Fast TCM Decoding: Phase Quantization and Integer Weighting

Frank Carden
New Mexico State University - Main Campus

Michael D. Ross
New Mexico State University - Main Campus

Brian T. Kopp
New Mexico State University - Main Campus

William P. Osborne
osborne@engr.siu.edu

Fast TCM Decoding: Phase Quantization And Integer Weighting

Frank Carden, *Senior Member, IEEE*, Michael D. Ross, *Member IEEE*, Brian T. Kopp, *Student Member IEEE*, and William P. Osborne, *Member IEEE*

***Abstract*—TCM, combining modulation and coding, achieves coding gains over conventional uncoded multilevel modulation without the attendant bandwidth expansion [2]. Since TCM was proposed by Ungerboeck [2], [3], [4] substantial work has been done in this area. A large portion of the TCM work has been in the area of high-speed data transmission over voice grade modems using quadrature amplitude modulation, QAM [6], [7], [8], [9]. QAM, not having a constant envelope, is unattractive for satellite communication channels employing a TWT with its nonlinear behavior as the power stage. Additional work has been done in utilizing M-ary PSK with TCM [10], [11], [12], [13], [14], [15], [16]. Simulations by Taylor and Chan [13] utilizing a 4-state convolutional code demonstrated the coding gain of a rate 2/3 coded 8-PSK modulation scheme. Wilson et. al. [14] obtained results for 16-PSK TCM using codes with 4 to 32 states and achieved coding gains of 3.5 to 4.8dB respectively, over 8-PSK and demonstrated that small memory codes achieved encouragingly good gains with simple design procedures.**

I. INTRODUCTION

Unlike maximum likelihood decoding of binary sequences employing the Viterbi algorithm, which requires Hamming distance calculations, ideal maximum likelihood decoding of TCM requires real number Euclidean metric calculations, which increases the complexity of the decoder and the processing time. If this is to be avoided by obtaining the metrics from lookup tables and employing integer arithmetic, the received signal and the metrics must be quantized. This work presents the use of phase quantization and integer weight metrics in the decoding of TCM, specifically with 8-PSK signalling and the 4-state code of Ungerboeck [3], [4] (Fig. 1, 2, and 3). The results show that a maximum likelihood decoder can use simplified weights with very little penalty in performance. The concept of integer weighting as a metric is given theoretical justification and incorporated into a phase quantized Euclidean decoder (QED). Extensive simulations are performed in order to compare the performance of the QED using the optimal log-likelikhood metrics and several suboptimal metrics, including integer weights. Finally, the performance of the QED is compared to that of an unquantized maximum likelihood decoder.

Paper approved by Dariush Divsalar, the editor for coding Theory and Applications of the IEEE Communications Society. Manuscript received March 6, 1990, revised October 15 1991. This work was supported by NASA Grant NAGW-1746.

The authors are on thestaff of the Center for Space Telemetering and Telecommunications Systems, New Mexico State University, Las Cruces, New Mexico.

IEEE Log Number 9400967.

II. THE GENERAL LIKELIHOOD FUNCTION

An ideal maximum likelihood decoder, receiving a TCM sequence through a continuous channel, and employing the Viterbi Algorithm [1], makes decisions based on the conditional probability density function $p(Z|S_m)$ which characterizes the channel. Here, Z is the received noisy sequence and S_m is one of many possible transmitted sequences. In the case of the memoryless channel, where the effect of noise is independent from one symbol to the next, the conditional probability density function of a sequence of length L is calculated directly from the probability functions of the individual elements of the sequence, that is:

$$p(Z|S_m) = \prod_{i=1}^{L} p(z_i|s_{mi}) \qquad (1)$$

where z_i is a vector and denotes an element of the sequence Z, and s_{mi} is a vector element of the sequence S_m.

It is computationally expedient, and mathematically equivalent, to use the log-likelihood function $-ln[p(Z|S_m)]$ which may be summed, rather than probability density functions which must be multiplied. The decoder would then select s_m to minimize:

$$-ln[p(Z|S_m)] = \sum_{i=1}^{L} -ln[p(z_i|s_{mi})] \qquad (2)$$

In the case of an additive white Gaussian noise channel, the log-likelihood function to be minimized is: $\sum_{i=1}^{L} |z_i - s_{mi}|^2$, that is the Euclidean metric between Z and S_m.

Equations (1) and (2) imply that the decoder must obtain a metric (a log-likelihood or a weight derived from the log-likelihood) for every symbol in its alphabet each time a signal vector is received. Time may be saved by obtaining the metrics from lookup tables, however this requires that the received signal be quantized in some way. It has already been shown [10] that when the in-phase and quadrature components are quantized independently, performance can be made arbitrarily close to that of an unquantized system by

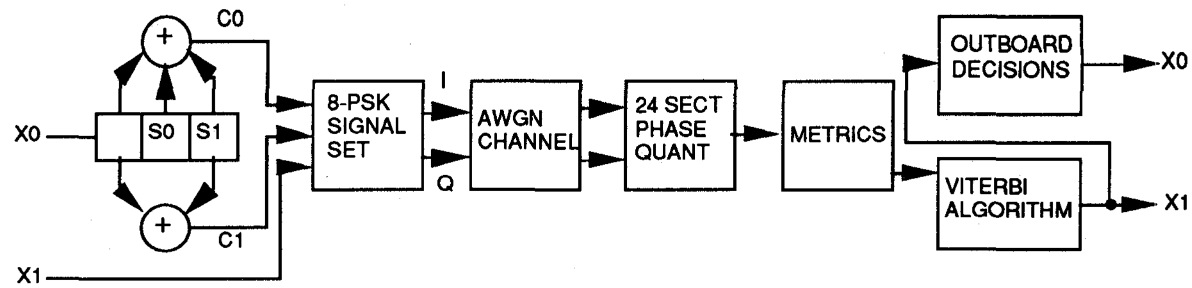

Fig. 1. 4-state 8-PSK system

specifying a sufficient number of quantization levels for each component. However, the total number of quantization points then becomes quite large. In fact, in the work of [10], the metrics were still calculated on line, even after quantization.

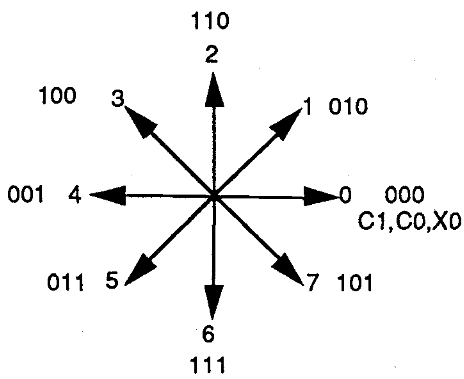

Fig. 2. 8-PSK Signal Constellation.

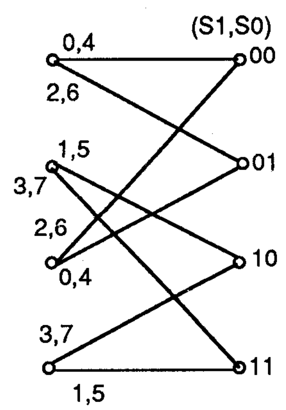

Fig. 3. 4-state 8-PSK Trellis Diagram.

III. PHASE QUANTIZATION

When PSK is received through a noisy channel, both the amplitude and phase of the received vector are used in making maximum likelihood decisions. However, it is often of interest to investigate the performance of systems which make maximum likelihood decisions using phase information only, as these systems are less complex in some applications. Also, some form of quantization is necessary to make use of a lookup table, as mentioned in the previous section. Therefore, phase quantization of PSK signal vectors, with hardlimiting is considered.

The term phase quantization is used here to mean that each received signal vector of a PSK sequence is quantized to the nearest of a set of quantization points located at even intervals on the circle of radius $\sqrt{E_s}$ as shown in Fig. 4. The effect on performance of this form of quantization with unquantized metrics has been demonstrated for 8-PSK with 16, 24, and 32 sectors, as well as for 16-PSK with 32, 48 and 64 sectors, with 4-state codes [17], [18], [19]. Here it was found that, for 8-PSK the performance difference between 16 sectors and 24 sectors was significant, while 32 sectors was only a slight improvement over 24. The results for 16-PSK were similar, with 48 sectors being significantly better than 32, and 64 sectors being only slightly better than 48. Later [21] similar results were found with 8-PSK and an eight state code. The performance of 24 sector 8-PSK was found to be within 0.6 dB of unquantized. Parsons and Wilson [20] have shown that the symmetric cutoff rate, R_0 is optimized when the thresholds of the quantization zones lie on the signal vectors, unlike the configurations discussed here. Fig. 5 presents a comparison of R_0 for the two cases (using 32 sector quantization) and the numerical difference in the actual values of R_0 is indeed very slight. Hence the choice between the two quantization threshold values has no effect on the overall quantizer decoder combination. and does not detract from the utility of integer weighting discussed in the following section.

IV. PROBABILITY CALCULATIONS AND WEIGHTING FUNCTIONS

With the discrete input, discrete output channel, the decoder is to find the sequence which maximizes the sequence likelihood function:

$$P(Z|S_m) = \prod_{i=1}^{L} P(z_i\,|s_{mi}) \qquad (3)$$

This is (1) except that the probability density functions p are replaced by the probabilities P, and Z indicates the sequence of discrete outputs of the quantizer instead of the sequence of continuous vectors from the Gaussian channel. As in the general problem, it is desirable to have weights which the decoder can sum along a sequence rather than probabilities (or

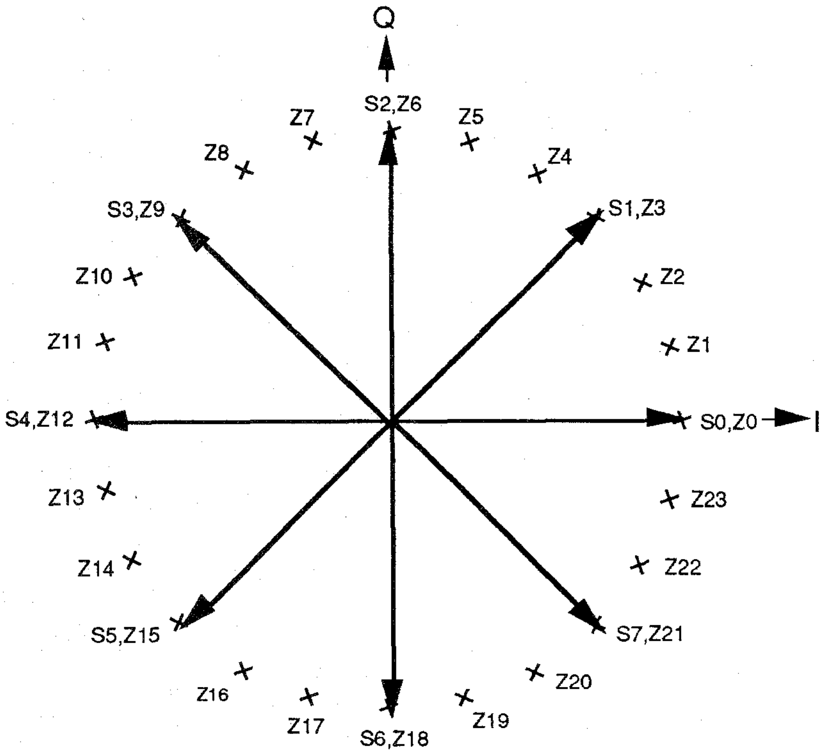

Fig. 4. 24-sector 8-PSK signal constellation

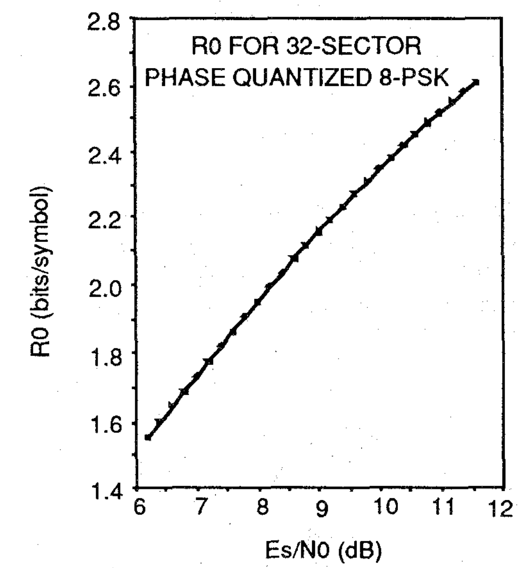

Fig. 5. R0 for 32-sector phase quantized 8-PSK with optimal spacing, and configuration of kind shown in Fig. 4.

probability densities) which must be multiplied. Suppose the decoder is made to select the sequence S to minimize:

$$P(Z|S) = \sum_{i=1}^{L} w(z_i, s_{mi}) \tag{4}$$

where $w(z_i, s_{mi})$ is an appropriately chosen weighting function or metric.

Such a decoder will yield maximum likelihood decisions (with respect to the quantizer output) if and only if $w(z,s) = a - b\ ln[P(z|s)]$ where a is any real constant and b is any real positive constant. It is of engineering interest to determine if there are simpler weighting functions which, although suboptimal, will yield acceptable degradation. Although the lookup table allows any arbitrary set of weights to be used, the use of simple weighting functions whose values can be represented by a minimal number of bits will greatly facilitate VLSI implementation. In any case, it is certain that $w(z,s)$ should decrease with $P(z|s)$.

For a phase quantized system, the phase sector probabilities are needed to determine the weighting function. Integrating the phase probability density function requires double integration, since all known forms of the density function include the $Q()$ function which exists only in integral form. Alternatively, sector probabilities may be obtained from $P[\phi] = P\ [0 \le \phi_z \le \phi]$ where ϕ_z is the phase of the unquantized signal received signal vector. $P[\phi]$ is found by integrating the Gaussian distribution over S_ϕ as shown in Fig. 6:

$$P[\phi] = \frac{1}{\pi N_0} \iint_{S_\phi} exp\left\{ -\frac{1}{N_0}\left[\left(x - \sqrt{E_s}\right)^2 + y^2 \right] \right\} dxdy \tag{5}$$

Making a change of variables, the above double integral reduces to the single integral:

$$P[\phi] = \frac{1}{2} - \frac{1}{2\pi} \int_0^{\pi-\phi} exp\left\{ -\frac{E_s}{N_0} \left[\frac{sin\theta}{tan\phi} + cos\theta \right]^{-2} \right\} d\theta \tag{6}$$

For the 8-PSK system of Fig. 1, and the 24-sector quantization scheme of Fig. 4, the set of probabilities to be calculated reduces to $P(z_d|s_0)$ for $0 \le d \le 6$, due to the symmetries involved, and the fact that the outboard decision eliminates symbol vectors removed from the received symbol vector by more than six sectors. Let w_d be the weight to associate with a symbol vector removed from the received vector by d sectors. Fig. 7 shows the log-likelihood function, $-ln[P(z_d|s_0)]$ for 24-sector 8-PSK, calculated at E_s/N_0 = 10dB. As can be seen, the log-likelihood function is approximately linear for $1 \le d \le 6$, the region of interest to the decoder. This suggests that simple integer weighting, letting $w_d = d$ should produce good results, and simulations show this is indeed the case.

Reasonable candidates for w_d include the following:

1. the log-likelihood function: $w_d = -ln[P(z_d|s_0)]$

2. n-bit representation of the log-likelihood function

w_d = nearest integer to

$$\left[(2^n - 1)\frac{ln[P(z_0|s_0)] - ln[P(z_d|s_0)]}{ln[P(z_0|s_0)] - ln[P(z_6|s_0)]}\right]$$

3. Euclidean metric: $w_d = |z_d - s_0|^2$

4. Integer weighting: $w_d = d$

All of the above weighting functions were tested through simulation and found to yield results which differed only slightly from the results obtained using the log-likelihood function, which is the optimal weighting function.

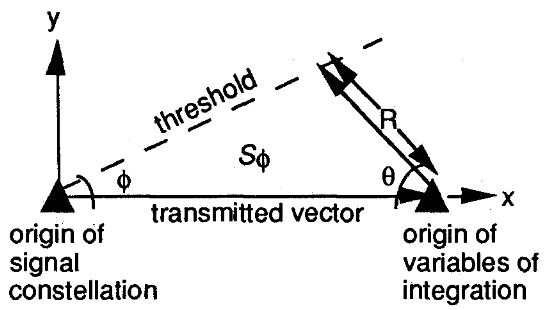

Fig. 6. Region of integration for sector probability calculation.

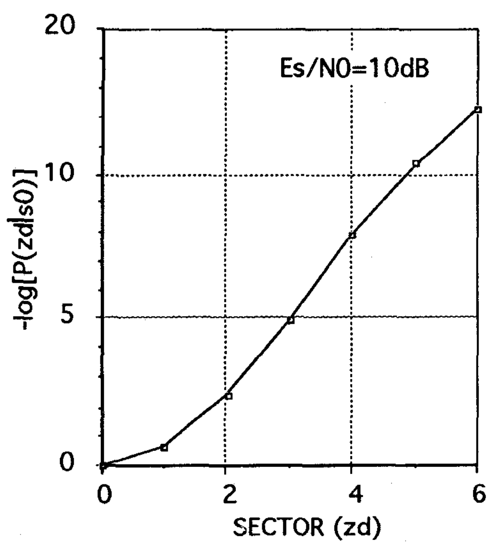

Fig. 7. Log-likelihoods for 24-sector phase quantizarion.

V. RESULTS AND CONCLUSION

Integer weighting is an effective and computationally efficient means of implementing Viterbi decoding on a trellis coded sequence of 8-PSK symbols. Use of 24-sector phase quantized 8-PSK with Euclidean metrics is found to result in a degradation of approximately 0.6dB as compared to unquantized 8-PSK. When phase quantization is used, integer weights are very nearly as good as log-likelihood weights, which are optimal (for phase quantization), but much more complicated.

Fig. 8 shows the results. The 3-bit representation of the log-likelihood function did not perform significantly worse than the log-likelihood function itself, and the Euclidean metric performed less than 0.05dB worse than the log-likelihood function. In principle, the log-likelihood function depends on the SNR, however, the numerical value of the weights varies little over the useful range of the decoder, ie, from 6dB to 12dB. In the results shown, the weights were calculated for an SNR of E_s/N_0 = 10dB, and then used for all SNR's. The use of simple integer weights results in a degradation of about 0.2 dB from the theoretically optimal log-likelihood weights, which shows that integer weighting is a viable solution for fast TCM decoding.

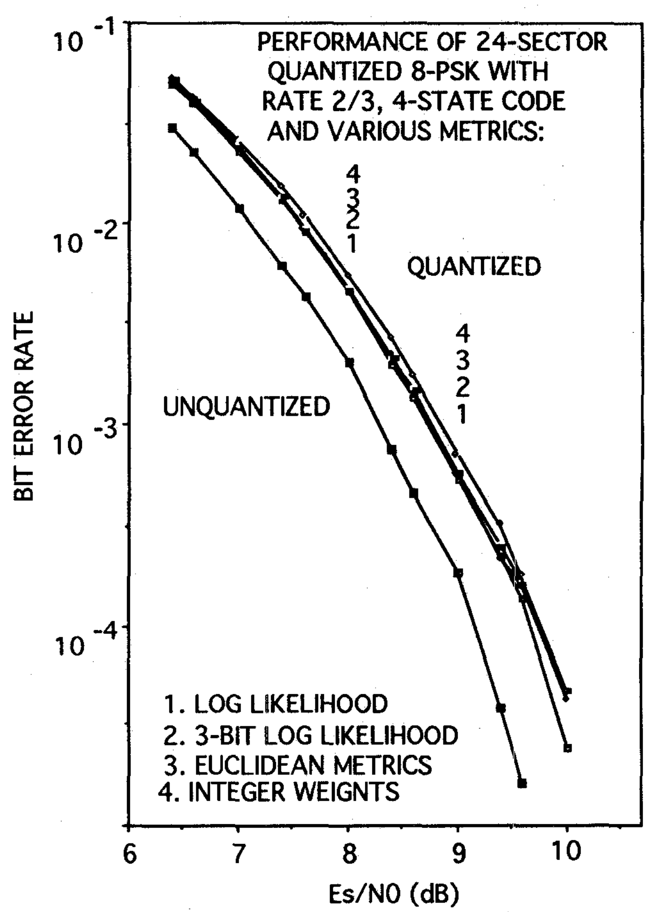

Fig. 8. Bit error rate performance for various metrics.

REFERENCES

[1] A.J. Viterbi, "Convolutional Codes and their Performance in Communication Systems," *IEEE Transactions on Communication Technology*, Vol. CT-19, pp. 751-771, October 1971.

[2] G. Ungerboeck, "Channel Coding with Multi-Level/Phase Signals," *IEEE Trans. Inform Theory*, Vol. IT-28, pp. 55-66, Jan. 1982.

[3] G. Ungerboeck, "Trellis Coded Modulation with Redundant Signal Sets, Part I: Introduction," IEEE Communications Magazine, Vol. 25, No. 2, pp. 5-11, February 1987.

[4] G. Ungerboeck, "Trellis Coded Modulation with Redundant Signal Sets, Part II: State of the Art," *IEEE Communications Magazine*, Vol. 25, No. 2, pp. 12-21, February 1987.

[5] G. Ungerboeck and I. Csajka, "On Improving Data Link Performance by Increasing the Channel Alphabet and Introducing Sequence Coding," presented at *Int. Symp. Inform Theory*, Ronneby, Sweden, 1976.

[6] H. K. Thapar, "Real Time Application of Trellis Coding to High-Speed Voiceband Data Transmission," *IEEE Journal on Selected Areas in Communication*, Vol. SAC-2 No. 5, pp. 648-658, Sept 1984.

[7] J. Paton and S. Qureshi, "Trellis Encoding: What it is and how it Affects Data Transmission," *Data Commun.*, pp. 143-152, May 1985.

[8] A. R. Calderbank and N. J. A. Sloan, "Four-Dimensional Modulation with an Eight-State Trellis Code," *AT&T Tech. Journal*, Vol. 64, No. 5, pp. 1006-1018, May/June 1985.

[9] R. Calderbank and James E. Mazo, "A New Description of Trellis Codes," *IEEE Trans. Inform. Theory*, Vol. IT-30 No. 6, pp. 784-791, Nov 1984.

[10] S. A. Rhodes, R. J. Fang, and P. Y. Chang, "Coded Octal Phase Shift Keying in TDMA Satellite Communications," *COMSAT Tech. Review*, Vol. 13, No. 3, pp. 221- 258, Fall 1983.

[11] E. Biglieri, "High Level Modulation/Coding for Nonlinear Satellite Channels," *IEEE Trans. on Comm*. Vol. COM-32, No. 5, pp. 616-626, May 1984.

[12] S. H. Leibowitz and S. A. Rhodes, "Performance of Coded 8-PSK Signaling for Satellite Communications," in Colnf. Rec., Int. Conf. Commun., Denver, CO< pp. 47.4.1 - 47.4.8, June 1981.

[13] D. P. Taylor and H. C. Chan, "A Simulation Study of Two Bandwidth-Efficient Modulation Techniques," *IEEE Trans. Commun.*, Vol. COM-29, pp. 267-275, Mar. 1981.

[14] S. G. Wilson, et al., "Rate 3/4 Convolutional Coding of 16-PSK: Code Design and Performance Study," *IEEE Trans. on Comm.*, Vol. COM-32, No. 12, pp. 1308-1315, Dec. 1984.

[15] D. Divsalar, M. Simon, "Trellis Coded Modulation for 4800-9600 bits/sec Transmission Over a Fading Mobile Satellite Channel," *IEEE Journal on Selected Areas in Communication*, Vol. SAC-5, No. 2, pp; 162-175, Feb. 87.

[16] A. J. Viterbi, J. K. Wolf, E. Zehavi, and Roberto Padovani, "A Pragmatic Approach to Trellis-Coded Modulation," *IEEE Commun. Mag.*, Vol. 27, No. 7, pp. 11-19, July 1989.

[17] Frank Carden, "A Quantized Euclidean Soft-Decision Maximum Likelihood Sequence Decoder: A Concept for Spectrally Efficient TM Systems," *Proceedings of the International Telemetering Conference*, Vol. XXIV, pp. 375-384, October 1988.

[18] Frank Carden and Brian Kopp, "A Quantized Euclidean Soft Decision Maximum Likelihood Sequence Decoder for TCM," *IEEE Military Communications Conference*, Vol. 2, pp. 679-682, October 1988.

[19] Frank Carden and Michael Ross, "A Spectrally Efficient Communication System Utilizing a Quantized Euclidean Decoder," *Proceedings of the International Telemetering Conference*, Vol. XXV, pp. 575-582, October 1989.

[20] R. D. Parsons, Jr., and S. G. Wilson, "Polar Quantizing for Coded PSK Transmission," *IEEE Transactions on Communications*, Vol. 38, No. 9, pp. 1511-1519.

[21] Michael D. Ross and Frank Carden, "A Quantized 8-PSK 8-State Decoder for Spectrally Efficient Communications," *Proceedings of the International Telemetering Conference*, Vol. XXV, pp. 489-495, October 1990.