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Diversity Combining in Antenna Array Base Station Receiver for DS/CDMA System

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***Abstract*— We evaluate using simulation studies the performance of several schemes for combining base station antenna array signals in wireless direct sequence code-division multiple access (DS/CDMA). The results indicate that under certain assumptions, on multiple access interference statistics, the probability of error of modified rank test (MRT) is lower than that of equal gain combining (EGC), if a few high power interfering users are present along with a low power user of interest. If there are a moderately large number of users, and if the received power of all the users are nearly the same, then EGC out performs MRT. In fact, under this condition, the performance of EGC is close to that of the optimal likelihood ratio test (LRT).**

***Index Terms*—Antenna array, diversity combining, DS/CDMA, rank test.**

I. Introduction

DIRECT SEQUENCE code-division multiple access (DS/CDMA) is an alternative to frequency division or time division multiple access scheme based cellular networks [1], [2]. In [1], for the IS-95 cellular standard, an antenna array 2-D noncoherent RAKE receiver with equal gain combining (EGC) as the decision rule was considered. Further details of this receiver can be found in [1].

The total received signal at the base station in the IS-95 mobile radio environment is given by [1]

$$\begin{aligned}\boldsymbol{x}(t) = \sum_{i=1}^{N}\sum_{l=1}^{L_i} \rho_i \sqrt{P_i}\psi_i \Big[& W^{(h)}(t-\tau_{l,i})\, a_i^I(t-\tau_{l,i}) \\ & + jW^{(h)}(t-T_0-\tau_{l,i}) a_i^Q(t-T_0-\tau_{l,i})\Big] \\ & \cdot [\cos(\theta_{l,i}) + j\sin(\theta_{l,i})]\boldsymbol{a}_{l,i} + \boldsymbol{n}(t) \end{aligned} \quad (1)$$

where N is the number of users in the system, L_i is the number of paths received from the ith user, ρ_i models the effects of path loss and log-normal shadowing, P_i is the transmitted power per symbol, ψ_i is a Bernoulli random variable with probability of success ν (assumed 0.375) that models the voice activity of the user, $W^h(t)$ is the hth orthogonal Walsh function, $h = 1, 2, \cdots, M$, T_0 is the time offset between the I and Q channels, $\boldsymbol{a}_{l,i}$ is the $S \times 1$ response vector of the cell site antenna array to signals in the lth path from the ith user, S denotes the number of elements in the array, $\tau_{l,i}$ is the time delay of the lth multipath component, ω_c is the carrier angular frequency, and $\theta_{l,i} = \omega_c \tau_{l,i}$. The product of the user pseudonoise (PN) code and the I or Q channel PN code is denoted as a_i^I and a_i^Q, respectively. $T_w/T_c = 256$ is the processing gain of the system where T_w is the symbol period and T_c is the chip period. $\boldsymbol{n}(t)$ is the additive complex Gaussian noise vector with zero mean and covariance $\sigma_n^2 \mathbf{I}\delta(t_1-t_2)$ ($\sigma_n^2/T_c = E(\rho_1^2 P_1)$ is assumed in the sequel).

Suppose we are interested in the signal sent by the first user. Equation (1) can be rewritten as

$$\begin{aligned}\boldsymbol{x}(t) = \sum_{l=1}^{L_1} \rho_1 \sqrt{P_1}\psi_1 \Big[& W^{(h)}(t-\tau_{l,1}) a_1^I(t-\tau_{l,1}) \\ & + jW^{(h)}(t-T_0-\tau_{l,1}) a_1^Q(t-T_0-\tau_{l,1})\Big] \\ & \cdot [\cos(\theta_{l,1}) + j\sin(\theta_{l,1})]\boldsymbol{a}_{l,1} + \boldsymbol{m}(t) + \boldsymbol{n}(t) \end{aligned} \quad (2)$$

where

$$\begin{aligned}\boldsymbol{m}(t) = \sum_{i=2}^{N}\sum_{l=1}^{L_i} \rho_i \sqrt{P_i}\psi_i \Big[& W^{(h)}(t-\tau_{l,i})\, a_i^I(t-\tau_{l,i}) \\ & + jW^{(h)}(t-T_0-\tau_{l,i})\, a_i^Q(t-T_0-\tau_{l,i})\Big] \\ & \cdot [\cos(\theta_{l,i}) + j\sin(\theta_{l,i})]\boldsymbol{a}_{l,i} \end{aligned} \quad (3)$$

is the multiple access interference (MAI).

Block diagrams for the receiver structure are given in [1, Figs. 2–4]. Using the optimum beamforming weights, the output of the beamformer for the kth multipath component of the first user (assumed as the desired user), assuming that the first Walsh symbol is transmitted, is given by [1]

$$|U_{k,1}^{(n)}|^2 = \begin{cases} |2A_1\sqrt{T_w} e^{j\theta_{k,1}} \boldsymbol{a}_{k,1} + \boldsymbol{m}_{k,1}^{(n)} + \boldsymbol{n}_{k,1}^{(n)}|^2, & n = 1 \\ |\boldsymbol{m}_{k,1}^{(n)} + \boldsymbol{n}_{k,1}^{(n)}|^2, & n \in (2, \cdots, M) \end{cases} \quad (4)$$

where $A_i = \rho_i \sqrt{P_i}\psi_i$, $\boldsymbol{m}_{k,1}^{(n)}$ is the multiple access interference signal vector and $\boldsymbol{n}_{k,1}^{(n)}$ is due to the additive white Gaussian noise (AWGN). $M = 64$ in the IS-95 CDMA standard. In [1], an equal gain combining of the path variables $\{|U_{k,1}^{(n)}|^2, k = 1, 2, \cdots, L_1\}$ was carried out. In this paper we consider a new rank-based algorithm for combining these variables.

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II. Diversity Combining Schemes and MAI Model

Assuming the number of paths for each user signal to be the same (i.e., $L = L_i$, $i = 1, \cdots, N$), the samples $|U_{k,1}^{(n)}|^2$, $n = 1, 2, \cdots, M$, $k = 1, 2, \cdots, L$, can be arranged in a matrix with M rows and L columns. The signal detection problem is to identify the unique row of samples that corresponds to the transmitted signal of user 1.

The EGC and a modified rank test (MRT) [3] implement the following decision rules: decide l as the signal row if

$$l = \arg \max_{k \in (1, M)} H_k \tag{5}$$

where $H_k = \sum_{l=1}^{L} |U_{l,1}^{(k)}|^2$ for EGC,

$$H_k = \sum_{j=1}^{L} V_{kj},\; V_{kj} = \begin{cases} R_{kj}, & \text{if } R_{kj} \geq M - p + 1 \\ 0, & \text{otherwise} \end{cases}$$

R_{kj} is the rank of $|U_{j,1}^{(k)}|^2$ among the samples $\{|U_{j,1}^{(l)}|^2, l = 1, 2, \cdots, M\}$, for MRT. Here $p \in 1, \cdots, M$ is a design parameter for MRT.

It has been shown in [1], and also verified by us, that for a large number of users ($N \gtrsim 40$), the MAI signal vector $\boldsymbol{m}_{k,1}^{(n)}$ can be modeled as spatially white Gaussian. However, this model no longer holds good for a small number of users. In fact, the individual components of $\boldsymbol{m}_{k,1}^{(n)}$ can be approximated to have a Laplace distribution when they are only a few simultaneous users present in the system (Fig. 1 shows the histogram for the 1st component of the I-channel, $N = 5$, similar results are obtained for Q-channel). The joint densities of these components cannot be assessed easily. Instead, a low value of the Forbenius norm, $\|e^N\|_F = \|\hat{\mathbf{R}}_{uu,k,1}^{(n),N} - \mathbf{I}\|_F$, indicates that the components can be assumed to be statistically uncorrelated. The covariance matrix of the MAI for $N = 5$, $\hat{\mathbf{R}}_{uu,k,1}^{(n),5}$, was estimated to be

$$\text{Real}(\hat{\mathbf{R}}_{uu,k,1}^{(n),5}) = \begin{bmatrix} 1.0056 & 0.0209 & 0.0087 & 0.0179 & -0.0025 \\ 0.0209 & 0.9760 & 0.0105 & 0.0190 & -0.0028 \\ 0.0087 & 0.0105 & 1.0395 & 0.0279 & 0.0091 \\ 0.0179 & 0.0190 & 0.0279 & 0.9832 & 0.0260 \\ -0.0025 & -0.0028 & 0.0091 & 0.0260 & 0.9957 \end{bmatrix}$$

$$\text{Imag}(\hat{\mathbf{R}}_{uu,k,1}^{(n),5}) = \begin{bmatrix} 0.0000 & 0.0044 & -0.0030 & -0.0032 & -0.0360 \\ -0.0044 & 0.0000 & -0.0003 & -0.0034 & 0.0091 \\ 0.0030 & 0.0003 & 0.0000 & 0.0073 & 0.0013 \\ 0.0032 & 0.0034 & -0.0073 & 0.0000 & -0.0028 \\ 0.0360 & -0.0091 & -0.0013 & 0.0028 & 0.0000 \end{bmatrix}.$$

The Forbenius norm of the error $\|e^5\|_F$ is estimated to be 0.0112 which is slightly higher than the estimated value of 0.0050 for $N = 40$. Although we have only verified that for $N = 5$, the components of the MAI vector are uncorrelated Laplace variables, we assume them to be independent in the following discussion.

We assume that at a given time period, a few number of users will have a high priority. Such users' signals will have a relatively large power as compared to a low priority user.

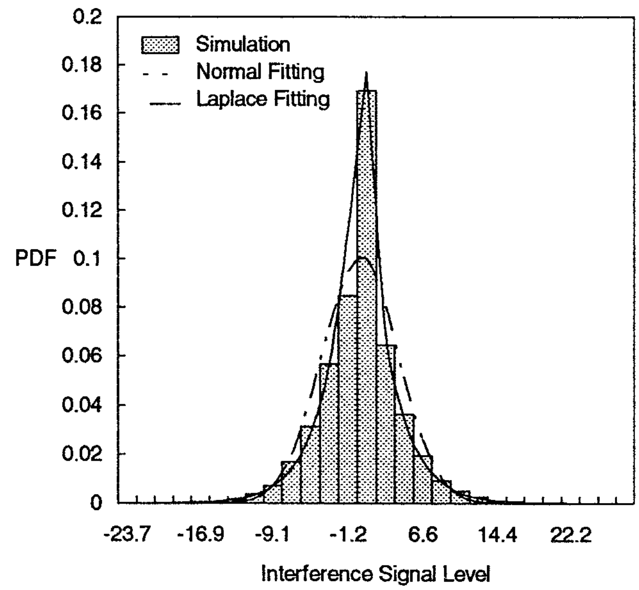

Fig. 1. I-channel: first antenna interference distribution for $N = 5$.

Hence, the MAI resulting from the high priority users can be modeled as Laplace. We studied the performances of the receivers mentioned above under this MAI assumption as well as their performances under the assumption of Gaussian MAI.

III. Performance Comparison

We consider the case corresponding to low Doppler frequency and ideal power control [1], i.e., the signal to noise ratio γ_s is a fixed quantity (as defined in [1, eq. (61)]). The performances of MRT and EGC are evaluated by finding the probability of bit error (P_b) in identifying the signal row. Let the ratio of the signal power of a user with high priority to the signal power of a user with low priority be denoted as μ and let ∂_i denote the ratio of the received power from the ith path to the received power from the first path. In order to estimate the probability of errors of EGC and MRT, we compute the corresponding variables H_k, $k = 1, \cdots, M$, by generating the random variables $\{U_{k,1}^{(n)}, k = 1, \cdots, L, n = 1, \cdots, M\}$ using the appropriate International Mathematical and Statistical Library (IMSL) routines. The real and imaginary values of the MAI in (4), for low number of users, are generated as Laplace distributions with mean 0 and variances

$$\frac{\sigma_{n_j}^2}{2T_c} = 2\left[\sum_{i=2}^{N} \mu\nu \sum_{l=1}^{L} E(\rho_{il}^2 P_{il}) + \sum_{\substack{l=1 \\ l \neq j}}^{L} E(\rho_{1l}^2 P_{1l})\right], \quad j = 1, \cdots, L. \tag{6}$$

Enough samples were simulated to obtain a confidence coefficient exceeding 0.95.

In Table I, the P_b of user 1 (low power user) corresponding to EGC, MRT with $p = 1, 6, 64$, are given with the number of priority users N_h being 5, $N = 6$, and $\mu = 10$. Here λ represents the ratio of the P_b of EGC to the P_b of MRT with $P = 6$. It can be seen that when the path strengths of the three

TABLE I
PROBABILITY OF BIT ERROR OF A LOW POWER USER FOR DIFFERENT TESTS WITH $\mu = 10$, $L = 3$, $S = 3$, $N_h = 5$, $N = 6$

Path Strength	Probability of bit error P_b				Ratio
	EGC	MRT			λ
		$p = 64$	$p = 6$	$p = 1$	
$\partial_2 = 1.0$, $\partial_3 = 1.0$	0.0023	0.0013	0.00076	0.0236	3.095
$\partial_2 = 0.5$, $\partial_3 = 0.5$	0.0005	0.0002	0.00008	0.0019	6.199
$\partial_2 = 0.7$, $\partial_3 = 0.2$	0.0006	0.0001	0.00007	0.0014	9.062

TABLE II
BER FOR $S = 1$, $N = 45$, $M = 64$, $\partial_l = 1$, $l \in (2, \cdots, L)$

	Analytical/ Simulated BER	Simulated BER	
L	EGC	MRT ($p = 64$)	LRT
2	1.20×10^{-2}	2.44×10^{-2}	9.44×10^{-3}
4	3.45×10^{-2}	7.12×10^{-2}	3.08×10^{-2}

paths are equal, then $\lambda = 3.095$. When the second and third path strengths are half the first path strength, $\lambda = 6.199$, and when $\partial_2 = 0.7$ and $\partial_3 = 0.2$, then $\lambda = 9.062$.

The results indicate that when the path strengths are equal, the performances of MRT ($p = 6$) is slightly better than that of EGC. However, under varying path strengths, the MRT ($p = 6$) achieves significant performance gain over EGC. When the MAI is Gaussian, as happens with a moderate to large number of users of same power, the EGC outperforms MRT. In fact, in such situations, the performance of EGC is close to that of the optimal likelihood rank test (see Tables II and III).

TABLE III
BER FOR $S = 3$, $L = 2$, $M = 64$, $\partial_2 = 1$

	Analytical / Simulated BER	Simulated BER	
N	EGC	MRT ($p = 64$)	LRT
132	9.54×10^{-3}	2.16×10^{-2}	8.22×10^{-3}
153	2.20×10^{-2}	3.82×10^{-2}	1.94×10^{-2}
192	5.53×10^{-2}	8.29×10^{-2}	5.13×10^{-2}

IV. CONCLUSION

We considered the EGC and an MRT for combining antenna array signals in wireless DS/CDMA under a specific user environment. The environment considered consists of a few high power interfering users along with a low power user of interest. Simulation results show that, for a small number of users, the MAI can be modeled as a Laplace density. Also, the results indicate that under this condition, the MRT does better than EGC for varying path strengths. Under the condition of a large number of simultaneous users having equal power, the EGC performs better than the MRT.

REFERENCES

[1] A. F. Naguib and A. Paulraj, “Performance of DS/CDMA with M-ary orthogonal modulation cell site antenna array,” *IEEE J. Select. Areas Commun.*, vol. 14, pp. 1770–1783, Dec. 1996.

[2] J. C. Liberti and T. S. Rappaport, “Analytical results for capacity improvement in CDMA,” *IEEE Trans. Veh. Technol.*, vol. 43, pp. 680–690, Aug. 1994.

[3] V. Annampedu and R. Viswanathan, “A rank order test for M-ary detection,” in *Proc. 31st Annu. Conf. on Information Sciences and Systems*, The John Hopkins Univ., Baltimore, MD, Mar. 1997, pp. 819–824.