both homogeneous and nonhomogeneous background noise are derived. General guidelines on how to obtain the false alarm probabilities for a network of N sensors are also provided in this section.

A. S + OS Distributed CFAR for Two Sensors

Consider a two-sensor distributed network as shown in Fig. 1. Here, Y_{ij} is the observation (excluding the test sample), where $i = 1,2$ indicates the numbering of the sensors, and $j = 1,2,\ldots,(m_i - 1)$ represents the sample number in the range cells available to the ith sensor. In general m_1 need not be equal to m_2. It is assumed that both the sensors scan the same search environment. The sample in the test cell for the ith sensor is denoted by X_{0i}, and the rank-ordered adjacent cell observations are denoted by $Y_{i(1)}, Y_{i(2)}, \ldots, Y_{i(m_i-1)}$ where $Y_{i(r)}$ denotes the rth largest order statistic of $\{Y_{i1},\ldots,Y_{im_i-1}\}$. A statistic Z_i from the ith sensor is sent to the fusion center. In our setup, $Z_1 = Y_{1(k)}$ and $Z_2 = Y_{2(l)}$, where k and l are appropriate integers. At the fusion center, two quantities, $X = X_{01} + X_{02}$, and a function $g(Z_1,Z_2)$, are computed. The MOS detector assumes $g(Z_1,Z_2) = \max(Z_1,Z_2) = R$, whereas for the mOS scheme, $g(Z_1,Z_2) = \min(Z_1,Z_2) = W$, where $\max(\cdot)$ and $\min(\cdot)$ are the maximum and the minimum of Z_1 and Z_2, respectively. Fusion center decides the presence or the absence of a target in the test cell by comparing X with $Tg(\cdot)$, where T is an appropriate scaling factor.

It is assumed that $Y_{i1}, Y_{i2}, \ldots, Y_{im_i-1}$ are independent identically distributed (IID) random variables (rv) that follow an exponential distribution. In the case of homogeneous noise, $E[Y_{ij}] = \lambda_0$, where λ_0 is the noise power and we denote the corresponding density and cumulative distribution functions as $f(y)$ and $F(y)$, respectively. Let CNR represent the clutter-to-noise power ratio. In the case of nonhomogeneous background, the expected value of Y_{ij} is λ_0 or $\lambda_0(1 + \mathrm{CNR})$, depending on whether the sample Y_{ij} is from noise-only region or from clutter, respectively. Assuming a Rayleigh fluctuating target, the test sample, X_{0i}, also has an exponential distribution with mean λ ([6, pp. 208–209]). The mean λ is unknown and depends on the target presence/absence, the clutter level, and the target

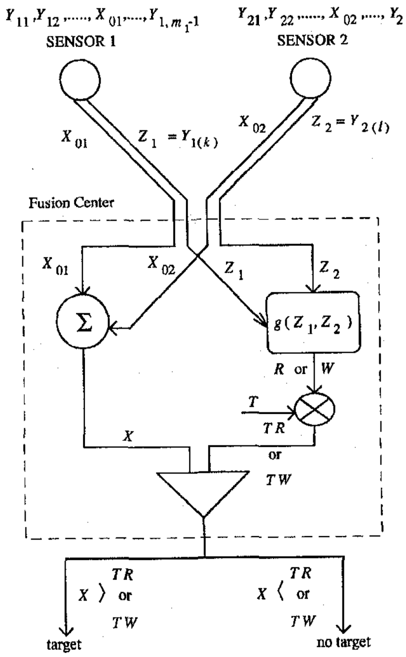


Fig. 1. Two-sensor S + OS distributed CFAR detector.

strength:

$$\lambda = \begin{cases} \lambda_0 \text{ or } \lambda_0(1+\text{CNR}), & \text{under } H_0 \\ \lambda_1 = \lambda_0(1+\text{SNR}), & \text{under } H_1 \end{cases} \tag{1}$$

where hypothesis H_1 represents the presence of a target and hypothesis H_0 means no target, and $\lambda_1 = \lambda_0(1+\text{SNR})$ represents the signal-plus-noise power, where SNR is the ratio of signal power to noise power. Under H_0, with clutter background, λ equals $\lambda_0(1+\text{CNR})$.

At the fusion center, applying a likelihood ratio test (LRT) to the hypotheses of (1) yields

$$LR = \frac{\prod_{i=1}^{2} f_{X_{0i}}(x_{0i} \mid H_1)}{\prod_{i=1}^{2} f_{X_{0i}}(x_{0i} \mid H_0)} \underset{H_0}{\overset{H_1}{\gtrless}} T_L \tag{2}$$

where T_L is an appropriate threshold. Simplifying (2) yields

$$X = \sum_{i=1}^{2} X_{0i} \underset{H_0}{\overset{H_1}{\gtrless}} \frac{\ln\left(T_L \dfrac{\lambda_1}{\lambda_0}\right)}{\left(\dfrac{1}{\lambda_0} - \dfrac{1}{\lambda_1}\right)}. \tag{3}$$

The above LRT cannot be realized since λ_1 and λ_0 are unknown. However, a CFAR test can be constructed if X is compared against a constant times $g(Z_1,Z_2)$, provided that $g(\cdot)$ is chosen in such a way that λ_0 is the scale parameter of the density of $g(\cdot)$. That is, the density of the random variable $g(Z_1,Z_2)/\lambda_0$ is independent of λ_0. The proposed CFAR test is based on

$$X = \sum_{i=1}^{2} X_{0i} \underset{H_0}{\overset{H_1}{\gtrless}} T g(Z_1, Z_2) \tag{4}$$

where T is a scaling parameter that is adjusted to yield a desired false alarm rate under homogeneous background noise. Since the left-hand side of (4) represents a sufficient statistic of the LRT, the proposed test combines X_{01} and X_{02} in an optimum manner. Because X_{0i} has an exponential distribution, X is a random variable with a gamma distribution whose parameters are 2 and $(1/\lambda)$. The general form of a gamma probability density function (pdf) with parameters α and β is

$$f(x) = \frac{1}{\Gamma(\alpha)} \beta^{\alpha} x^{\alpha-1} e^{-\beta x}, \qquad x \geq 0, \quad \alpha > 0, \quad \beta > 0 \tag{5}$$

where $\Gamma(\alpha)$ is the gamma function. From (4) we can describe the probability of false alarm P_f as

$$P_f = E_{(Z_1,Z_2)}[P(X \geq T g(Z_1,Z_2) \mid H_0] \tag{6}$$

where $E_{(Z_1,Z_2)}[\cdot]$ represents the expectation with respect to Z_1, Z_2. Hence,

$$\begin{aligned} P_f &= \int_0^{\infty} (P(X \geq Tg \mid H_0, g(Z_1,Z_2) = g)) f_{G|H_0}(g)\, dg \\ &= \int_0^{\infty} \left(\int_{Tg}^{\infty} \frac{1}{\lambda_0^2} x e^{-(1/\lambda_0)x}\, dx \right) f_G(g)\, dg \end{aligned} \tag{7}$$

where we have used the fact that X and $g(Z_1,Z_2)$ are statistically independent and that $f_{G|H_0}(g) = f_G(g)$.

B. Two Sensors and Homogeneous Background Noise

We denote the probability of false alarm in the case of homogeneous background noise for MOS by P_{fMH}, and by P_{fmH} for mOS.

1) *MOS Detector Performance*: For the MOS detector, $g(Z_1,Z_2) = R$ is the estimate of the noise power of the test cells. We use (7) to derive an expression which indicates the relationship between P_{fMH} and T. The pdf of R can be expressed as ([16, pp. 139–140])

$$f_R(r) = f_{Z_1}(r) F_{Z_2}(r) + f_{Z_2}(r) F_{Z_1}(r) \tag{8}$$

where ([17, pp. 10–12])

$$\begin{aligned} f_{Z_i}(r) &= k_i \binom{m_i - 1}{k_i} [F(r)]^{k_i - 1} [1 - F(r)]^{(m_i - 1) - k_i} f(r) \\ F_{Z_i}(r) &= \sum_{q=k_i}^{m_i - 1} \binom{m_i - 1}{q} [F(r)]^q [1 - F(r)]^{(m_i - 1) - q} \end{aligned} \tag{9}$$

$k_1 = k$ and $k_2 = l$. With the assumed exponential densities for the Y_{ij}s, we can write (9) as

$$f_{Z_i}(r) = k_i \binom{m_i - 1}{k_i} \left[1 - \exp\left(-\frac{r}{\lambda_0}\right)\right]^{k_i - 1} \times \left[\exp\left(-\frac{r}{\lambda_0}\right)\right]^{(m_i-1)-k_i} \frac{1}{\lambda_0} \exp\left(-\frac{r}{\lambda_0}\right)$$

$$F_{Z_i}(r) = \sum_{q=k_i}^{m_i-1} \binom{m_i - 1}{q} \left[1 - \exp\left(-\frac{1}{\lambda_0} r\right)\right]^{q} \times \left[\exp\left(-\frac{1}{\lambda_0} r\right)\right]^{(m_i-1)-q} \tag{10}$$

where exp(·) represents the exponential function $e^{(\cdot)}$. Using (8) through (10) yields

$$\begin{aligned} f_R(r) = {} & \frac{1}{\lambda_0} k \binom{m_1 - 1}{k} \sum_{s=l}^{m_2-1} \binom{m_2 - 1}{s} \\ & \times \left[1 - \exp\left(-\frac{r}{\lambda_0}\right)\right]^{(k+s-1)} \\ & \times \left[\exp\left(-\frac{r}{\lambda_0}\right)\right]^{[M-(k+s)+1]} \\ & + \frac{1}{\lambda_0} l \binom{m_2 - 1}{l} \sum_{q=k}^{m_1-1} \binom{m_1 - 1}{q} \\ & \times \left[1 - \exp\left(-\frac{r}{\lambda_0}\right)\right]^{(l+q-1)} \\ & \times \left[\exp\left(-\frac{r}{\lambda_0}\right)\right]^{[M-(l+q)+1]} \end{aligned} \tag{11}$$

where $M = [(m_1 - 1) + (m_2 - 1)]$. In order to find P_{fMH} we evaluate the inner integral of (7) and write

$$P_{fMH} = \int_0^\infty \exp\left(-\frac{T}{\lambda_0} r\right) f_R(r)\, dr + \frac{T}{\lambda_0} \int_0^\infty r \exp\left(-\frac{T}{\lambda_0} r\right) f_R(r)\, dr. \tag{12}$$

Upon denoting the first and the second terms in (12) by Φ and Λ, we have

$$P_{fMH} = \Phi + \Lambda. \tag{13}$$

After performing the appropriate integration and straightforward simplifications, we obtain

$$\Phi = k \binom{m_1 - 1}{k} S_1 + l \binom{m_2 - 1}{l} S_2 \tag{14}$$

where

$$S_1 = \sum_{s=l}^{m_2-1} \binom{m_2 - 1}{s} \Gamma(k+s) \frac{\Gamma[T + M - (k+s) + 1]}{\Gamma(T + M + 1)}$$

$$S_2 = \sum_{q=k}^{m_1-1} \binom{m_1 - 1}{q} \Gamma(l+q) \frac{\Gamma[T + M - (l+q) + 1]}{\Gamma(T + M + 1)}. \tag{15}$$

While evaluating S_1 and S_2 numerically, the individual gamma functions in (15) may assume large values. Hence, for numerical purposes, (15) needs to be rewritten using the identities:

$$\binom{m_2 - 1}{s} \Gamma(k+s) = \left(\prod_{i=0}^{s-1} [(m_2 - 1) - i]\right) \times \left(\prod_{j=0}^{k-2} [s + (k-1) - j]\right) \tag{16}$$

$$\frac{\Gamma[T + M - (k+s) + 1]}{\Gamma(T + M + 1)} = \prod_{r=0}^{(k+s)-1} \frac{1}{(T + M - r)} \tag{17}$$

as

$$S_1 = \sum_{s=l}^{m_2-1} \left(\prod_{i=0}^{s-1} \frac{[(m_2 - 1) - i]}{(T + M - i)}\right) \left(\prod_{j=0}^{k-2} \frac{[s + (k-1) - j]}{[T + M - (j+s)]}\right) \times \left(\frac{1}{[T + M - (k+s-1)]}\right)$$

$$S_2 = \sum_{q=k}^{m_1-1} \left(\prod_{i=0}^{q-1} \frac{[(m_1 - 1) - i]}{(T + M - i)}\right) \left(\prod_{j=0}^{l-2} \frac{[q + (l-1) - j]}{[T + M - (j+q)]}\right) \times \left(\frac{1}{[T + M - (l+q-1)]}\right). \tag{18}$$

To evaluate the second term of (13) we use (16) and (17) and the identities

$$(x - y)^n = \sum_{i=0}^{n} (-1)^i \binom{n}{i} x^{n-i} y^i \tag{19}$$

$$\int_0^\infty \left(\frac{x}{\beta}\right)^{\alpha-1} \exp\left(-\frac{1}{\beta} x\right) \left(\frac{1}{\beta}\right) dx = \Gamma(\alpha) \tag{20}$$

to obtain

$$\Lambda = T\left\{k \binom{m_1 - 1}{k} S_3 + l \binom{m_2 - 1}{l} S_4\right\} \tag{21}$$

where

$$S_3 = \sum_{s=l}^{m_2-1}\sum_{d=0}^{(k+s)-1}\left(\prod_{h=0}^{s-1}\frac{[(m_2-1)-h]}{s-h}\right)\times\left(\prod_{a=0}^{d-1}\frac{[(k+s)-1]-a}{d-a}\right)\times\left(\frac{(-1)^d}{[T+M-(k+s)+d+1]^2}\right)$$

$$S_4 = \sum_{q=k}^{m_1-1}\sum_{c=0}^{(l+q)-1}\left(\prod_{u=0}^{q-1}\frac{[(m_1-1)-u]}{q-u}\right)\times\left(\prod_{\nu=0}^{c-1}\frac{[(l+q)-1]-\nu}{c-\nu}\right)\times\left(\frac{(-1)^c}{[T+M-(l+q)+c+1]^2}\right). \quad (22)$$

Therefore,

$$P_{fMH} = k\binom{m_1-1}{k}(S_1+TS_3) + l\binom{m_2-1}{l}(S_2+TS_4). \quad (23)$$

Using (23), T can be adjusted by a numerical search to achieve a desired P_{fMH}. Using (4)–(6) and the definition of the detection probability, it can be seen that the probability of detection, P_{dMH}, is obtained from P_{fMH} by replacing T with $(T/1+\text{SNR})$.

2) *mOS Detector Performance*: The pdf of W is given by ([16, pp. 139–140])

$$f_W(w) = f_{Z_1}(w) + f_{Z_2}(w) - [f_{Z_1}(w)F_{Z_2}(w) + f_{Z_2}(w)F_{Z_1}(w)]. \quad (24)$$

The expression within the brackets is the pdf of the $\max(Z_1, Z_2) = R$. Therefore

$$f_W(w) = f_{Z_1}(w) + f_{Z_2}(w) - f_R(w). \quad (25)$$

From (7) we can write

$$P_{fmH} = \int_0^\infty \left[\int_{Tw}^\infty \frac{1}{\lambda_0^2} x\exp[-(1/\lambda_0)x]\,dx\right] f_W(w)\,dw. \quad (26)$$

Using (25), (26), and (12)

$$P_{fmH} = \int_0^\infty \left[\int_{Tw}^\infty \frac{1}{\lambda_0^2} x\exp[-(1/\lambda_0)x]\,dx\right] \times (f_{Z_1}(w) + f_{Z_1}(w))\,dw - P_{fMH}. \quad (27)$$

Upon evaluation of the inner integral in (27),

$$P_{fmH} = \int_0^\infty \left[\left(1+\frac{Tw}{\lambda_0}\right)\exp\left(-\frac{Tw}{\lambda_0}\right)\right] \times (f_{Z_1}(w) + f_{Z_2}(w))\,dw - P_{fMH}. \quad (28)$$

The computational steps involved in finding P_{fmH} from (28) are very much similar to the ones stated earlier for P_{fMH}. We simply state the final expression for P_{fmH} as

$$P_{fmH} = T\left\{k\binom{m_1-1}{k}S_5 + l\binom{m_2-1}{l}S_6\right\} + \prod_{i=0}^{k-1}\frac{((m_1-1)-i)}{(T+(m_1-1)-i)} + \prod_{j=0}^{l-1}\frac{((m_2-1)-j)}{(T+(m_2-1)-j)} - P_{fMH} \quad (29)$$

where

$$S_5 = \sum_{i=0}^{k-1}\prod_{c=0}^{i-1}(-1)^i\frac{[(k-1)-c]}{(i-c)[T+(m_1-1)-k+i+1]^2}$$

$$S_6 = \sum_{j=0}^{l-1}\prod_{h=0}^{j-1}(-1)^j\frac{[(l-1)-h]}{(j-h)[T+(m_2-1)-l+j+1]^2} \quad (30)$$

and P_{fMH} is given by (23). To compute the probability of detection, T is replaced with $(T/1+\text{SNR})$ in (29) and (30), and P_{fMH} by P_{dMH} in (29).

C. Two Sensors and Nonhomogeneous Background Noise

Samples in the reference window of a search radar are considered to be from a nonhomogeneous background noise when signal returns are either from a multiple-target environment, or from a region with nonuniform clutter within the range cells. The effect of this nonhomogeneity on signal detection appears either as an increase in the probability of false alarm, or as target masking.

In this section we analyze the false alarm and the detection performances of the MOS and the mOS detectors in a multiple-target situation or regions of clutter transitions. We assume a clutter model with a step-type behavior. That is, at a sensor, all the reference cells to the left of the point of discontinuity of the step have a common mean noise (clutter) power and all the cells to the right have another common mean noise (clutter) power. We did not consider the situation when signal returns are from clutter plus multiple targets region. We use the symbols P_{fM} and P_{dM} to denote the probabilities of false alarm and detection of the MOS detector, and use the symbols P_{fm} and P_{dm} to denote the corresponding quantities for the mOS detector.

1) *MOS Detector Performance*: First consider the multiple targets environment with test samples from the noise-only region. We use INR as the ratio of interfering target power to noise power. The pdf and the CDF of R are now different from those in

the homogeneous background noise case. To compute P_{fM}, we use an equivalent form of (7)

$$P_{fM} = \int_0^\infty \left(\frac{T}{\lambda_0}\right)^2 r \exp\left(-\frac{T}{\lambda_0} r\right) F_R(r)\, dr. \qquad (31)$$

Clearly,

$$F_R(r) = F_{Z_1}(r) F_{Z_2}(r). \qquad (32)$$

To write expressions for $F_{z_i}(r)$, $i = 1,2$, let b_i be the number of samples from interfering targets with cumulative distribution function (cdf) $F_1(\cdot)$ for the ith sensor. Then, $[(m_i - 1) - b_i]$ of the observations are due only to noise and have cdf $F(\cdot)$. $F_1(\cdot)$ is given by

$$F_1(r) = 1 - \exp\left(-\frac{1}{\lambda_0(1+\mathrm{INR})} r\right). \qquad (33)$$

It can be shown that for the ith sensor [18]

$$\begin{aligned} F_{Z_i}(r) = \sum_{h=k_i}^{(m_i-1)} \sum_{j=\max(0,h-b_i)}^{\min[h,((m_i-1)-b_i)]} & \binom{(m_i-1)-b_i}{j} \\ & \times [F(r)]^j [1-F(r)]^{[(m_i-1)-b_i-j]} \\ & \times \binom{b_i}{h-j} [F_1(r)]^{(h-j)} [1-F_1(r)]^{[b_i-(h-j)]} \end{aligned} \qquad (34)$$

where, $k_1 = k$, $k_2 = l$. Hence

$$\begin{aligned} F_R(r) = \sum_{i,j,s,\nu} & \binom{(m_1-1)-b_1}{j} \binom{(m_2-1)-b_2}{\nu} \binom{b_1}{i-j} \\ & \times \binom{b_2}{s-\nu} \left[\exp\left(-\frac{1}{\lambda_0} r\right)\right]^{[M-(b_1+b_2)-(j+\nu)]} \\ & \times \left[\exp\left(-\frac{r}{\lambda_0(1+\mathrm{INR})}\right)\right]^{[(b_1+b_2)+(j+\nu)-(i+s)]} \\ & \times \left[1-\exp\left(-\frac{r}{\lambda_0}\right)\right]^{(j+\nu)} \\ & \times \left[1-\exp\left(-\frac{r}{\lambda_0(1+\mathrm{INR})}\right)\right]^{[i+s-(j+\nu)]} \end{aligned} \qquad (35)$$

where

$$k \le i \le (m_1 - 1), \qquad \max(0, i-b_1) \le j \le \min(i, [(m_1-1)-b_1]) \qquad (36)$$

$$l \le s \le (m_2 - 1), \qquad \max(0, s-b_2) \le \nu \le \min(s, [(m_2-1)-b_2]).$$

Substituting (35) in (31), and after some mathematical simplifications,

$$P_{fM} = \sum_{i,j,s,\nu,p,n} \left\{ \frac{(-1)^{p+n}\mathrm{PRD}}{\left[\frac{[M-(b_1+b_2)-(j+\nu)]+p}{T} + 1 + \frac{[(b_1+b_2)-(i+s)+(j+\nu)]+n}{T(1+\mathrm{INR})}\right]^2} \right\} \qquad (37)$$

where i, j, s, and ν are given by (36), $0 \le p \le (j+\nu)$, and $0 \le n \le [(i+s)-(j+\nu)]$. Also in (37), the symbol PRD is given by

$$\begin{aligned} \mathrm{PRD} = & \left(\prod_{a=0}^{j-1} \frac{[((m_1-1)-b_1)-a]}{a+1}\right) \\ & \times \left(\prod_{c=0}^{\nu-1} \frac{[((m_2-1)-b_2)-c]}{c+1}\right) \\ & \times \left(\prod_{h=0}^{(i-j)-1} \frac{(b_1-h)}{h+1}\right) \left(\prod_{d=0}^{(s-\nu)-1} \frac{(b_2-d)}{d+1}\right) \\ & \times \left(\prod_{q=0}^{p-1} \frac{(j+\nu)-q}{q+1}\right) \left(\prod_{r=0}^{n-1} \frac{[(i+s)-(j+\nu)-r]}{r+1}\right). \end{aligned} \qquad (38)$$

For calculating P_{dM} we replace T with $(T/1+\mathrm{SNR})$ in (37).

The false alarm performance in the region of clutter power transitions can be analyzed using (37). In this situation b_i is the number of clutter cells in the reference window for the ith sensor. For step-type clutter, there exists a single transition from a noise-only region to a region with higher clutter-plus-noise power. Since the reference samples are rank-ordered at each sensor, then if b_i is less than $(m_i - 1)/2$, the test cell is in the clutter-free region. Otherwise, it is in the clutter. When test samples are from the clutter-free region, P_{fM} is obtained by replacing INR with CNR. In the case when the test samples are from the clutter region, P_{fM} is obtained by changing INR to CNR and T to $(T/1+\mathrm{CNR})$ in (37).

2) *mOS Detector Performance*: In this case

$$\begin{aligned} F_W(w) &= F_{Z_1}(w) + F_{Z_2}(w) - F_{Z_1}(w)F_{Z_2}(w) \\ &= F_{Z_1}(w) + F_{Z_2}(w) - F_R(w). \end{aligned} \qquad (39)$$

Using (39) and (31), with $F_W(\cdot)$ replacing $F_R(\cdot)$,

$$\begin{aligned} P_{fm} = \int_0^\infty & \left(\frac{T}{\lambda_0}\right)^2 r \exp\left(-\frac{T}{\lambda_0} r\right) \\ & \times [F_{Z_1}(w) + F_{Z_2}(w)]\, dw - P_{fM}. \end{aligned} \qquad (40)$$

First, we consider P_{fm} for the multiple-target environment. We use (33) and (34) to obtain $F_{Z_i}(w)$,

$i = 1,2$, and substitute the result in (40). Then

$$P_{fm} = \sum_{i,j,p,n} \left\{ \frac{(-1)^{p+n} \binom{(m_1-1)-b_1}{j} \binom{b_1}{i-j} \binom{j}{p} \binom{i-j}{n}}{\left[\frac{(m_1-1)-b_1]-j+p}{T} + 1 + \frac{[b_1-(i-j)]+n}{T(1+\text{INR})}\right]^2} \right\} + \sum_{s,\nu,j_1,j_2} \left\{ \frac{(-1)^{j_1+j_2} \binom{(m_2-1)-b_2}{\nu} \binom{b_2}{s-\nu} \binom{\nu}{j_1} \binom{s-\nu}{j_2}}{\left[\frac{[(m_2-1)-b_2]-\nu+j_1}{T} + 1 + \frac{[b_2-(s-\nu)]+j_2}{T(1+\text{INR})}\right]^2} \right\} - P_{fM} \quad (41)$$

where i, j, s, and ν are given by (36), $0 \le p \le j$, $0 \le n \le (i-j)$, $0 \le j_1 \le \nu$, and $0 \le j_2 \le (s-\nu)$. The detection performance analysis for the multiple targets case, and the false alarm analysis for the clutter power transition situation can both be done by adjusting the appropriate parameters in (41), as discussed in Section IIC1.

D. Extension to an N-Sensor Network

We generalize the proposed distributed CFAR detector of Section IIA to the case of N distributed sensors. We follow the notation developed in Section IIA, except that the numbering of the sensors is extended to N, so $i = 1,2,\ldots,N$. Each sensor transmits the statistic $Z_i = Y_{i(k_i)}$ and the test sample X_{0i} to the fusion center.

At the fusion center, let the noise power estimate $g(Z_1,\ldots,Z_N)$ be the kth OS of the $\{Z_i,\ i = 1,\ldots,N\}$. For notational convenience let $g(Z_1,Z_2,\ldots,Z_N) = Z_{(k)} = V$. The fusion center test is given by

$$X = \sum_{i=1}^{N} X_{0i} \underset{H_0}{\overset{H_1}{\gtrless}} TV \quad (42)$$

where T is an appropriate scaling constant. The probability of false alarm, P_{fNH}, of the test (42) is written as

$$P_{fNH} = \int_0^\infty (P(X \ge T\nu \mid H_0)) f_{V|H_0}(\nu)\, d\nu = \int_0^\infty (1 - F_{X|H_0}(T\nu)) f(\nu)\, d\nu \quad (43)$$

where

$$F_{X|H_0}(T\nu) = 1 - \left[\sum_{i=0}^{N-1} \frac{(T\nu)^i}{\lambda_0^i i!}\right] e^{-(1/\lambda_0)T\nu}. \quad (44)$$

To compute (43), we need to determine the pdf of the random variable V. An expression for the density of V can be obtained using the permanent (defined like the determinant of a square matrix except that all signs are positive) of the square N-by-N matrix $\mathbf{V}$ [19]:

$$\mathbf{V} = \begin{bmatrix} F_{Z_1}(\nu) & \cdots & F_{Z_N}(\nu) \\ \vdots & & \vdots \\ F_{Z_1}(\nu) & \cdots & F_{Z_N}(\nu) \\ f_{Z_1}(\nu) & \cdots & f_{Z_N}(\nu) \\ 1-F_{Z_1}(\nu) & \cdots & 1-F_{Z_N}(\nu) \\ \vdots & & \vdots \\ 1-F_{Z_1}(\nu) & \cdots & 1-F_{Z_N}(\nu) \end{bmatrix} \begin{matrix} \left.\right\} (k-1) \text{ rows} \\ \\ \left.\right\} (N-k) \text{ rows} \end{matrix}. \quad (45)$$

Thus

$$f_V(\nu) = \frac{1}{(N-k)!(k-1)!} \overset{+}{|}\mathbf{V}\overset{+}{|} \quad (46)$$

where $F_{Z_i}(\nu)$ and $f_{Z_i}(\nu)$ are given by (10) and $\overset{+}{|}\mathbf{V}\overset{+}{|}$ denotes the permanent of the matrix $\mathbf{V}$. For multiple-target environment, $F_{Z_i}(\nu)$ is evaluated as in (34), assuming there are b_i number of interfering targets within the resolution cells of the ith sensor.

III. NUMERICAL RESULTS

In this section we discuss the numerical results obtained from an evaluation of the performance equations of the MOS and the mOS detectors (Section II), the central order statistic detector (Appendix A), the distributed CFAR and the AND (Appendix B), and the OR (Appendix C) detectors. We mention below various ranges of parameters over which the comparisons are made. Section IIIA provides detector comparisons under homogeneous background noise and Section IIIB provides the same under interfering target or nonhomogeneous background situation.

For a two-sensor network, our numerical analysis is carried out for the following specific values of the various parameters. The number of cells (noise plus

TABLE I
Calculated Values for Constant Multiplier T for $P_f = 10^{-6}$

$m_1 = 12$	$k = 8$	$m_2 = 14$	$l = 9$	$M = 24$
Distributed	OS-CFAR	New	Scheme	COS-CFAR
AND Fusion Rule $P_{f_1} = P_{f_2} = 10^{-3}$	OR Fusion Rule $P_{f_1} = P_{f_2} = 5.0 \times 10^{-7}$	$Max(Z_1, Z_2)$	$Min(Z_1, Z_2)$	$Y_{(17)}$
$T_1 = 9.55527$ $T_2 = 9.72534$	$T_1 = 36.26238$ $T_2 = 34.2302$	$T = 21.96$	$T = 47.30013$	$T_C = 23.6455$

test) at sensor 1 is $m_1 = 12$, and at sensor 2, $m_2 = 14$. Thus, the total number of reference cells is

$$M = (m_1 - 1) + (m_2 - 1) = 24.$$

The noise estimate at sensor 1 is the kth order statistic where $k = 8$, and at sensor 2, the $l(= 9)$th order statistic is used. The pth order statistic is used for the COS-CFAR where $p = k + l = 17$.

We solved (23) and (29) for T through a numerical search such that $P_{fMH} = P_{fmH} = 10^{-6}$, for the MOS and the mOS detectors (see Table I). For COS-CFAR, (50) was used to determine T_C.

In the case of the AND fusion rule, (54) was solved numerically to fix T_1 and T_2 at values corresponding to $P_{f_1} = P_{f_2} = 10^{-3}$, so that the overall designed false alarm rate, P_{fAH}, is set at 10^{-6} (Table I). We chose $P_{f_1} = P_{f_2}$, since the sensors have no a priori knowledge about the number of interferers, and any asymmetric design values of P_{f_1} and P_{f_2} (for example 10^{-4}, 10^{-2}) will not be optimum for all situations. Furthermore, as Fig. 10 indicates, in the case of 6 and 7 clutter cells at sensors 1 and 2, when $P_{f_1} = 10^{-4}$ and $P_{f_2} = 10^{-2}$, there is no significant false alarm performance improvement over the case when $P_{f_1} = P_{f_2} = 10^{-3}$. The marginal gain in the false alarm rate observed is at the expense of degradation in detection performance. Similarly, we solved (58) to fix the values for T_1 and T_2 in the case of the OR fusion rule (Table I). To obtain an overall false alarm rate of 10^{-6}, we set $P_{f_1} = P_{f_2} = 5.0 \times 10^{-7}$.

A. Detector Comparisons. Homogeneous Background Noise

The detection performances of all the CFAR detectors, in the case of homogeneous background noise, are shown in Fig. 2. The better performance of the MOS detector followed by the mOS, particularly for SNR in the range of 10–20 dB, over the OS-CFAR with the AND and the OR fusion rules, can be observed. Fig. 2 also indicates that the performances of the COS-CFAR and the MOS detector are very close to each other.

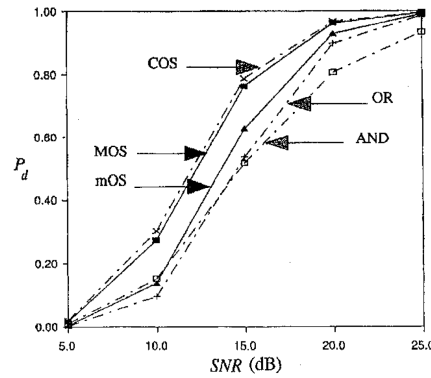


Fig. 2. Probability of detection versus SNR when background noise is homogeneous.

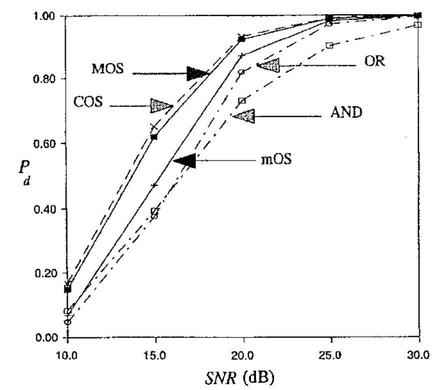


Fig. 3. Probability of detection versus SNR (number of interfering targets at each sensor is $b_1 = b_2 = 2$, and INR = SNR).

B. Detector Comparison. Effect of Interfering Targets

For the order statistic based schemes, the maximum number of tolerable interfering targets depends on the selected rank. For example, if there are N reference samples and the selected rank is k, then $(N - k)$ interfering targets can be tolerated by an order statistic based processor. Figs. 3 and 4 show P_d as a function of SNR when the number of interfering targets at sensors 1 and 2, (b_1, b_2), are (2 2), and (3 4), respectively. For the COS-CFAR, the number of interfering targets b is assumed to be $b_1 + b_2$ throughout this numerical study. Fig. 3 shows that the MOS detector has a marginally lower P_d as compared with the COS-CFAR, but has a much better performance than the other schemes. Fig. 4 shows that the performance of the MOS scheme is competitive with that of the COS-CFAR. Notice that in Fig. 4, $(b_1\ b_2) = (3\ 4)$, so that $b = 7$. These are the maximum tolerable number of interfering targets in order not to have a significant degradation in detection performance. Whereas, in Fig. 3, the number of interfering targets are less than the maximum tolerable value.

To study the effect of an increase in the number of interfering targets on the detection performance, let us consider Figs. 5–8. In Fig. 5, the number of interfering targets at sensor 1 is $b_1 = 2$, the number of interfering

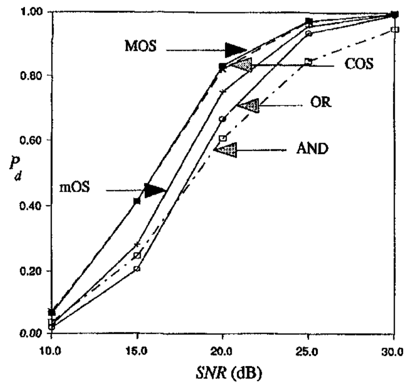


Fig. 4. Probability of detection versus SNR (with maximum number of tolerable interfering targets, that is, at sensor 1 $b_1 = 3$, and at sensor 2 $b_2 = 4$ INR = SNR).

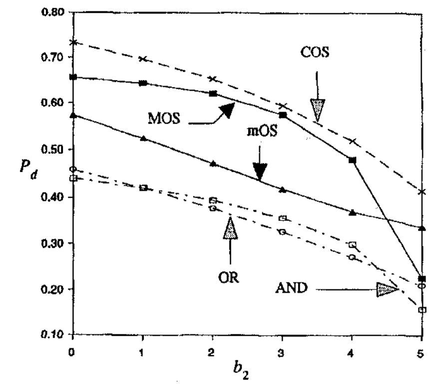


Fig. 5. Probability of detection versus number b_2 of interfering targets at sensor 2 (number of interfering targets at sensor 1 is $b_1 = 2$, and INR = SNR = 15 dB).

targets in sensor 2 (b_2) ranges from 0 to 5, and for the COS-CFAR, $b = 2 + b_2$. For b_2 from 0 through 4, the MOS detector has a considerably better detection performance than the mOS detector. It also performs much better than the OS-CFAR with the AND and the OR fusion rules. Notice that this corresponds to the case

$$b_1 = 2 < (m_1 - 1) - k \qquad \text{and} \qquad b_2 \le (m_2 - 1) - l = 4$$

i.e., b_1 and b_2 are within the tolerable ranges. At $b_2 = 5$, we observe a sharp drop in P_d for the MOS scheme. Also, for $4 \le b = 2 + b_2 \le 6$, the performance of the MOS detector is close to that of the COS-CFAR. Fig. 6 shows P_d as a function of b_2 for $b_1 = 2 < [(m_1 - 1) - k]$ and $b_1 = 4 > [(m_1 - 1) - k]$. There is a significant drop in P_d for the MOS scheme when $b_1 = 4$. The same is true for the mOS detector when $b_2 = 5 > [(m_2 - 1) - l] = 4$ and $b_1 = 4$. In Fig. 7, $b_1 = 3 = [(m_1 - 1) - k]$, and in Fig. 8, $b_1 = 4 > [(m_1 - 1) - k]$. We observe that in Fig. 7, the sharp drop in P_d for the MOS detector occurs when $b_2 = 5 > [(m_2 - 1) - l]$. It also shows that the performance of the MOS scheme is very close to that of the COS-CFAR. Also, notice that the P_d drop for the mOS detector is not as drastic as for the MOS scheme. But, in Fig. 8 for $b_1 = 4 > [(m_1 - 1) - k]$, a significant degradation in P_d also occurs for the mOS test. It is interesting to notice the similarity of the performance characteristics between the mOS scheme and the OR rule and also between the MOS detector and the AND rule. In fact, a counting rule can be considered as a discrete analog of an order statistic based rule [20]. The correspondence becomes an equivalence if the rules are based on fixed thresholds and a nonequivalence if the rules are based on adaptive thresholds, as in the present problem.

The performances of the detectors in multiple targets environment that are presented in Figs. 3–8 show the typical behavior of the order statistic based schemes. That is, their performances depend on the selected rank. For all combinations satisfying:

$$b_1 \le (m_1 - 1) - k \qquad \text{and} \qquad b_2 \le (m_2 - 1) - l$$

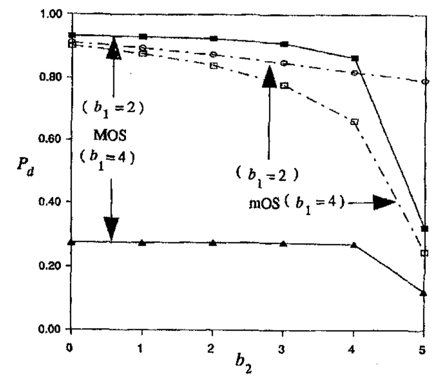


Fig. 6. Probability of detection versus number b_2 of interfering targets at sensor 2 (for two values of number b_1 of interfering targets at sensor 1, and INR = SNR = 20 dB).

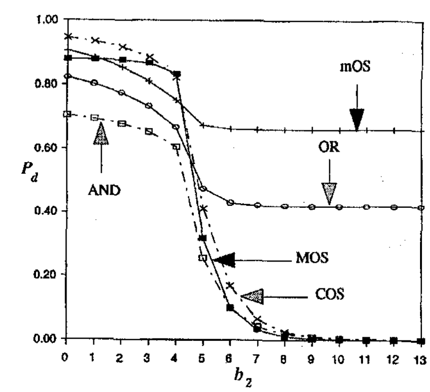


Fig. 7. Probability of detection versus number b_2 of interfering targets at sensor 2 (number b_1 of interfering targets at sensor 1 is 3, and INR = SNR = 20 dB).

the MOS detector followed by the mOS scheme outperforms the OS-CFAR detectors with the AND and the OR fusion rules. The figures also indicate that the detection performance of the MOS detector is close to that of the COS-CFAR. Figs. 8 and 9 show that as long as $b_i \le [(m_i - 1) - k_i]$, for either $i = 1$ or 2, but not both, the mOS detector has a better detection performance than the MOS detector. But, this is at the expense of an increase in the false alarm rate as discussed next.

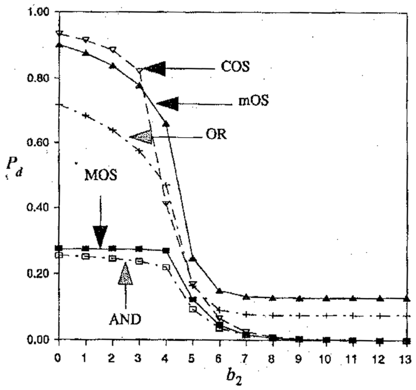


Fig. 8. Probability of detection versus number b_2 of interfering targets at sensor 2 (number b_1 of interfering targets at sensor 1 is 4, and INR = SNR = 20 dB).

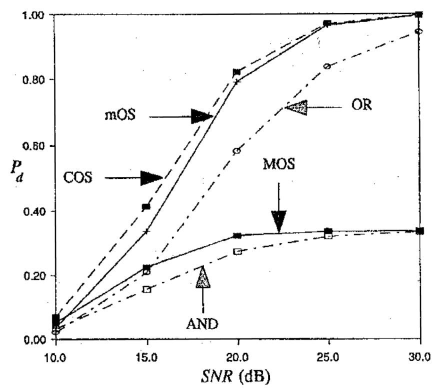


Fig. 9. Probability of detection versus SNR (number b_1 of interfering targets at sensor 1 is 2, and at sensor 2 $b_2 = 5$. INR = SNR.)

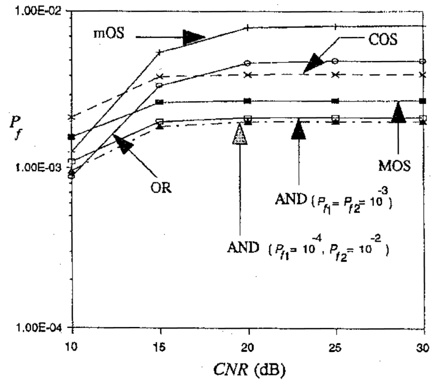


Fig. 10. False alarm performance of each detector (both test cells in clutter, sensor 1 has 6 clutter cells and sensor 2 has 7).

Fig. 10 shows the maximum increase in the false alarm probability corresponding to the worst case situation when there are 6 ($= b_1$) and 7 ($= b_2$) clutter cells at sensors 1 and 2, respectively, and the test samples are from the clutter region for the different schemes. It can be seen that for CNR between 10–12 dB, the different graphs follow each other closely, and for CNR = 15 dB or greater, the performances of the AND fusion rule followed by the MOS scheme are better than the others. It should be emphasized that although the false alarm performance of the AND fusion rule is marginally better than that of the MOS detector, as seen earlier, the latter has a considerably better detection performance.

IV. SUMMARY AND CONCLUSION

In this study we have developed a new S + OS CFAR test using distributed sensors. Our problem formulation has assumed that the test cells of different sensors all have statistically identical noise (clutter), and that if a target is present in the surveillance regions, all the test cells have statistically identical target returns. This requirement implies that all sensors see the same test SNR. In the S + OS scheme, each sensor transmits its test sample and a designated order statistic of its surrounding observations to a fusion center, where the sum of the samples of the test cells is compared with a constant multiplied by a function of the order statistics. For a two-sensor network, the functions considered are the mOS and MOS. For detecting a Rayleigh fluctuating target in Gaussian noise, closed-form expressions for the false alarm and the detection probabilities are obtained. Extension to an N-sensor network is also considered, and general equations for the false alarm probabilities under homogeneous and nonhomogeneous background noise are presented. Performances of these two schemes are compared with those of the distributed CFAR with the AND rule and the OR rule and a COS-CFAR test.

We conclude from the study of a two-sensor network that for the homogeneous background noise, the detection performance of the proposed MOS scheme is very close to that of the COS-CFAR, and is considerably better than those of the OS-CFAR with the AND and the OR fusion rules, particularly at SNR ranging from 10 to 20 dB. In multiple targets situation, the results indicate the following. As long as the number of interfering targets in the two sensors, namely b_1 and b_2, are such that

$$b_1 \leq (m_1 - 1) - k \qquad \text{and} \qquad b_2 \leq (m_2 - 1) - l$$

where $(m_1 - 1)((m_2 - 1))$ is the number of reference samples available at sensor 1 (sensor 2), the MOS detector has a performance closer to the COS-CFAR detector, which is much better than that of the distributed OS-CFAR detector with the AND or the OR fusion rule.

APPENDIX. COMPETING DETECTORS

For a two-sensor network we derive performance equations for different competing detectors. Section A develops the probability of error expressions for the central OS (COS-CFAR) detector discussed in Section I. In Sections B and C we consider the distributed

CFAR scheme when binary decisions of the sensors are sent to the fusion center. At the fusion center, AND or OR rule is used to make a final decision with regard to the presence or the absence of a target. Here, we consider the case when each sensor is an OS-CFAR detector. Barkat and Varshney [12] have considered the distributed CA-CFAR detector. Distributed OS-CFAR scheme is also investigated by Uner and Varshney [14]. In their work, the overall probability of detection for a given fusion rule is maximized by optimizing the constant multiplier and the rank order of the local detectors simultaneously, in a homogeneous background noise. However, the probability of detection of an OS detector in a homogeneous environment is relatively insensitive to the exact rank order used, except for a very small or a very large number. Also, the rank order in an OS detector is usually fixed depending on the expected maximum number of targets [3]. We assume some representative values for the rank orders k and l in the numerical evaluation. We derive expressions for he probabilities of false alarm and detection when the AND or the OR rule is used at the fusion center, assuming independent observations for both cases of homogeneous and nonhomogeneous background noise. The expressions for the AND and the OR rules, for the interfering target case, are new and not available elsewhere.

A. Central OS-CFAR

In a centralized procedure, each sensor transmits all of its observations to a fusion center where a decision is made. A central scheme in general has a better performance in comparison with a decentralized one, since more information is sent to the fusion center. We consider the performance of a COS-CFAR detector in order to assess how good the MOS and the mOS detectors are.

In the COS-CFAR considered here, each sensor of the two sensor network in Fig. 1 sends all the reference samples and the test sample to the fusion center. Hence, the number of the reference samples at the fusion center is $M = (m_1 - 1) + (m_2 - 1)$. Let $Y_1, Y_2, \ldots, Y_M$ denote the reference observations at the fusion center, and $Y_{(1)}, Y_{(2)}, \ldots, Y_{(M)}$ represent the rank-ordered samples. Let $Z = Y_{(p)}$, $p = k + l$, where k and l are defined below (9). The COS-CFAR test is similar to the one in (4):

$$X = \sum_{i=1}^{2} X_{0i} \underset{H_0}{\overset{H_1}{\gtrless}} T_c Z \tag{47}$$

where T_c is an appropriate multiplying constant.

From (47), the probability of false alarm, P_{fCH}, for a homogeneous background noise can be expressed as

$$P_{fCH} = \int_0^{\infty} (1 - F_{X|H_0}(T_C z)) f_Z(z)\, dz \tag{48}$$

where from (10), $f_Z(z)$ can be written by changing k_i to p, $(m_i - 1)$ to M. Also, since X is distributed as a gamma random variable with parameters 2 and $1/\lambda$,

$$F_{X|H_0}(T_C z) = 1 - \left[\sum_{i=0}^{1} \frac{(T_C z)^i}{\lambda_0^i i!}\right] \exp\left(-\left(\frac{1}{\lambda_0}\right) T_C z\right). \tag{49}$$

Using (49) in (48) and upon evaluating the integral,

$$P_{fCH} = p\binom{M}{p} \left\{ \sum_{i=0}^{1} \sum_{j=0}^{p-1} (-1)^j \binom{p-1}{j} \times \left(\frac{T_C^i}{[T_C + M - p + j + 1]^{i+1}} \right) \right\}. \tag{50}$$

In a multiple targets environment, (48) and (49) can be used to write the probability of false alarm P_{fC} as

$$P_{fC} = \int_0^{\infty} \left(\frac{T_C}{\lambda_0}\right)^2 z \exp\left(-\frac{T_C}{\lambda_0} z\right) F_Z(z)\, dz \tag{51}$$

where, $F_Z(z)$ is the cdf of Z when there are a total of b number of interfering targets appearing in the resolution cells of all the sensors. We use (34) to write an expression for $F_Z(z)$. Simplification of (51) then yields

$$P_{fC} = \sum_{i,j,h,d} \left\{ \frac{(-1)^{h+d} \mathrm{PRC}}{\left[\frac{(M-b) - j + h}{T_C} + 1 + \frac{b - (i-j) + d}{T_C(1 + \mathrm{INR})}\right]^2} \right\} \tag{52}$$

where $0 \le h \le j$, $0 \le d \le (i - j)$, $p \le i \le M$, $\max(0, i - b) \le j \le \min(i, (M - b))$, and the symbol PRC is given by

$$\mathrm{PRC} = \left(\prod_{q=0}^{j-1} \frac{(M-b) - q}{q+1}\right) \left(\prod_{r=0}^{(i-j)-1} \frac{b-r}{r+1}\right) \times \left(\prod_{s=0}^{h-1} \frac{j-s}{s+1}\right) \left(\prod_{\nu=0}^{d-1} \frac{(i-j) - \nu}{\nu+1}\right). \tag{53}$$

B. Distributed OS-CFAR Detector with AND Fusion Rule

Let P_{FAH} be the overall false alarm probability when the test samples are from homogeneous background noise. Then [1, 3, 12]

$$P_{FAH} = \prod_{i=1}^{2} P_{f_i} \tag{54}$$

where

$$P_{f_1} = \left(\prod_{n=0}^{k-1} \frac{[(m_1 - 1) - n]}{[T_1 + (m_1 - 1) - n]} \right) \quad (55)$$

$$P_{f_2} = \left(\prod_{j=0}^{l-1} \frac{[(m_2 - 1) - j]}{[T_2 + (m_2 - 1) - j]} \right) \quad (56)$$

and T_1, T_2 are the threshold used by the OS tests at sensor 1 and sensor 2, respectively. To compute P_{DAH}, the overall probability of detection, we replace T_i with $(T_i/1 + \mathrm{SNR})$ in (54).

When signal returns are from multiple-target environment, the reference window for the ith sensor contains b_i interference-plus-noise samples and $[(m_i - 1) - b_i]$ noise-only observations. Considering the case that the test samples are due only to noise, the overall false alarm probability for a multiple-target case is obtained as

$$P_{FA} = \sum_{i,j,i_1,i_2} \left\{ \frac{(-1)^{i_1+i_2} \binom{(m_1-1)-b_1}{j} \binom{b_1}{i-j} \binom{j}{i_1} \binom{i-j}{i_2}}{\left[\frac{[(m_1-1)-b_1] - j + i_1}{T_1} + 1 + \frac{[b_1 - (i-j)] + i_2}{T_1(1+\mathrm{INR})} \right]} \right\} \times \sum_{s,\nu,j_1,j_2} \left\{ \frac{(-1)^{j_1+j_2} \binom{(m_2-1)-b_2}{\nu} \binom{b_2}{s-\nu} \binom{\nu}{j_1} \binom{s-\nu}{j_2}}{\left[\frac{[(m_2-1)-b_2] - \nu + j_1}{T_2} + 1 + \frac{[b_2 - (s-\nu)] + j_2}{T_2(1+\mathrm{INR})} \right]} \right\} \quad (57)$$

where $0 \le i_1 \le j$, $0 \le i_2 \le (i-j)$, $0 \le j_1 \le \nu$, $0 \le j_2 \le (s-\nu)$ and i, j, s, and ν are given by (36). The overall detectionprobability P_{DA} is obtained when T_i is replaced with $(T_i/1 + \mathrm{SNR})$ in (57).

C. Distributed OS-CFAR Detector with OR Fusion Rule

Let P_{FR} be the false alarm probability when the OR rule is used at the fusion center. Then

$$P_{FR} = \sum_{i=1}^{2} P_{f_i} - \prod_{i=1}^{2} P_{f_i}. \quad (58)$$

In the case of homogeneous background noise, P_{FRH} is obtained by using (54)–(56) and (58). From P_{FRH} we compute P_{DRH}, the probability of detection, by replacing T_i with $(T_i/1 + \mathrm{SNR})$.

For multiple-target situation, the false alarm probability P_{FR} is obtained using (58) with

$$P_{f_1} = \sum_{i,j,i_1,i_2} [\cdot], \qquad P_{f_2} = \sum_{s,\nu,j_1,j_2} [\cdot]$$

where $\sum_{i,j,i_1,i_2}[\cdot]$ and $\sum_{s,\nu,j_1,j_2}[\cdot]$ are as stated in (57).

REFERENCES

[1] Rohling, H. (1983)
Radar CFAR thresholding in clutter and multiple target situations.
IEEE Transactions on Aerospace and Electronic Systems, **AES-19** (July 1983), 608–621.

[2] Rohling, H. (1984)
New CFAR processor based on order statistic.
In *Proceedings of the IEEE International Radar Conference*, Paris, 1984, 38–42.

[3] Gandhi, P., and Kassam, S. A. (1988)
Analysis of CFAR processors in nonhomogeneous background.
IEEE Transactions on Aerospace and Electronic Systems, **24** (July 1988), 427–445.

[4] Viswanathan, R., and Eftekhari, A. (1992)
A selection and estimation test for multiple targets in clutter detection.
IEEE Transactions on Aerospace and Electronic Systems, **28** (Apr. 1992), 505–519.

[5] Barkat, M., and Varshney, P. K. (1987)
A weighted cell-averaging CFAR detector for multiple target situations.
In *Proceedings of the Twenty-First Annual Conference on Information Sciences and Systems*, The Johns Hopkins University, Baltimore, MD, Mar. 1987, 118–123.

[6] Nitzberg, R. (1992)
Adaptive Signal Processing for Radar.
Norwood, MA: Artech House, 1992.

[7] Ritcy, J. A. (1986)
Censored mean-level detector analysis.
IEEE Transactions on Aerospace and Electronic Systems, **22** (July 1986), 443–454.

[8] Barkat, M., Himonas, S. D., and Varshney, P. K. (1989)
CFAR detection for multiple target situations.
IEE Proceedings, **136**, Pt. F (Oct. 1989), 193–209.

[9] Levanon, N. (1988)
Detection loss due to interfering targets in order statistic CFAR.
IEEE Transactions on Aerospace and Electronic Systems, **24** (Nov. 1988), 678–681.

[10] Schmieder, D. E., and Weathersby, M. R. (1983)
Detection performance in clutter with variable resolution.
IEEE Transactions on Aerospace and Electronic Systems, **AES-19** (July 1983), 622–630.

[11] Dasarathy, B. V. (1994)
Decision Fusion.
Los Alamitos, CA: IEEE Computer Society Press, 1994.

[12] Barkat, M., and Varshney, P. K. (1989)
Decentralized CFAR signal detection.
IEEE Transactions on Aerospace and Electronic Systems, **25** (Mar. 1989), 141–148.

[13] Barkat, M., and Varshney, P. K. (1991)
Adaptive cell-averaging CFAR detection in distributed sensor networks.
IEEE Transactions on Aerospace and Electronic Systems, **27** (May 1991), 424–429.

[14] Uner, M. K., and Varshney, P. K. (1993)
Decentralized CFAR detection based on order statistics.
In *Proceedings of the 36th Midwest Symposium on Circuits and Systems*, Detroit, MI, Aug. 1993, 146–149.

[15] Blum, R. S., and Kassam, S. A. (1993)
Distributed cell-averaging CFAR detection of dependent signal returns.
In *Proceedings of 1993 IEEE International Symposium on Information Theory*, Jan. 1993, p. 12.

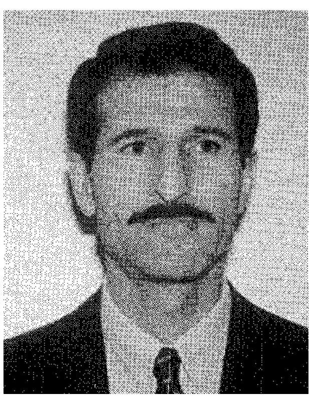
[16] Papoulis, P. (1984)
Probability, Random Variables, and Stochastic Processes.
New York: McGraw-Hill, 1984.

[17] Arnold, B. C., Balakrishnan, N., and Nagaraja, H. N. (1992)
A First Course in Order Statistics.
New York: Wiley, 1992.

[18] Peterson, S. R., Lee, Y. H., and Kassam, S. A. (1988)
Some statistical properties of α-trimmed mean and standard type M-filters.
IEEE Transactions on Acoustics, Speech and Signal Processing, **36** (May 1988), 707–713.

[19] Vaughan, R. J., and Venables, W. N. (1972)
Permanent expressions for order statistic densities.
Journal of the Royal Statistical Society, **34** (1972), 308–310.

[20] Viswanathan, R., and Ansari, A. (1989)
Distributed detection of a signal in generalized Gaussian noise.
IEEE Transactions on Acoustics, Speech and Signal Processing, **37** (May 1989), 775–778.



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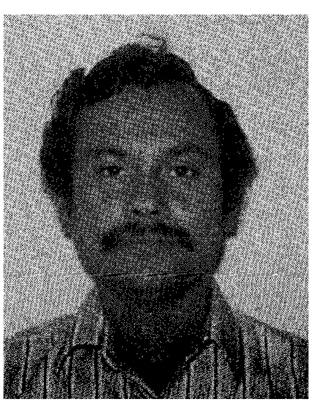

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