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Impact of Channel Errors on Decentralized Detection Performance of Wireless Sensor Networks: A Study of Binary Modulations, Rayleigh-Fading and Nonfading Channels, and Fusion-Combiners

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Impact of Channel Errors on Decentralized Detection Performance of Wireless Sensor Networks: A Study of Binary Modulations, Rayleigh-Fading and Nonfading Channels, and Fusion-Combiners

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Abstract—**We provide new results on the performance of wireless sensor networks in which a number of identical sensor nodes transmit their binary decisions, regarding a binary hypothesis, to a fusion center (FC) by means of a modulation scheme. Each link between a sensor and the fusion center is modeled independent and identically distibuted (i.i.d.) either as slow Rayleigh-fading or as nonfading. The FC employs a counting rule (CR) or another combining scheme to make a final decision. Main results obtained are the following: 1) in slow fading, a) the correctness of using an average bit error rate of a link, averaged with respect to the fading distribution, for assessing the performance of a CR and b) with proper choice of threshold, ON/OFF keying (OOK), in addition to energy saving, exhibits asymptotic (large number of sensors) performance comparable to that of FSK; and 2) for a large number of sensors, a) for slow fading and a counting rule, given a minimum sensor-to-fusion link SNR, we determine a minimum sensor decision quality, in order to achieve zero asymptotic errors and b) for Rayleigh-fading and nonfading channels and PSK (FSK) modulation, using a large deviation theory, we derive asymptotic error exponents of counting rule, maximal ratio (square law), and equal gain combiners.**

Index Terms—**Asymptotic error, counting rule, equal gain combiner, FSK, large deviations, maximal ratio combiner, PSK, Rayleigh-fading, square law combiner, wireless sensor networks.**

I. INTRODUCTION

PERFORMANCES of decentralized detection (DD) systems employing a set of geographically separated sensors have been investigated for the past couple of decades [1]–[5]. Tsitsiklis had established fundamental results on the optimal decision rules for processing at the sensors and at the fusion center [1], [2]. In these earlier studies, the transmission links from sensors to a fusion center (FC) were assumed to be error

free. However, because of recent interest in wireless sensor networks (WSN), many authors have analyzed the performance of these DD systems in which transmissions from sensors to FC are subject to channel fading and noise [6]–[20]. Because a WSN may contain a large number of sensors, a number of these studies deal with asymptotic (infinite number of sensors) issues [6]–[15]. In a power constrained WSN, Chamberland and Veeravalli have shown that fading reduces the overall performance, but the quality of sensor observations has a greater impact on the overall probability of error than fading [7]. In [8] and [9], the same authors show that for independent and identical Gaussian or exponentially distributed sensor observations, identical binary-threshold processing at all the sensors yields asymptotically optimal results. Under power and bandwidth constraints and the transmission of local sensor observations using analog relay amplifier processing, Jayaweera shows that in the case of detection of deterministic signals, it is better to combine many not-so-good local decisions than relying on one (or a few) very-good local decision(s) [10], [12]. Assuming a type-based random access between sensors and the fusion center and a network power constraint, Liu and Sayeed [13], Mergen *et al.* [14], and Anandkumar and Tong [15] have studied asymptotic error exponents. A general principle in these approaches is reporting of the counts that occurred in each interval of a quantizer by each sensor, which also allows for noncoherent detection at the fusion center [15].

The performance of a wireless sensor DD system depends on many factors such as decision fusion rules [16], channel error control coding, sensor quality, etc. Chen *et al.* [17] and Niu *et al.* [18] have formulated the parallel fusion problem with a fading channel layer and derived the optimal likelihood ratio (LR) based fusion rule, along with three other suboptimal fusion rules. Performance analysis was carried out only for the case of a finite number of sensors. If sufficient error control coding makes a sensor link to be highly reliable, then the earlier analysis of DD systems with error-free links would be applicable. However, fusion of binary decisions transmitted over fading channels that employ no error control codes may have important applications in low-cost and low-power WSN. Moreover, if only a counting rule based on decisions received at the FC is considered, then the previous analyses of decentralized detection in error free sensor links can be applied simply by replacing the distributions of sensor decisions with the corresponding distributions of sensor decisions received at the FC. But, we consider

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other fusion strategies mentioned above, as well as square law combining, all of which require a new analysis that takes into account specific channel model, modulation, and fusion rule. In this paper we provide new results on the binary-hypothesis detection performance of a WSN, when each link between a sensor and the fusion center is modeled as either independent and identically distributed (i.i.d.) slow Rayleigh-fading or a non-fading binary modulated signal received in AWGN. The sensors quantize their observations to binary decisions and transmit them using a basic binary modulation scheme such as, FSK, PSK or ON/OFF keying (OOK). FSK or OOK is more suitable in a fading channel because of the applicability of noncoherent detection, which does not require the tracking of carrier phase at the FC. While phase tracking may be difficult to achieve in a fading channel, the PSK modulation may still be achievable under certain conditions (PSK was the only modulation considered in [17]). The FC combines the received signals in order to make a final decision on the presence or absence of a phenomenon of interest (POI). Our discussions in the paper differ from existing literature in two aspects: 1) results show how significantly the link condition could affect the probability of false alarm of a sensor/fusion decision, when observed at the fusion center and 2) previous analyses of asymptotic (large number of sensors) error exponents for the Neyman–Pearson criterion were based on Stein's lemma, which is applicable only to optimal likelihood ratio tests. Since suboptimal fusion rules are also considered in this paper, the theory of large deviation of the sample mean is employed to arrive at the error exponents. Moreover, the variation of miss error exponent as a function of false alarm error exponent is studied, as both errors are allowed to approach zero asymptotically. Since it is not possible to predict large sensor-network performance, based on the performance analysis for a small, finite number of sensors carried out in [17], such an analysis is meaningful.[1]

The paper is organized as follows. In Section II we evaluate the effect of channel errors on the reliability of a sensor decision at the fusion center. For a counting rule (CR), the variations of false alarm and detection probabilities at the fusion center, as a function of channel signal-to-noise ratio (SNR), are studied. In Section III, for various other combining schemes, using a theory of large deviation, we evaluate the rates at which the asymptotic probabilities of errors of a final decision approach zero. Both slow Rayleigh-fading and nonfading cases are considered. Conclusions from this study are presented in Section IV.

II. EFFECT OF CHANNEL ERRORS ON THE RELIABILITY OF DECISION AT THE FUSION CENTER

Consider a wireless sensor network consisting of N sensors, which is deployed to assess the presence or absence of a phenomenon of interest (POI) in a geographical area of interest. Sensor i gathers information pertaining to the POI and makes a decision U_i ($U_i = 1$ for deciding the presence of POI and $U_i = 0$ otherwise) and sends its binary decision to a fusion

center through an unreliable communication channel or link. We assume identical binary quantizers, existence of noninterfering and identical parallel links between the sensors and the fusion center, and conditioned on the hypothesis, i.i.d. observations across the sensors. Therefore, conditioned on the hypothesis, the sensors have identical and independent distributions for their decisions. Let U_{i0} denote the decision of the ith sensor, as received at the fusion center. Hence, the following probabilities $P_{fi} = P(U_i = 1|\text{POI absent})$, $\alpha_i = P(U_{i0} = 1|\text{POI absent})$, $P_{di} = P(U_i = 1|\text{POI present})$, $\beta_i = P(U_{i0} = 1|\text{POI present})$ are all independent of i. Assuming that the link error event is statistically independent of the decision made by the sensor, the false alarm probability of the received decision from the ith sensor is described by the following equation:

$$\alpha = P_f(1 - P_{C1}) + (1 - P_f)P_{C0} \tag{1}$$

where P_{C1} (P_{C0}) is the probability of bit error of the ith link, when the ith sensor transmits bit "1" ("0"). For symmetric channels, $P_{C1} = P_{C0} = P_C$ so that (1) simplifies to

$$\alpha = P_f + (1 - 2P_f)P_C. \tag{2}$$

The probability of detection of the received decision, β can be obtained from (1) by replacing P_f with P_d. For symmetric channels

$$\beta = P_d + (1 - 2P_d)P_C. \tag{3}$$

Let the link bit error, $P_C < 1/2$ (if it is greater than 1/2, then the decision rule of the receiver for the ith link at the fusion center could be complemented to yield a value less than 1/2). If $P_f < 1/2$, then $\alpha > P_f$. That is, the false alarm probability of the decision received at the fusion center is higher than the false alarm probability of the decision made by the sensor. As the link becomes very unreliable, the probability α approaches 1/2. Similarly, when $P_d > 1/2$, $\beta < P_d$. Only when $P_d < 1/2$, the link error "increases" β to be larger than P_d (of course this is achieved with a concomitant increase in the false alarm probability). Given the unreliable nature of the communication link between a sensor and the fusion center, we next examine its impact on the reliability of the decision made by the fusion center.

Under very general conditions, an optimum fusion rule for combining the decisions of the sensors takes the form of a counting rule [1], [2]. This result is also valid for combining decisions received through noisy links, as long as the links are also i.i.d.. In the remainder of this section, the performance of a counting rule (CR) in a slow Rayleigh-fading channel is considered.

A. Performance Analysis of CR for Finite N

For a wireless sensor network, slow Rayleigh-fading channel is an appropriate model in certain applications. The slow fading characterization implies that channel characteristics do not change over several successive bit intervals and within this period, the received signal amplitude in a sensor link at the FC can be assumed to be a sample of a Rayleigh random variable. Using standard results on reception in slow Rayleigh-fading
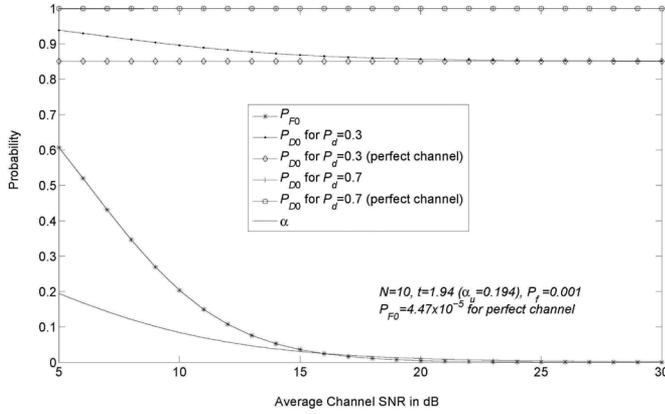
Fig. 1. Probability of false alarm/detection versus average channel SNR for noncoherent FSK.

channels, we can write the following relations (see [21, pp. 818]):

$$E(P_C) = \begin{cases} \frac{1}{2+\gamma_0} & \text{noncoherent FSK} \\ \frac{1}{2}\left(1 - \sqrt{\frac{\gamma_0}{1+\gamma_0}}\right) & \text{PSK} \end{cases} \quad (4)$$

where $E(P_C)$ is the average channel error probability and γ_0 is the average SNR of the Rayleigh-fading channel. By using (4) in (2) and (3), with P_C replaced by $E(P_C)$, the corresponding probabilities α and β can be obtained. For OOK, the probability of error $E(P_{C1})$ is not equal to the probability of error $E(P_{C0})$. With noncoherent detection, using the probabilities of bit errors given in (see [23, eq. (9.5.9)]) and (1), we get

$$\alpha_{\text{OOK}} = P_f\left(e^{\left(-b_0^2/2(1+\gamma_0)\right)}\right) + (1-P_f)e^{\left(-b_0^2/2\right)} \quad (5)$$

$$\beta_{\text{OOK}} = P_d\left(e^{\left(-b_0^2/2(1+\gamma_0)\right)}\right) + (1-P_d)e^{\left(-b_0^2/2\right)} \quad (6)$$

where b_0 is the normalized threshold used by the noncoherent detector. Let P_{F0}, P_{D0} denote the false alarm probability and the detection probability, respectively, of a counting rule at the fusion center, which decides that a POI is present when $\sum_{i=1}^{N} U_{i0}$ is greater than or equal to a threshold t. While computing the overall false alarm (and detection) error probability of a CR, the required α and β are obtained by using the average channel error probability of each link in (2) and (3). The correctness of this statement is proved in the Appendix. The threshold t for the CR ranges between 1 and N, with 1 corresponding to the Boolean OR rule and N corresponding to the Boolean AND rule. Let t be equal to $N\alpha_u$, where α_u is the maximum tolerable probability of false alarm (which depends on a minimum value of γ_0, $\gamma_{\min}$, for a specific sensor quality, P_f) for an individual link. Such a choice of t guarantees the probability of false alarm of a counting rule (P_{F0}) to approach zero asymptotically with increasing N, as discussed in Section II-B. For $N = 10$ and noncoherent FSK, Fig. 1 shows the variations of P_{F0} and the probability of detection of a counting rule (P_{D0}) for different average channel SNR values (γ_0). The figure also shows α as a function of γ_0. With γ_0 slightly greater than the assumed minimum guaranteed channel SNR ($\gamma_{\min} = 5$ dB), P_{F0} is several decades higher than P_f (from (2) and (4), a sensor with a low P_f of 0.001 is completely

masked by a high channel error of approximately 0.194, leading to a high P_{F0}). As γ_0 approaches infinity, α approaches P_F and P_{F0} approaches the value that would be obtained had the link been error free. Therefore, for a specific N, it is essential that the link reliability is greater than a certain minimum value in order that an acceptable P_{F0} is achieved. On the other hand, when $P_d < 1/2$, P_{D0} decreases with increases in channel SNR, i.e., higher P_{D0} is achieved when the link is less reliable! (of course this is achieved with a concomitant increase in the false alarm probability). But, when $P_d > 1/2$, P_{D0} increases as γ_0 increases. In general, except for weak observation SNR of sensors, the effect of link errors on the detection probability is less severe, assuming that the sensor detection probability will be larger than 0.1. A similar observation was made in [7] under the condition of a large number of sensors.

B. Asymptotic Error Exponents for the CR

For the counting rule with the threshold $t = N\alpha_u$, the condition $\alpha < \alpha_u < \beta$ guarantees that both the probability of false alarm (P_{F0}) and probability of miss ($P_{M0} = 1 - P_{D0}$) go to zero as N increases without bound. This can be seen from the following argument. According to the law of large numbers, as $N \to \infty$, the distribution of $S_D = \sum_{i=1}^{N} U_{i0}$ under no POI hypothesis (H_0) becomes degenerate at the value $N\alpha$. Hence, the probability that this sum exceeds $N\alpha_u$ goes to zero, if $\alpha_u > \alpha$. Similarly, under POI hypothesis (H_1), the condition $\beta > \alpha_u$ guarantees that $P_{M0} \to 0$, as $N \to \infty$. The constraints on α and β will be satisfied as long as the sensor signal-to-noise ratio (θ) is above a certain minimum value and the link bit error rate is below a certain value. For example, when detecting a constant signal in AWGN, the detection probability and the false alarm probability at a sensor are related by $P_d = Q\left(Q^{-1}(P_f) - \sqrt{\theta}\right)$, where $Q(\cdot)$ is the complementary cumulative distribution function of standard normal variate. Using this expression and (2)–(3) the required values of θ and P_C to guarantee $\beta > \alpha_u$ can be determined. The two asymptotic error exponents are defined as $\rho_0 = \lim_{N\to\infty} -(\log(P_{F0})/N)$ and $\rho_1 = \lim_{N\to\infty} -(\log(P_{M0})/N)$. Whereas an application of the central limit theorem (CLT) to S_D leads to incorrect error exponents, an application of a large deviation theory leads to the correct error exponents for the CR with the threshold, $t = N\alpha_u$ (see example 1.2, statement at the bottom of page 1, and equation (1.4) in [22]):

$$\rho_0 = -\log\left[\left(\frac{\alpha}{\alpha_u}\right)^{\alpha_u}\left(\frac{1-\alpha}{1-\alpha_u}\right)^{1-\alpha_u}\right] \quad (7)$$

$$\rho_1 = -\log\left[\left(\frac{\beta}{\alpha_u}\right)^{\alpha_u}\left(\frac{1-\beta}{1-\alpha_u}\right)^{1-\alpha_u}\right]. \quad (8)$$

For each of the three modulation schemes, given that a minimum channel SNR ($\gamma_{\min}$) is achievable, we now examine the sensor quality (in terms of its probability of detection P_d, at a given P_f) requirement for achieving zero asymptotic errors. For all modulations the CR threshold is set at $t = N\alpha_u$. With noncoherent detection of binary FSK, the false alarm probability (α_{FSK}) and the detection probability (β_{FSK}) at the fusion center are obtained by using (4) in (2)–(3). Assuming that

TABLE I
MIN. P_d REQUIRED FOR ACHIEVING ASYMPTOTICALLY ZERO P_{M0} (α_u IS TAKEN TO BE α ATTAINED WITH FSK AT A PARTICULAR $\gamma_{\min}$, OOK COLUMN IS BASED ON (9) FOR b_0.) (a) $P_f = 0.1$ (b) $P_f = 0.001$

$\gamma_{\min}(dB)$	α for FSK at $\gamma_{\min}$; same as min. P_d for FSK	PSK	OOK
-5	0.4454	0.3039	0.6207
0	0.3667	0.2172	0.2837
5	0.255	0.1513	0.1713
10	0.1667	0.1186	0.1295

a

$\gamma_{\min}(dB)$	α for FSK at $\gamma_{\min}$; same as min. P_d for FSK	PSK	OOK
-5	0.4319	0.2554	0.0059
0	0.3347	0.1427	0.0027
5	0.1943	0.0651	0.0017
10	0.0842	0.0242	0.0013

b

$P_f < 1/2$ and $\alpha_u < 1/2$, α_{FSK} is bounded within the interval $[P_f, 1/2]$ and there exists a $\gamma_0 = \gamma_{\min}$ such that α_{FSK} will be less than α_u, for $\gamma_0 > \gamma_{\min}$. The behavior of β_{FSK} depends on the value of P_d. For $P_d < 1/2$, β_{FSK} is bounded within the interval $[P_d, 1/2]$. Therefore, in order that $P_{D0} \to 1$ with increasing N, $\min(\beta_{\text{FSK}}) = P_d$ has to be greater than α_u (α_u is determined by P_f and $\gamma_{\min}$). This establishes a minimum sensor quality requirement. When $P_d > 1/2$, β_{FSK} is bounded within the interval $[1/2, P_d]$. Under the assumption that $\alpha_u < 1/2$, P_d is greater than α_u, for $\gamma_0 > \gamma_{\min}$. $P_d > 1/2$ is sufficient to ensure that P_{M0} asymptotically approaches zero.

For PSK appropriate probabilities are obtained by using (4) in (2)–(3). Again, $P_d > \alpha_u$ is a necessary condition for the asymptotic errors to go to zero. However, for a given P_f and minimum channel SNR, $\gamma_{\min}$, maximum of α_{PSK} is smaller than that of FSK. Therefore, the minimum P_d required at the sensor is smaller in the case of PSK (see Table I, which shows the P_d requirements at the sensor for different minimum channel SNRs, for $P_f = 0.1$ and $P_f = 0.001$). Of course, PSK requires the tracking of the carrier phase.

For OOK, both α_{OOK} and β_{OOK} (see (5) and (6)) are monotonic increasing functions of the SNR, with $\alpha_{\text{OOK}} \in \left[e^{(-b_0^2/2)}, e^{(-b_0^2/2)} + P_f \left(1 - e^{(-b_0^2/2)} \right) \right]$. Since b_0 is a parameter of choice, same α_u could be assumed for OOK and FSK. If γ_0 is assumed unknown, then b_0 cannot be obtained as a function of γ_0; in this case, given specific P_f and α_u values, b_0 can be chosen so that α_u is equated to the upper bound of α_{OOK}, i.e.,

$$b_0 = \sqrt{2 \log \left(\frac{1 - P_f}{\alpha_u - P_f} \right)}. \qquad (9)$$

OOK requires less energy than the other two schemes; the energy saved depends on how often a sensor decides the presence of a POI. However, another measure of comparison is to compute the minimum required P_d for each case (Table I). Because of the method of selection of b_0, for $P_f = 0.001$, the minimum required P_d for OOK is much less than those of FSK and PSK (using (6) and equating $\beta_{\text{OOK}} = \alpha_u$ at $\gamma_0 = \gamma_{\min}$ to solve for P_d). However, a penalty in terms of asymptotic rate at which P_{F0} approaches zero is paid. The false alarm error exponent is
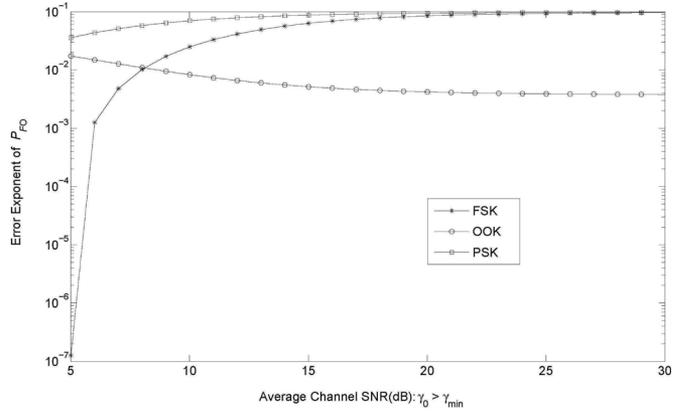

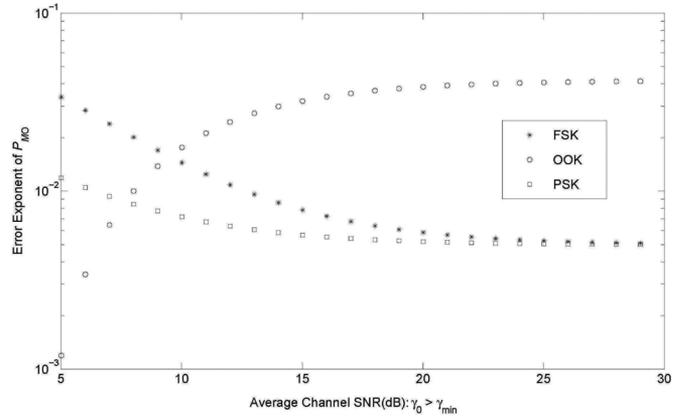Fig. 2. False alarm error exponents for counting rule with $P_f = 0.001$.

Fig. 3. Miss error exponents for counting rule with $P_d = 0.3$ and $P_f = 0.001$.

so small (not reported here) when compared to those of FSK and PSK, this method of choice of b_0 makes OOK modulation unfavorable. To overcome this problem, an alternate method is to let α_{OOK} be bounded below $\alpha_u - \Delta$, where $0 < \Delta < \alpha_u$, and then obtain b_0 by replacing α_u with α_u-Δ in (9) (clearly, $(\alpha_{\text{OOK}})_{\max} = \alpha_u - \Delta < \alpha_u$).

With this choice of b_0, the $\min(P_d)$ required for OOK is obtained by equating β_{OOK} at $\gamma_{\min}$ to α_u:

$$\min(P_d) = \frac{\alpha_u - e^{(-b_0^2/2)}}{\left(e^{(-b_0^2/2)} \right)^{(1/1+\gamma_{\min})} - e^{(-b_0^2/2)}}. \qquad (10)$$

For example, with $\alpha_u = \min(P_d)_{\text{FSK}} = 0.1943$, $\Delta = 0.0915$ and $P_f = 0.001$, b_0 equals 2.1371, and with $\gamma_{\min} = 5$ dB, $\min(P_d)_{\text{OOK}}$ equals 0.1943. With the FC counting threshold set at $N\alpha_u$, this method of selection of b_0 gives a larger ρ_0. Fig. 2 shows that ρ_0 for OOK is still somewhat smaller when compared to the error exponents for FSK or PSK. Comparing FSK and PSK, it can be seen that ρ_0 for PSK is larger. The variation of miss error exponent depends on the value of P_d (Fig. 3 is for $P_d < 1/2$). The miss error exponents for both FSK and PSK decrease with increases in average SNR, only when $P_d < 1/2$. This is due to the fact that the probability β, in both these two cases, decreases with increases in channel SNR. In the case of OOK, ρ_1 increases with increases in average SNR, for both $P_d > 1/2$ and $P_d < 1/2$. For $P_d < 1/2$, the

error exponent for OOK is better than that of FSK (or PSK), for moderate to large SNR values. Though not shown here, similar comparative performances are observed to be true for $P_f = 0.1$ and $P_f = 0.01$. Also, for any given modulation, specific P_d and SNR values, the error exponent (for both miss and false alarm) increases, when P_f is decreased. This is reasonable because a lower P_f at a specified P_d implies a better sensor. Hence, with a proper selection of threshold for noncoherent OOK detection, it is possible to obtain performance comparable to that of FSK (higher error exponent for P_{M0}, but smaller error exponent for P_{F0}), with the added benefit of energy conservation.

Even though noncoherent detection of FSK is the preferred choice for fading channels, some observations regarding coherent FSK performance can be made now: for large SNR and slow-Rayleigh fading, coherent PSK has a 3 dB advantage in SNR over coherent FSK; hence, for large SNR, the graph for coherent FSK is exactly the graph for PSK shifted to the right by 3 dB (Figs. 2 and 3). Similarly, for large SNR, the graph for differential PSK (DPSK) modulation is exactly the graph for noncoherent FSK shifted to the left by 3 dB.

III. ASYMPTOTIC ERROR EXPONENTS OF LIKELIHOOD RATIO, MRC, EGC, CR, AND SQUARE LAW COMBINING

For a finite N and a slow Rayleigh-fading channel, it was pointed out in [17] that the maximal ratio combining (MRC) of the received signals from different sensors does not provide the best detection performance. Here, using a large deviation theory, we evaluate the asymptotic error rates of MRC, equal gain combining (EGC), and optimal likelihood ratio (LR) rule for PSK and square law combining (SLC) for FSK. Consideration of SLC for FSK is motivated by the result that for a slow Rayleigh-fading channel, the SLC combining of diversity branches with FSK signals is analogous to the MRC combining of diversity branches with PSK signals [21]. Whereas these combiners are optimal in diversity context, in decentralized detection setup, as discussed later, both the MRC for PSK and the SLC for FSK turn out to be suboptimal. Slow Rayleigh-fading channel is considered first followed by the nonfading case. Error exponents of the CR, for Rayleigh-fading, were already obtained in II. For OOK, although SLC is a possible combiner, its performance is not addressed here. For this modulation, the asymptotic performance of a CR in a Rayleigh channel was thoroughly addressed in Section II. Whereas, in the case of CR, there exists a minimum P_d requirement for the errors to go to zero asymptotically, as will be seen below, no additional requirement, beyond the "natural" constraint of $P_d > P_f$, is required for all the other combiners. Without any loss of generality, let $P_f > 0$ and $P_d < 1$.

A. Slow Rayleigh-Fading Channel

1) Maximal Ratio Combining: In a wireless sensor network of N sensors, consider the situation that K out of N sensors decide "1" (the presence of a POI) and that the remaining $N - K$ sensors decide "0" otherwise. To illustrate the MRC output, assume that the first set of K sensors had decided binary "1". If the sensors use PSK signaling to transmit their data, then upon matched filtering, the maximal ratio combiner output for the Rayleigh faded PSK signals received in a zero mean AWGN,

conditioned on the above assumption on sensors' decisions, is given by [17], [21],

$$S_{\text{MRC}} = \sum_{j=1}^{K} h_j^2 - \sum_{j=K+1}^{N} h_j^2 + \sum_{j=1}^{N} h_j n_j \quad (11)$$

where $h_j > 0$ is the channel gain of the jth link. Implementation of MRC requires the knowledge of the channel states, $\{h_j\}$. $\{h_j^2, j = 1, 2, \ldots, N\}$ are all i.i.d. as exponential with mean σ_R^2 and $\{n_j, j = 1, 2, \ldots, N\}$ are all i.i.d. zero mean Gaussian noise with variance σ^2, which are independent of $\{h_j\}$. Under the hypothesis of no POI (H_0), the unconditional S_{MRC}, for i.i.d. sensors and i.i.d. links, can be treated as the sum of N i.i.d. samples of the form shown below:

$$S_{\text{MRC}} = \sum_{j=1}^{N} (y_j + h_j n_j),$$

$$\text{where } y_j = \begin{cases} h_j^2 & \text{with probability } P_f \\ -h_j^2 & \text{with probability } (1 - P_f). \end{cases} \quad (12)$$

For large N, using large deviations, we can find the rate with which the false alarm error probability at the fusion center approaches zero [22]:

$$\lim_{N \to \infty} P_{F0} = \lim_{N \to \infty} P(S_{\text{MRC}} \geq NC_{\text{MRC}} | H_0)$$

$$= \lim_{N \to \infty} P\left(\sum_{i=1}^{N} Z_i \geq 0 | H_0\right)$$

$$= \lim_{N \to \infty} e^{-ND_{\text{FMRC}}} + R_N \quad (13)$$

where R_N goes to zero faster than the exponential term in (13), as N increases without bound, Z_i are i.i.d. variables specified by $Z_i = U_i h_i^2 - (1 - U_i) h_i^2 + n_i h_i - C_{\text{MRC}}$ and C_{MRC} is a constant threshold value. In order for the false alarm error to go to zero in the limit as N goes to infinity, C_{MRC} has to be chosen to yield a negative expected value of Z_i. Hence

$$C_{\text{MRC}}^* = \frac{C_{\text{MRC}}}{\sigma_R^2} > (2P_f - 1). \quad (14)$$

By computing the moment generating function (MGF) of Z_i, the error exponent in (13) can be computed (see [22, Theorem 3.1 on p. 7 and eq. (2.2), (2.3)]):

$$D_{\text{FMRC}} = \log\Bigg(\inf_{((-1+\sqrt{1+4a})/2a) > \tau > 0}$$

$$\times \left\{ e^{-\tau C_{\text{MRC}}^*} \left[\frac{(1 - P_f)}{1 - a\tau^2 + \tau} + \frac{P_f}{1 - a\tau^2 - \tau} \right] \right\} \Bigg) \quad (15)$$

where the expression within the curly bracket is the MGF of Z_i, $\gamma = (\sigma_R^2/\sigma^2)$, $a = (1/2\gamma)$ and γ is the SNR (average) of the Rayleigh channel.

Remark: For all the combiners in this section, the miss error exponent can be obtained from the corresponding false alarm error exponent by replacing P_f with P_d and τ with -τ in the

expression within the curly brackets. The miss error exponent for MRC dictates

$$C^*_{\mathrm{MRC}} < (2P_d - 1). \tag{16}$$

In order that both the errors approach zero asymptotically, C^*_{MRC} has to satisfy both the constraints, (14) and (16).

If desired, results for MRC combining of coherent FSK signals can easily be obtained from (12). For coherent FSK, denoting the two filter outputs corresponding to the frequency f_1 (for decision H_1 by a sensor) and the frequency f_0 (for decision H_0 by a sensor), as S_1 and S_0, respectively, it is clear that S_1 is similar to (12) with n_j replaced by n_{1j} (corresponding to noise in filter f_1), $y_j = 0$ with probability $(1 - P_f)$, and $y_j = h_j^2$ with probability P_f, whereas S_0 is similar to (12) with n_j replaced by n_{0j} (corresponding to noise in filter f_0), $y_j = 0$ with probability P_f, and $y_j = h_j^2$ with probability $(1 - P_f)$. The final decision is arrived at by comparing $S_1 - S_0 - NC_{\mathrm{CSFK}}$ to zero. $S_1 - S_0$ has the noise term $h_j (n_{1j} - n_{0j})$, with the variance of $(n_{1j} - n_{0j})$ being $2\sigma^2$, which is twice the variance of n_j in PSK (for diversity reception, similar result is well known, see [21, p. 826]). Hence, the error exponent for coherent FSK at a specific SNR is the error exponent of PSK at the SNR, which is 3 dB below the specified SNR.

2) Equal Gain Combining: For equal gain combining, the equation for S_{EGC}, an analog of (12), for the MRC, is given by

$$S_{\mathrm{EGC}} = \sum_{j=1}^{N} (E_j + n_j)$$

$$\text{where } E_j = \begin{cases} h_j & \text{with probability } P_f \\ -h_j & \text{with probability } (1 - P_f). \end{cases} \tag{17}$$

The false alarm error exponent is given by

$$D_{\mathrm{FEGC}} = -\log\left(\inf_{\tau > 0} \left\{ e^{(-\tau C^*_{\mathrm{EGC}} + \tau^2/2\gamma)} \right.\right.$$
$$\left.\left. \times \left(1 + \frac{\sqrt{\pi}\tau}{2} e^{\tau^2/4} \left(2P_f - 1 + \mathrm{erf}\left(\frac{\tau}{2}\right) \right) \right) \right\} \right) \tag{18}$$

where $\mathrm{erf}(x) = (2/\sqrt{\pi}) \int_0^x e^{-t^2} dt$ and $((2P_d - 1)/2)\sqrt{\pi} > C^*_{\mathrm{EGC}} = (C_{\mathrm{EGC}}/\sigma_R) > ((2P_f - 1)/2)\sqrt{\pi}$.

3) Square Law Combining of FSK Signals: Let f_{0k}, f_{1k} be the frequencies by which sensor k sends bits, $U_k = 0$, $U_k = 1$, respectively. After square law combining of the N branch signals, let L_0, L_1 denote the normalized square law outputs, normalized with respect to the noise variance σ^2, that detect the

combined energies in frequencies, f_{0*}, f_{1*}, respectively. Under H_0, L_0, and L_1 can be represented by the following equations:

$$L_0 = \sum_{k=1}^{N} y_{0k} = \sum_{k=1}^{N} (1 - U_k) X_{1k} + U_k X_{0k}$$

$$L_1 = \sum_{k=1}^{N} y_{1k} = \sum_{k=1}^{N} U_k X_{1k} + (1 - U_k) X_{0k} \tag{19}$$

where X_{1k} is distributed as exponential with mean $(\gamma + 1)$ and X_{0k} is distributed as exponential with mean 1, when the frequency f_{1k} was transmitted by the sensor k, and $\gamma = (\sigma_R^2/\sigma^2)$ is the average SNR. The distributions are interchanged when the frequency f_{0k} is sent. U_k, X_{1k}, X_{0k} are all mutually statistically independent and are mutually independent across the index k. The square law combining makes a decision by comparing $(L_1 - L_0)$ with the threshold, NC_{SLC}. Hence, see (20), shown at the bottom of the page, where $(2P_f - 1)\gamma < C^*_{\mathrm{SLC}} = (C_{\mathrm{SLC}}/\sigma^2) < (2P_d - 1)\gamma$.

4) Optimal LR Rule: Using [17] and by denoting λ_i as the output of the LR statistic based on the MF output y_i

$$\lambda_i = \frac{P_d e^{-((y_i - h_i)^2/2\sigma^2)} + (1 - P_d) e^{-((y_i + h_i)^2/2\sigma^2)}}{P_f e^{-((y_i - h_i)^2/2\sigma^2)} + (1 - P_f) e^{-((y_i + h_i)^2/2\sigma^2)}}. \tag{21}$$

The LR test decides the presence of a POI when $\sum_{i=1}^{N} \log(\lambda_i)$ exceeds a constant. The error exponents can be numerically computed. For a fixed average channel SNR and for each combiner, we observe how the error exponents vary with thresholds. Figs. 4–6 show the variations of the miss error exponent against the false alarm error exponent for $P_f = 0.1$, $P_d = 0.7$ and for various values of SNR. As expected, the curves for the LR fusion rule stay above all the others. However, the implementation of LR rule requires the error probabilities associated with a sensor decision and the channel state information, and hence is complex. From Fig. 4, we observe that for PSK in Rayleigh-fading channels, at very low SNR of -5 dB, the MRC outperforms EGC and CR. This is consistent with the fact that the optimal likelihood ratio test is approximated by MRC, as indicated in [17]. At high SNRs of 10 dB (Fig. 5), CR exhibits the best performance when compared to the other two suboptimal rules. For moderate SNRs, equal gain combining has a better performance than maximal ratio combining and counting rule.

Individual decisions required in CR are based on coherent detection of PSK signals and hence requires the tracking of carrier phases of individual sensor-to-fusion links. In addition, the maximal ratio combining fusion rule requires the channel state information, viz., channel coefficients $\{h_j\}$. Hence, considering both the complexity of implementation and the performance, equal gain combining is the best choice for low to moderate SNR, whereas counting rule is the best choice for large SNR values.

$$D_{\mathrm{FSLC}} = -\log\left(\inf_{(1/1+\gamma) > \tau > 0} \left\{ e^{-\tau C^*_{\mathrm{SLC}}} \left(\frac{P_f}{(1 - \tau(1+\gamma))(1+\tau)} + \frac{1 - P_f}{(1 + \tau(1+\gamma))(1-\tau)} \right) \right\} \right) \tag{20}$$
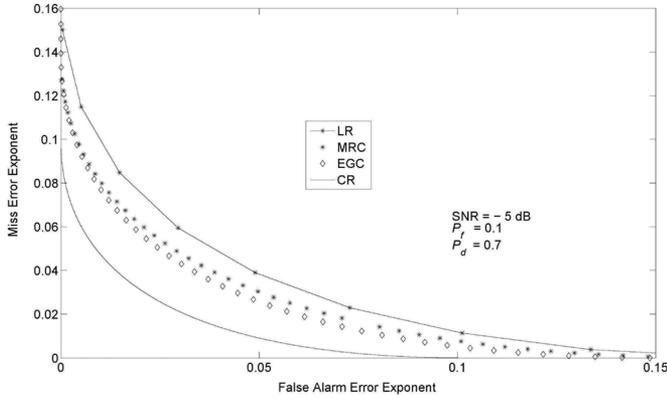
Fig. 4. Error Exponents, PSK in Rayleigh-Fading, Miss versus False Alarm, SNR = −5 dB.
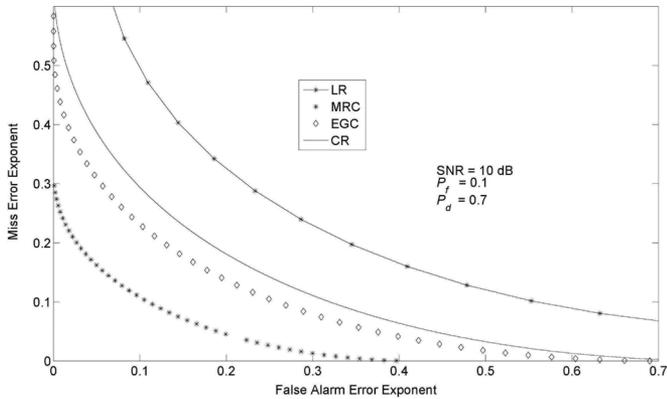

Fig. 5. Error Exponents, PSK in Rayleigh-Fading, Miss versus False Alarm, SNR = 10 dB.
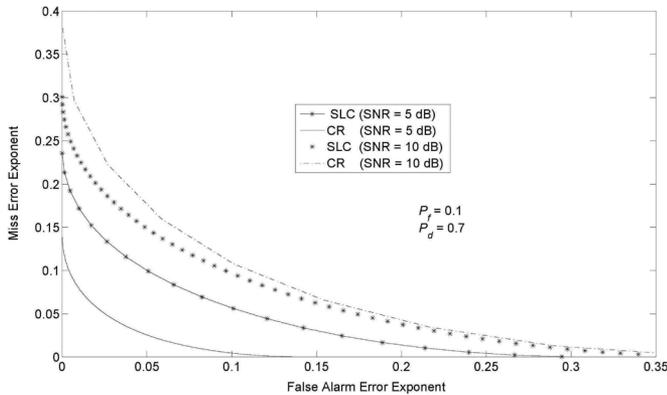

Fig. 6. Asymptotic error exponents, FSK in Rayleigh-fading, miss versus false alarm.

These relative performances are similar to those obtained for the case of a small number of sensors (see [17]; the Chair-Varshney rule is equivalent to a CR for the i.i.d. sensors and channels). As expected, a larger P_d at a given P_f (i.e., a better quality sensor) leads to increased exponents for both types of errors (for the sake of brevity, results are not shown, but are available in [24]).

For binary FSK in Rayleigh-fading channels, from Fig. 6 and other observations not shown here (see [24]), we can conclude

that square law combining outperforms counting rule for SNR values of 0 dB and 5 dB. Only from moderate SNR of 10 dB to very high SNR values, does counting rule outperform square law combining. In general, the best error exponents achieved with FSK are below those achieved with PSK. Considering that noncoherent FSK does not require carrier phase tracking, when FSK is chosen as the modulation scheme, square law combining with FSK is a good choice for low to moderate SNR. At high SNR, counting rule is preferred over square law combining.

B. Asymptotic Performance in AWGN Channel

Considering that significant direct line of sight propagation could exist between the sensors and the FC in certain applications, it will be of interest to know the asymptotic error exponents of different combiners in such situations. For the case of PSK signals in AWGN, we consider EGC (which is equivalent to MRC for this channel) and CR, whereas for FSK signals, we consider SLC and CR.

For EGC, (17) applies with h_j replaced by 1. The false alarm error exponent is given by

$$
\begin{aligned}
&D_{\text{F-NF-EGC}} \\
&= -\log\left(\inf_{\tau>0}\left\{e^{-\tau(C+1)}e^{\tau^2/2\gamma}\left[(1-P_f)+P_f e^{-2\tau}\right]\right\}\right)
\end{aligned}
\tag{22}
$$

where $2P_f - 1 < C < 2P_d - 1$ and γ is the SNR. For CR, (2)–(3) apply with $P_C = Q\left(\sqrt{2\gamma}\right)$ for PSK and $P_C = 0.5e^{-\gamma/2}$ for noncoherent FSK. For SLC, (19) applies but with X_{1k}, X_{0k} being distributed as noncentral chi-square with 2-degrees of freedom and mean $2(1+\gamma)$, and exponential with mean 2, respectively, when the frequency f_{1k} was transmitted by the sensor k. The distributions will be interchanged when the frequency f_{0k} was sent. The false alarm error exponent is given by

$$
\begin{aligned}
D_{\text{F-NF-SLC}} = -\log\Bigg(&\inf_{0<\tau<1/2}\Big\{\exp(-\tau C_{\text{NF-SLC}}) \\
&\times\left(\frac{P_f e^{(2\gamma\tau/1-2\tau)}+(1-P_f)e^{-(2\gamma\tau/1+2\tau)}}{1-4\tau^2}\right)\Big\}\Bigg)
\end{aligned}
\tag{23}
$$

where $2(2P_f - 1)\gamma < C_{\text{NF-SLC}} < 2(2P_d - 1)\gamma$. For PSK and FSK signals received in a zero mean AWGN channel, we show representative asymptotic error exponents of suboptimal fusion rules in Figs. 7 and 8, respectively ($P_f = 0.1$, $P_d = 0.7$ and SNR values of 0 and 10 dB). We have observed for PSK, at SNR of 0 dB and for all combinations $P_f \in (0.001, 0.1)$, $P_d \in (0.1, 0.7)$, equal gain combining outperforms counting rule. However, with increase in channel SNR, the performance of counting rule gets better when compared to equal gain combiner. In the case of FSK, for all combinations of $P_f \in (0.001, 0.1)$, $P_d \in (0.1, 0.7)$ and SNR values considered, square law combining is better than counting rule for small SNR values. Only for large SNR, counting rule outperforms square law combining.
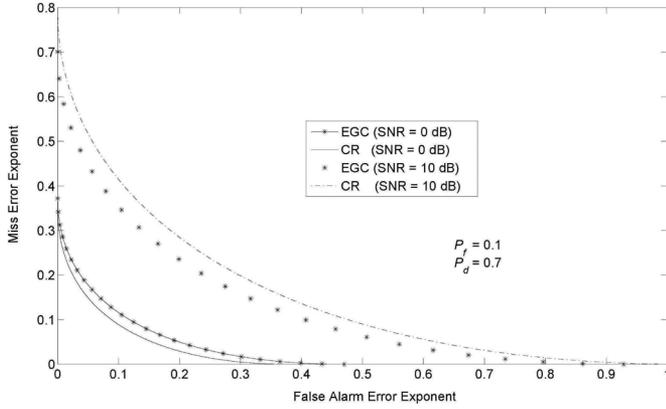
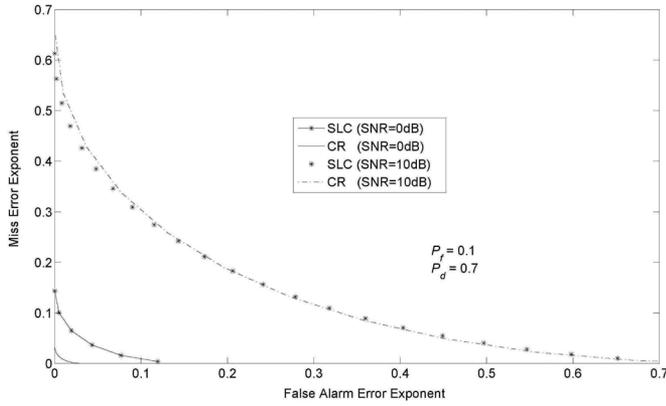Fig. 7. Asymptotic error exponents, PSK in AWGN, miss versus false alarm.

Fig. 8. Asymptotic error exponents, FSK in AWGN, miss versus false alarm.

IV. CONCLUSION

In this paper we have analyzed the impact of the quality of wireless sensor links on decentralized detection performance of wireless sensor networks. For a counting rule (CR) at the fusion center (FC) and a finite number of sensors (N), we have shown that the probability of false alarm of the CR could be several decades higher than the probability of false alarm of the sensor, depending on the channel SNR. Moreover, for a counting rule, slow Rayleigh channel, and a large number of sensors, the OOK with the proper choice of individual sensor decision threshold at the FC, in addition to providing energy saving, provides error performance comparable to that of FSK. For PSK signals, the relative performances of counting rule, maximal ratio combining and equal gain combining, for very large N and Rayleigh-fading or AWGN channel, resemble those seen earlier for finite N and Rayleigh-fading by Chen, Jiang, Kasetkesam and Varshney, viz., equal gain combining performs the best for low and moderate SNR, with the counting rule achieving best performance for large SNR. For FSK signals, in both AWGN and Rayleigh- fading channels, square law combining shows better performance over counting rule at low SNR values, with the converse being true for high SNR values. Extension of the analysis to nonidentical sensors and nonidentical sensor-fusion links will be meaningful. Also, the present work was based on the situation where the sensors transmit their data to the fusion center without any relay nodes. Further performance analysis involving amplify forward or decode forward relays in the network will be worth investigating.

APPENDIX

False Alarm Probability of CR With Independent Fading Links: We prove that the average link error probability can be used for each link while computing the overall false alarm (and detection) error probability of a CR.[2] Let $\gamma_1, \gamma_2, \ldots, \gamma_N$ be the instantaneous SNRs of the received signals corresponding to the individual links between a sensor and the fusion center. Then, for a specified counting rule at the fusion center with the threshold t, the false alarm probability of the FC decision can be written as $P_{F0} = E(f(P_f, \gamma_1, \gamma_2, \ldots, \gamma_N, t))$, where the expectation operation is with respect to the distribution of the instantaneous SNRs, $\gamma_1, \gamma_2, \ldots, \gamma_N$ and $f(\cdot)$ describes the function that determines the conditional false alarm probability of the CR, conditioned on the instantaneous SNRs. Hence,

$$P_{F0} = E\left(\sum_{s=t}^{N} P\left(s \text{ received counts in favor of } H_1 \text{ with } p_j \right.\right.$$
$$\left.\left. \text{the corresponding probability for the } j^{\text{th}} \text{count} \right) \right)$$

where p_j depends on γ_j and P_f. Equivalently

$$P_{F0} = E\left[\sum_{s=t}^{N} \sum_{\substack{i_1+i_2+\cdots i_N = s \\ i_k \in \{0,1\}}} \prod_{j=1}^{N} p_j^{i_j} (1-p_j)^{1-i_j} \right]$$
$$= \sum_{s=t}^{N} \sum_{\substack{i_1+i_2+\cdots i_N = s \\ i_k \in \{0,1\}}} \prod_{j=1}^{N} E\left(\left(p_j^{i_j}\right)(1-p_j)^{1-i_j} \right)$$

since $(\gamma_j, \; j = 1, 2, \ldots, N)$ are independent. Using the following relation:

$$E\left(p_j^{i_j}(1-p_j)^{1-i_j} \right) = \begin{cases} E(p_j), & \text{if } i_j = 1 \\ E(1-p_j), & \text{if } i_j = 0 \end{cases}$$

in the previous equation, we get

$$P_{F0} = \sum_{s=t}^{N} \sum_{\substack{i_1+i_2+\cdots i_N = s \\ i_k \in \{0,1\}}} (E(p_j))^{i_j} (1 - E(p_j))^{1-i_j}.$$

The above equation shows that the average probability can be used for the jth link in order to arrive at P_{F0}. If the links are identical, then $E(p_j)$ is independent of j and P_{F0} becomes a sum of Binomial probabilities.

[2]A reviewer had pointed out a proof using the optimality of the CR. For i.i.d sensors and i.i.d channels, assuming that the instantaneous channel SNR is unavailable, the probability mass function of a decision received will be the mass function of the decision, averaged with respect to the distribution of the instantaneous channel SNR, and the LRT becomes a CR based on the received decisions. The proof here does not assume optimality of the CR and is also valid for nonidentical links.

REFERENCES

[1] J. N. Tsitsiklis, "Decentralized detection," in *Advances in Statistical Signal Processing*. Greenwich, CT: JAI, 1993, vol. 2, pp. 297–344.

[2] J. N. Tsitsiklis, "Decentralized detection by a large number of sensors," *Math. Control. Signal. Syst.*, vol. 1, pp. 167–182, 1988.

[3] P. K. Varshney, *Distributed Detection with Data Fusion*. New York: Springer, 1997.

[4] R. Viswanathan and P. K. Varshney, "Distributed detection with multiple sensors: Part I fundamentals," *Proc. IEEE*, vol. 85, pp. 54–63, Jan. 1997.

[5] R. S. Blum, S. A. Kassam, and H. V. Poor, "Distributed detection with multiple sensors: Part II advanced topics," *Proc. IEEE*, vol. 85, pp. 64–79, Jan. 1997.

[6] Y. Sung, L. Tong, and A. Swami, "Asymptotic locally optimal detector for large-scale sensor networks under the Poisson regime," *IEEE Trans. Signal Process.*, vol. 53, no. 6, pp. 2005–2017, Jun. 2005.

[7] J.-F. Chamberland and V. V. Veeravalli, "The impact of fading on decentralized detection in power constrained wireless sensor networks," in *Proc. Int. Conf. Acoust., Speech, Signal Process. (ICASSP)*, May 2004, vol. 3, pp. iii: 837–iii: 840.

[8] J.-F. Chamberland and V. V. Veeravalli, "Decentralized detection in sensor networks," *IEEE Trans. Signal Process.*, vol. 51, no. 2, pp. 407–416, Feb. 2003.

[9] J.-F. Chamberland and V. V. Veeravalli, "Asymptotic results for decentralized detection in power constrained wireless sensor networks," *IEEE J. Sel. Area. Commun.*, vol. 22, no. 6, pp. 1007–1015, Aug. 2004.

[10] S. K. Jayaweera, "Large system performance of power-constrained distributed detection with analog local processing," in *Proc. IEEE Int. Conf. Wireless Networks, Commun., Mobile Comput.*, Jun. 2005, vol. 2, pp. 1083–1088.

[11] S. K. Jayaweera, "Large sensor system performance of decentralized detection in noisy, bandlimited channels," in *Proc. IEEE Vehicular Tech. Conf.*, May–Jun. 2005, vol. 2, pp. 1096–1100.

[12] S. K. Jayaweera, "Large system decentralized detection performance under communication constraints," *IEEE Commun. Lett.*, vol. 9, no. 9, pp. 769–771, Sep. 2005.

[13] K. Liu and A. Sayeed, "Type-based decentralized detection in wireless sensor networks," *IEEE Trans. Signal Process.*, vol. 55, no. 5, pp. 1899–1910, May 2007.

[14] G. Mergen, V. Naware, and L. Tong, "Asymptotic detection performance of type-based multiple access over multiaccess fading channels," *IEEE Trans. Signal Process.*, vol. 55, no. 3, pp. 1081–1092, Mar. 2007.

[15] A. Anandkumar and L. Tong, "Type-based random access for distributed detection over multi access fading channels," *IEEE Trans. Signal Process.*, vol. 55, no. 10, pp. 5032–5043, Oct. 2007.

[16] S. A. Aldosari and J. M. F. T. Moura, "Fusion in sensor networks with communication constraints," in *Proc. 3rd Int. Symp. Information Process. Sensor Networks*, Apr. 2004, pp. 108–115.

[17] B. Chen, R. Jiang, T. Kasetkesam, and P. K. Varshney, "Channel aware decision fusion in wireless sensor networks," *IEEE Trans. Signal Process.*, vol. 52, no. 12, pp. 3454–3458, Dec. 2004.

[18] R. Niu, B. Chen, and P. K. Varshney, "Fusion of decisions transmitted over Rayleigh fading channels in wireless sensor networks," *IEEE Trans. Signal Process.*, vol. 54, no. 3, pp. 1018–1027, Mar. 2006.

[19] B. Chen and P. K. Willet, "On the optimality of the likelihood-ratio test for local sensor decision rules in the presence of nonideal channels," *IEEE Trans. Inf. Theory*, vol. 51, no. 2, pp. 693–699, Feb. 2005.

[20] A. Kashyap, "Comments on on the optimality of the likelihood-ratio test for local sensor decision rules in the presence of nonideal channels," *IEEE Trans. Inf. Theory*, vol. 52, no. 3, pp. 1274–1275, Mar. 2006.

[21] J. Proakis, *Digital Communications*. New York: McGraw-Hill, 2002.

[22] R. R. Bahadur, "Some limit theorems in statistics," in *Regional Conference Series in Applied Mathematics*. Philadelphia, PA: SIAM, 1971.

[23] M. Schwartz, W. R. Bennett, and S. Stein, *Communication Systems and Techniques*. New York: McGraw-Hill, 1966.

[24] V. R. Kanchmarthy and R. Viswanathan, "Further impacts on the quality of wireless sensor links on decentralized detection performance," in *Proc. Conf. Information Sciences and Systems (CISS)*, Princeton, NJ, 2006, pp. 44–49.

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