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On the Calculation of the Exponential Bound Parameter for Phase Quantized 8-PSK

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***Abstract*—The exponential bound parameter is numerically evaluated for 16-zone quantized 8-PSK for signal vectors bisecting decision regions and for signal vectors lying on decision boundaries. For decision regions of equal span, signal vectors lying on decision boundaries are superior, confirming the result of Parsons and Wilson. Signal vectors bisecting decision regions are superior if span of decision regions not containing signal vectors is optimized. Using the partial derivative, it is shown that the optimal configuration depends on SNR. Furthermore, the optimal configuration is not necessarily one in which the decision boundaries are straight lines.**

I. Introduction

During the past decades, forward error correcting codes have become widely accepted for digital communications over additive white Gaussian noise (AWGN) channels, particularly satellite channels, which are power and bandwidth limited. More recently, the need for spectral efficiency on these channels has motivated the development of spectrally efficient coding schemes, such as trellis coded modulation (TCM), which use non-binary signaling. Often there is the additional requirement of constant envelope signaling, leading to the use of 8, 16, or 32 PSK. Theoretical predictions of information rates on AWGN channels generally assume the use of coding, and maximum likelihood detection using soft decisions. This in turn assumes a continuous signal vector space for the receiver; however, the design of practical decoders requires that the receiver space be quantized in some way.

Phase-quantized receivers are practical to build, and are an intuitively logical choice for use with PSK signaling. The merits of designing quantizers to optimize R_0 have been well established [1]. Parsons and Wilson [2] have investigated the design of phase only quantizers to optimize R_0 for PSK signaling. The purpose of this paper is to expand on the work of Parsons and Wilson, and to show that additional options exist for increasing R_0.

II. The Exponential Bound Parameter

It has been shown [1,3,4] that the attainable performance of codes using outputs from a quantized channel is bounded by:

$$P_e \le C_r 2^{-N[R_0 - R]} \tag{1}$$

provided that $R_0 > R$. Here, N is the block length of the code, R is the number of data bits per symbol, R_0 is the exponential bound parameter, and C_R is an empirically determined constant.

For a discrete channel having M equiprobable inputs $\{s_i\}$, and a finite set of outputs $\{z_j\}$, R_0 is calculated from the transition probabilities $P(z_j|s_i)$ as follows:

$$R_0 = -\log_2\left\{ \frac{1}{M^2} \sum_j \left[\sum_{i=0}^{M-1} \sqrt{P(z_j|s_i)} \right]^2 \right\} \tag{2}$$

Quantization degrades the channel but is often a practical necessity. The degradation is minimized when the quantizer is designed to maximize the exponential bound parameter.

The exponential bound parameter may be optimized by applying Lee's criterion [5], which states that if ρ is a point on the boundary between two decision regions D_a and D_b of an optimal quantizer then it is necessary that:

$$\sum_{m=0}^{M-1} \left[\frac{1}{\sqrt{P(a|m)}} \sum_{i=0}^{M-1} \sqrt{P(a|i)} \right] f(r|m) = \sum_{m=0}^{M-1} \left[\frac{1}{\sqrt{P(b|m)}} \sum_{i=0}^{M-1} \sqrt{P(b|i)} \right] f(r|m) \tag{3}$$

where $f(x|m)$ is the probability density function of the received vector given that signal m was transmitted, $P(a|m)$ is the probability that the quantizer will select D_a, given that m was transmitted, and $P(b|m)$ is the probability that the quantizer will select D_b, given that m was transmitted. Lee's criterion indicates a local maximimum of R_0, and is a necessary but not sufficient for a global maximimum.

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III. Exponential Bound Parameter For Phase Only Quantizers

Parsons and Wilson [2] have investigated the design of phase only quantizers, for M-ary PSK with M=4, 8, and 16; using quantizers of M, $2M$ and $4M$ zones, and have shown that Lee's criterion is met by quantizers in which the signal vectors lie on boundaries of decision regions as shown in Fig. 1, as opposed to quantizers in which the signal vectors bisect decision regions as shown in Fig. 2. This result is derived for 16 sector 8-PSK and then extended to 32-sector 8-PSK.

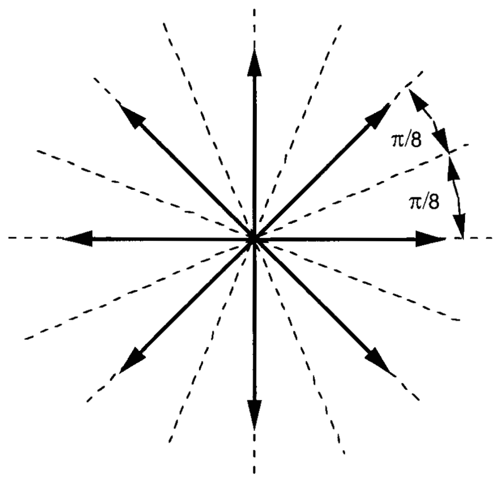

Fig. 1. First configuration for 16-sector 8-PSK.

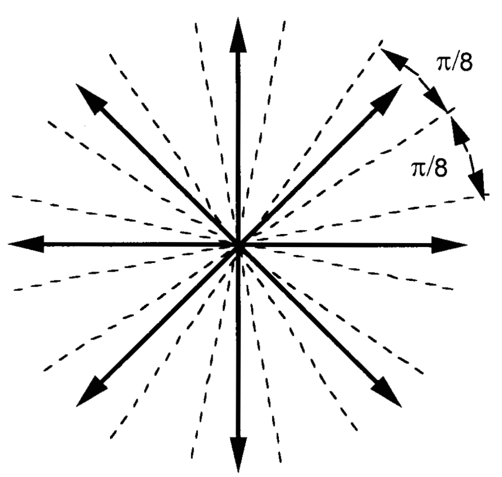

Fig. 2. Second configuration for 16-sector 8-PSK.

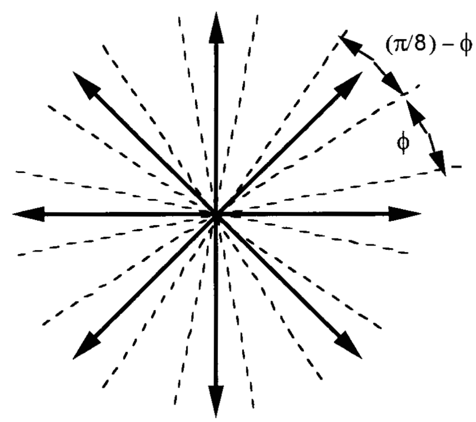

Fig 3. Third configuration for 16-sector 8-PSK.

As Lee's criterion does not guarantee a global opimimum, further optimization of R_0 was sought and obtained using the configuration of Fig. 3. In this configuration the 8 sectors which encompass a signal vector have span of ϕ, the 8 which do not, have span of $\frac{\pi}{8} - \phi$. The optimal value of ϕ depends on the signal-to-noise ratio, and by selecting an appropriate value of ϕ, it is possible to make R_0 for configuration 3 exceed R_0 for the configuration 1. Fig. 4 shows R_0 for the configuration 3 as a function of ϕ for $\frac{E_S}{N_0}$ = 9, 10, and 11dB. Fig. 5 shows R_0 as a function of $\frac{E_S}{N_0}$ for the three configurations of 16-sector 8-PSK. Here, ϕ is chosen to optimize R_0 at $\frac{E_S}{N_0} = 10dB$. Note that if configuration 3 is optimized, it has the greatest value of R_0 and the difference between configuration 3 and either 1 or 2 is greater than that between 1 and 2.

When R_0 is optimized over ϕ, which can be accomplished by calculating $\frac{d}{d\phi}R_0$, we find that Lee's criterion is not met at all points on the boundaries. Thus we suspect that further optimization would be obtained from a configuration in which the decision boundaries are not necessarily straight lines, although obtaining the exact configuration may be analytically intractable, and the gains minimal.

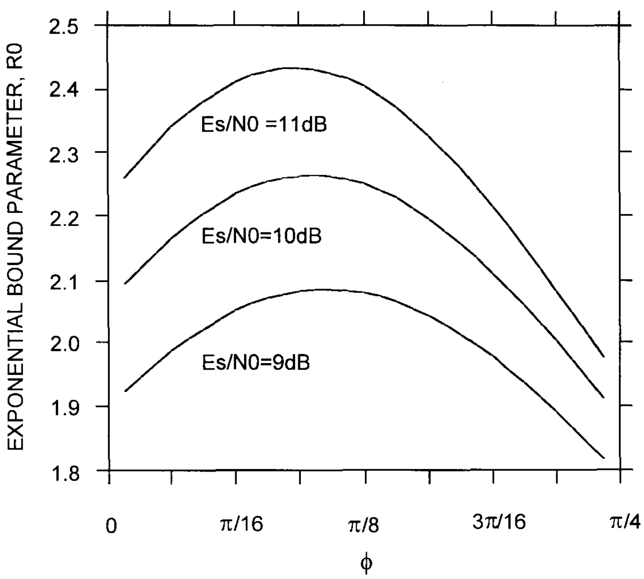

Fig. 4. Optimization of R_0 for 16-sector 8-PSK.

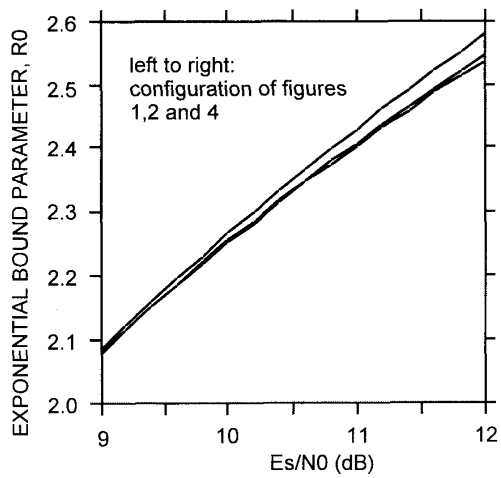

Fig. 5. R_0 for 3 configurations of 16-sector 8-PSK.

V. Conclusion

We have looked at the theoretical optimization of R_0 for phase quantized PSK signaling and have also calculated numerical values of R_0 for a few useful configurations. For 16-sector 8-PSK, the optimal configuration is not necessarily one in which the decision boundaries are straight lines. With the constraint that the decision boundaries be straight lines, the optimal configuration is one in which half of the decision boundaries contain signal vectors, while the span of the null decision regions is optimized for SNR. For 32-sector 8-PSK, the value of R_0 differs insignificantly among a number of configurations, and is in fact very close to the value of R_0 for unquantized phase. Therefore, we conclude that in the construction of systems to implement phase-quantized 8-PSK, minute gains obtainable by fine adjustments in decision boundaries of the quantizer will be overshadowed by practical considerations in hardware development.

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