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Throughput Performance of an Adaptive ARQ Scheme in Rayleigh Fading Channels

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***Abstract*— Using a simulation study we analyze the throughput performance of Yao's adaptive ARQ scheme in time-varying channels. The simulation takes into account the Rayleigh amplitude and the fast or the slow fading characteristics of a wireless channel, under a representative M-FSK modulation and Reed-Solomon coding scheme. We show that, for a specific set of design parameters, Yao's adaptive procedure works well for all channel fading rates, except for moderately slow rates. By observing variations of packet error rates at a specified SNR we provide an explanation for these varied behaviors under different channel fading rates.**

***Index Terms*— Adaptive ARQ, Rayleigh fading, go-back-*N*, throughput, packet error.**

I. Introduction

IN A WIRELESS SYSTEM such as cellular mobile radio, the signal strength at the receiver changes on a time-continuous basis. The probability of packet error in such a channel therefore varies with time. In [1] Yao presented a scheme that dynamically adapts to changing conditions of a channel. The scheme is a hybrid of regular go back-*N* (basic GBN) and *n*-copy GBN. However, the throughput performance analysis of the scheme was presented only for a static, non-fading channel. Yao's results for throughput in [1] as well as some minor corrections in [2], [3] show that the adaptive scheme follows closely the better of basic GBN and 2-copy GBN over a wide range of packet error rate values. Yao's scheme requires the specification of three parameters for its operation (see Section II). This study investigates whether Yao's scheme, with a fixed set of design parameters, provides improved throughput performance in a Rayleigh fading channel.

Several other issues of Yao's scheme have been examined in the literature, such as sub-optimal estimation of its design parameters [4], [5], exact Markov chain analysis for a static channel [2], [3] and a time-varying channel [6]. The two-state model for packet errors [6] provides a simplified analysis, but it does not fully capture the dynamics of a Rayleigh fading channel. Also, [5] provided an approximate and partial study of the performance of Yao's scheme in a moderately fast fading channel using a fading threshold model for the packet error probability. A few other adaptive packet transmission strategies for time-varying channels are discussed in [7], [8].

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II. Adaptive ARQ Scheme and Packet Error Simulation Model

In this section we provide a brief description of Yao's scheme followed by a description of the simulation model we have used for the generation of packet errors in a Rayleigh fading channel.

A. Yao's Adaptive ARQ Scheme

The idea behind Yao's adaptive scheme is to maintain the throughput efficiency of the basic GBN scheme during good channel conditions, but switch to *n*-copy GBN when channel conditions deteriorate [1]. The channel estimation is based only on a count of the acknowledgements (acks) and negative acknowledgements (nacks) from the receiver; no other channel state information is available at the transmitter. When the transmitter is in the basic mode, if the consecutive nacks from the receiver exceeds a predefined threshold α, then the transmitter assumes that the channel condition is bad and switches to *n*-copy mode. Similarly, the transmitter counts the total number of consecutive acks in the *n*-copy mode and compares it to another threshold β. If this count is above β, then the transmitter shifts back to the basic mode. The parameters α, β and *n* in *n*-copy are the design parameters for the Yao's scheme. In this paper, we primarily consider n=2 and (α=2, β=10) ([4], [5] specifies (α=2, β=24) or (α=3, β=45) as a better choice for static channel, but we comment on these choices later in section III). The GBN window size is set to N=10, which is the value used in [1]. The value of N is typically computed as the round trip delay between transmitter and receiver, normalized to the packet transmission time. In a land mobile cellular channel, N is generally small (close to 1 or 2) as the round trip delay is small, but larger values of N are necessary in a satellite mobile personal communications system operating at low elevation angle or in particular protocols used (a GPRS in GSM may delay the acknowledgment [9]).

B. Packet Error Simulation

Packet error rates in a fading channel can be modeled in two ways. One approach is to model the packet errors as a Markov chain. The second approach, which we use here, is to simulate the samples of the fading envelope and derive probabilities of packet errors for time consecutive packets generated by these samples. A drawback of this approach is that it requires the assumption of a particular modulation and coding scheme. In our simulation of packet errors we assume an *M*FSK modulation, an (N_c,K_c) Reed-Solomon (RS (N_c,K_c)) code, and L codewords per packet. The packet duration T_p equals L times the codeword duration. The signal-to-noise ratio (SNR) of different symbols in a packet could change from symbol to symbol, or from packet to packet, depending on the characterization of the channel as fast, or slow, respectively [10]. A channel with a maximum Doppler spread of f_m Hz can be generally classified as fast fading if $f_mT_p>1$ and as slow fading if $f_mT_p \ll 1$. In our simulation we classify fading rates in four classes: slow (f_mT_p=0.00336), moderately slow (f_mT_p=0.0335), moderately fast (f_mT_p=0.31) and fast (f_mT_p=2). For each case, T_p is fixed at 3.84 ms and the value of f_m is varied accordingly.

For an RS(N_c,K_c) code with t=(N_c-K_c)/2 symbol error correction capability, a decoding error occurs when more than t symbol errors occur in a codeword. In the simulation, the SNR (instantaneous) for each symbol is obtained from the Rayleigh channel simulator as explained later in this section. Because the SNR for each symbol is in general different (SNR depends on the fading rate of the channel), the symbol error probability for each symbol demodulated in a codeword could vary. For noncoherent detection of M-FSK, the symbol error probability is related to signal-to-noise ratio, γ, according to the equation

$$P_s = \sum_{n=1}^{M-1} (-1)^{n+1} \binom{M-1}{n} \frac{e^{\frac{-\gamma n}{n+1}}}{n+1} \qquad (1)$$

If X_i denotes the symbol error event for the i^{th} symbol in the k^{th} codeword (X_i=1 denotes an error, X_i=0 denotes no error), the decoded codeword error probability is given by

$$P_c(k) = P(\sum_{i=1}^{N_c} X_i > t | k^{th} codeword) \qquad (2)$$

Using (1) for the probability of error for the i^{th} symbol in the k^{th} codeword, $P_c(k)$ can be computed through the generalized binomial probability calculation described in [11]. The packet error rate (PER) is given by

$$P_p = 1 - \prod_{k=1}^{L} (1 - P_c(k)) \qquad (3)$$

The packet error rate given by (3) is actually a conditional probability, conditioned on the specific SNRs of the symbols that constitute a packet. However, for convenience, we call it a packet error rate, instead of conditional packet error rate. For the purpose of the simulation, P_p is obtained for a given packet and then a uniform random number Z from (0, 1) is generated. If $Z<P_p$, then the packet is declared to be in error. Otherwise, error-free reception is assumed and the throughput count is incremented. Because the signal fading and noise are independent, given that the symbols in a packet assume a given set of amplitude levels, the independent noise process causes the packet error.

Rayleigh distributed amplitude samples are simulated using the Gans model as described in [10]. We first generate frequency samples of the desired envelope and then derive the time (amplitude) samples from the frequency samples by using IFFT. The frequency and the time resolutions of the samples are given respectively as

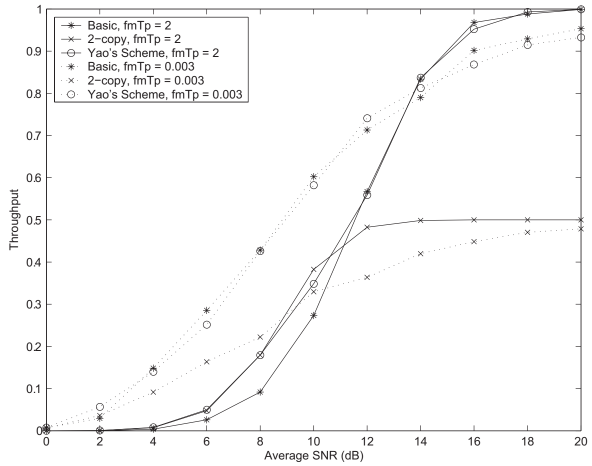

Fig. 1. Throughput vs average SNR, fast fading and slow fading channels.

$$\Delta f = 2f_m/N_f \qquad (4)$$

$$\Delta T = N_f/(2f_m(N_t - 1)) \qquad (5)$$

Here N_t is the number of time samples for the simulation and N_f is the total number of frequency samples. ΔT should be small enough so as to catch extreme and sudden variations in the signal level. Moreover, the number of time samples needs to be much greater than the frequency samples, as a time resolution of 1/2f_m, for the case of N_t=N_f would not be small enough. The values used in the simulation are as follows: f_m=520.83Hz (fast fading): N_t=32768, N_f=1024, f_m=80.73Hz (moderately fast fading): N_t=8192, N_f=256, f_m=8.77Hz (moderately slow fading): N_t=32768, N_f=128, f_m=0.873Hz (slow fading): N_t=32768, N_f=64. Amplitude-squared sample with a specified average SNR is obtained by squaring time sample and then dividing it by the average noise power of the channel. The average noise power is calculated by dividing the mean square of the time samples from the simulation by the average SNR of the channel, which is predefined in the simulation.

C. Simulation of Adaptive ARQ and Packet Errors

We simulated a discrete-event packet generation and error detection scheme by running the adaptive algorithm of Yao concurrently with the above mentioned time-consecutive packet error simulation procedure. At least 10,000 consecutive packets were generated for each average SNR in order to arrive at the throughput estimate.

III. Throughput Performance

We have analyzed the throughput performance of basic GBN, 2-copy GBN (simply referred below as basic and 2-copy, respectively), and Yao's scheme based on the simulation model presented in the previous section. In the simulation, we primarily considered the following parametric values: L=1,

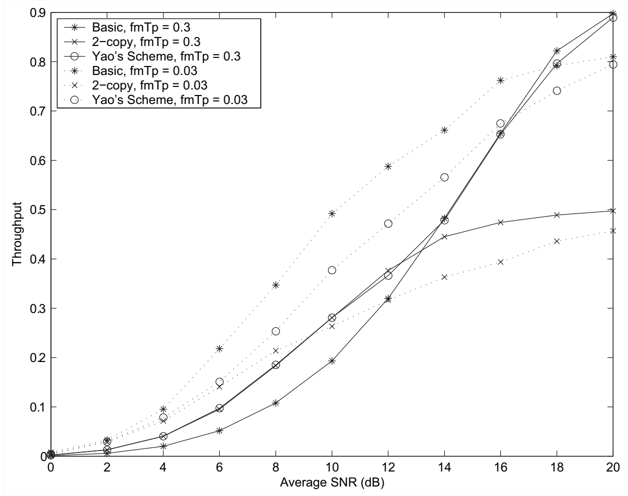

Fig. 2. Throughput vs average SNR, moderately fast fading and moderately slow fading channels.

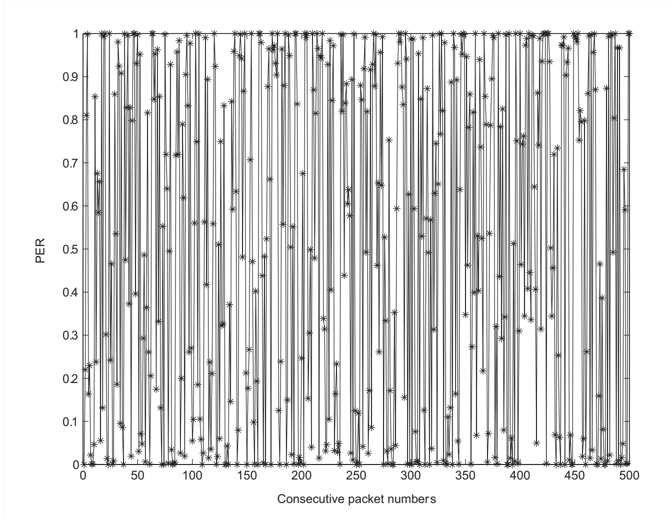

Fig. 3. PER variation with time in fast fading channel for SNR= 8 dB.

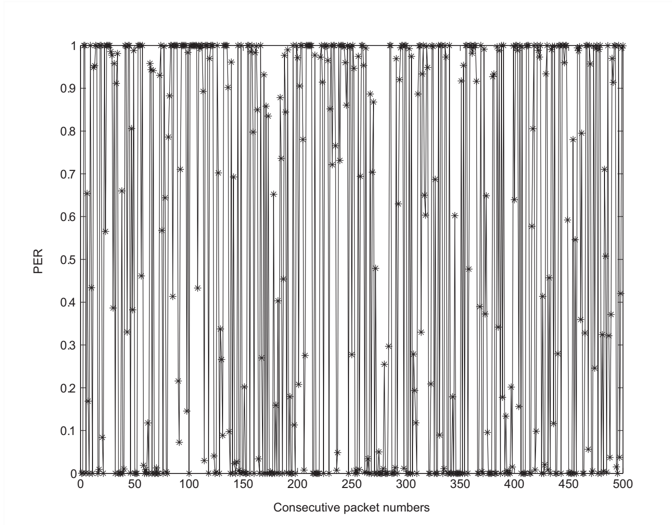

Fig. 4. PER variation with time in moderately fast fading channel for SNR= 8 dB.

M=32, N=10, N_c=31, K_c=7, T_p=3.84ms. Throughputs versus average SNR of the channel for various fading rates and (α=2, β=10, n=2) are plotted in Figs. 1-2. From Figs. 1 and 2 we observe that the throughput of the adaptive scheme follows the better of basic and 2-copy in the fast and moderately fast fading cases. In the moderately slow fading case, Fig. 2 shows that the throughput of the adaptive scheme follows that of 2-copy at very low SNR, but at moderate and large SNR values, it lies between the basic and 2-copy curves. The adaptation is not as good as that seen in the two previous cases. If α is increased to 3 from 2, then the adaptive scheme is forced to stay in basic mode longer, and hence its throughput is a little closer to that of basic for moderate and large SNRs. Of course, this is achieved at the expense of a slightly decreased throughput for SNR values in the range of 11 dB to 13 dB. We verified these observations for α=3 but the graphs are not shown here. For slow fading, the throughput of the adaptive scheme is very close to that of the basic scheme and both exhibit better performance than 2-copy (Fig. 1). In the static, non-fading channel, Yao's result and the corrected version [3] show that the 2-copy scheme performs better than basic when the packet error lies approximately above 0.1. We can explain the very different behaviors seen in Figs. 1 and 2 by looking at temporal variations of packet error rates for various fading rates.

Snapshots of PER for a span of 500 consecutive packets, for SNR=8dB and for fast, moderately fast, moderately slow and slow fading rates are shown in Figs. 3-6 (individual packets are indicated by *). The PER transits between the intervals, (0.9, 1.0) and (0.0, 0.1), in a span of 2-3 packet time periods, for the moderately fast and fast fading channel cases, and 10-20 packet time periods, for the slow fading channel case. PER variation is more rapid in the fast fading case. The point of interest is that, even though the average PER for the channel would be, e.g., 0.3, for moderate slow and slow channels, the instantaneous error rate lies near the two extremes of 1 or 0 for most of the time. The channel conditions therefore are either too bad for any scheme to be useful, or in the other extreme, they are in the range in which the basic scheme provides the best average throughput. With α=2, β=10, when the adaptive scheme is in the 2-copy mode and when the channel packet error rate falls close to 0, the receiver could count 10 consecutive acks with high probability thereby returning the algorithm to the basic mode quickly. Since the PER transition from 0 to 1 is quite rapid (within 10 to 20 packets) and since the durations for which PER is close to 1 or 0 are much larger than 20, the overall throughput of any scheme in slow fading is primarily determined by the duration of the channel in the low error state (PER close to 0). This explains why the observed throughput of Yao's scheme with these design parameters follows closely the throughput of basic in a slow fading channel. In a moderate slow fading channel, even though the transitions between the low error and the high error states are more frequent than in a slow fading channel, they are still small enough to allow the basic scheme to have better throughput than 2-copy. However, the inability of the adaptive scheme to quickly return to the basic mode, when the PER is in or near the low error state, causes

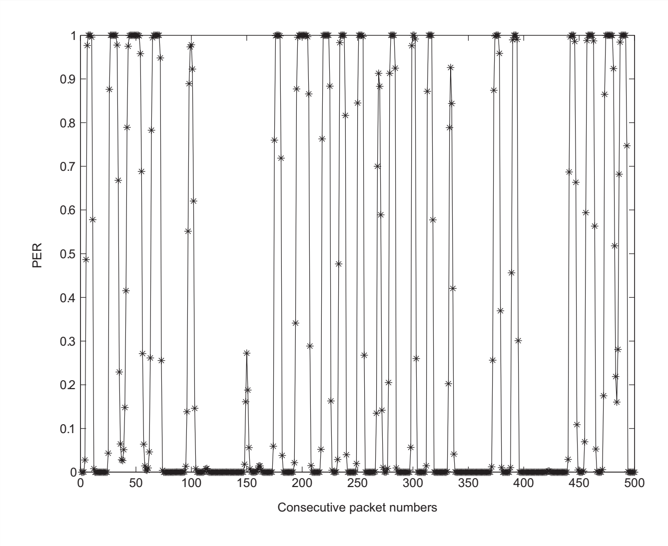

Fig. 5. PER variation with time in moderately slow fading channel for SNR= 8 dB.

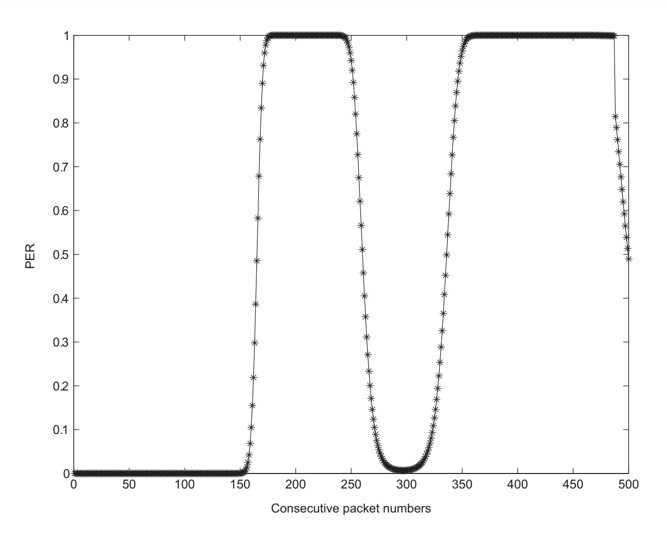

Fig. 6. PER variation with time in slow fading channel for SNR= 8 dB.

the throughput of Yao's scheme to be below that of basic. In contrast, for moderate fast and fast fading conditions and SNR values in the range of 6 dB-11dB, rapid fluctuations in PER lead to a better throughput characteristic for 2-copy as compared to basic. During these frequent fluctuations, Yao's scheme does not encounter 10 consecutive acks with high probability and therefore it stays in the 2-copy mode. This explains why the observed throughput graph of Yao's scheme is close to that of 2-copy when SNR assumes values in the range of 6 dB to 11 dB. For large SNR values exceeding 14 dB, the throughput of Yao's scheme follows that of basic because the PER stays close to 0 with high probability. If β=24, but α=2, then the transmitter spends a higher portion of the time in the 2-copy mode, thereby causing a decrease in the throughput. Moreover, for these values of α and β, in moderate slow fading, the throughput of Yao's scheme is very much below that of basic, when compared to that under (α=2, β=10). We also examined the cases of (α=3, β=45) and (α=2, β=6). In the former case, the observed performance, when compared to (α=2, β=10), is poor under moderate slow and slow fading rates, but is similar under the other two fading rates. In the latter case, the observed performance, when compared to (α=2, β=10), is better under moderate slow fading, but is somewhat poor under moderate fast and fast fading rates. The setting, (α=2, β=10), seems to be a reasonable choice for an overall best performance.

By decreasing the packet size to RS (15, 3) we observed an improvement in the achieved throughput. Also, the relative performances of the three schemes are not affected by the error correction capability of the code. For the case of RS (31, 7), as L is increased to 10, the number of PER samples falling in the range (0.1, 0.9), for various fading rates, is decreased as compared to L=1 case. Therefore, it is expected that 2-copy, and hence Yao's scheme, would perform better than basic only over a shorter SNR range and only under fast fading.

IV. Conclusion

We have shown by simulation of the throughput performance of Yao's adaptive scheme (α=2, β=10, n=2) for a specific M-FSK modulation with RS coding in a Rayleigh fading channel that the scheme adapts well for all fading rates, except for moderately slow rates.

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