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Formation of Buyer-Seller Trade Networks in a Quality-Differentiated Product Market

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Abstract: We examine the formation of buyer-seller links in an environment where exchange can only take place if such a link exists. Sellers can produce products of different qualities and multiple sellers of uniform or mixed quality can form an association to pool their customers setting uniform prices (called a sellers' association). Buyers may form a trade link with either an individual seller or with a sellers' association. We show which buyer-seller links will form and find the set of links (or networks) which are stable. Additionally, we show how these buyer-seller networks influence the price paid for the good or service exchanged. Specifically, we give conditions on the network for which the seller (or sellers' association) is guaranteed a non-negligible positive payoff and conditions on the network for which the buyer is guaranteed a non-negligible positive surplus.

Keywords: Trade Patterns and Pricing, Trade Networks, Sellers' Association

JEL Classification: D40, D51, D82

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1. Introduction

In the past decade, there has been a growing literature establishing theories about economic networks. Among various forms, trade networks are certainly important ones, as they are essential not only for exchange patterns but also for market organizations. While previous studies have created valuable insights towards understanding the emergence of trade networks, many interesting issues still remain unexplored. For example, are there incentives for sellers to form a coalition to set price cooperatively? If the answer is positive, can such a coalition consist of sellers of different quality? In the absence of incomplete information, is it possible to have a trade-network mismatch with the coexistence of unconsumed buyers (buyers who wish to purchase but cannot) and unsold goods or services? How do the equilibrium prices depend on the trade network, especially, the presence of unconsumed buyers or inactive sellers (sellers who wish to sell but do not)? For which types of trade networks can a seller set the price above his reservation price? For which types of trade networks must a seller set price below his customer's reservation price?

To address these questions, we develop a model of trade networks which specifies relationships or links between buyers and sellers producing goods or services of differentiated quality, where exchange can only take place if a link exists. Such a link may represent a formal contract between a buyer and seller or it may represent an informal tie where for instance buyers only buy goods from members of their family, ethnic, or social group.[1] We allow multiple sellers to form a coalition to pool their customers setting uniform prices (called a "sellers' association") and such an association may consist of members of mixed quality. There are many examples of buyer-seller links where both sellers' quality differences and sellers' associations are important. For instance, a trading company in a developing country is like a sellers' association in that it generally represents a group of domestic sellers. The trading company specializes in forming links with international buyers and is thus able to match such buyers with the

[1]Uzzi (1996) examines buyer-seller ties in the New York apparel industry where many business ties are also social ties.

sellers that it represents. These trading companies are common in the clothing industry where domestic sellers have different abilities regarding how fast they can fill an order and the ability to fill an order quickly is highly valued by the buyer. Similarly, in the health care industry, groups of buyers may be represented by an HMO who forms contracts (links) with hospitals and/or hospital chains of different qualities to provide health care service to the HMO's members.[2]

In this paper, we show which buyer-seller links will form and find the set of links (or networks) which can sustain. We also give conditions under which a sellers' association may form and buyers may decide to link with a sellers' association of mixed quality. Further, we show how these buyer-seller networks influence the price paid for the good or service exchanged. Specifically, we give conditions on the network for which the seller (or sellers' association) is guaranteed a non-negligible positive surplus and conditions on the network for which the buyer is guaranteed a non-negligible positive surplus.

Next, we briefly describe the model and results. There are a number of buyers and sellers. Each seller produces one good which may be of either high or low quality. The model is one of perfect information and thus the quality of the good is known to everyone. Buyers are homogeneous and each buyer would like to purchase at most one good. Sellers may form a sellers' association where all products in the association are sold at the same price regardless of quality. Such an association is costly to maintain. Buyers and sellers can trade goods only if there is a trade link connecting them. Buyers can form links with individual sellers or with sellers' associations where such trade links are costly to maintain. For simplicity a buyer can have at most one trade link, while a seller or sellers' association may have multiple trade links. A graph (or compilation of trade links) is called stable if no buyer or seller

[2] See Hsing (1999), Dicken and Hassler (2000) and Cawthorne (1995) for discussion of trading companies in the clothing industry and Town and Vistnes (2001) and Mathewson and Winter (1996) for the HMO insurance. While many other examples abound, an obvious case is that a household may contract (or link) with a cleaning service to have the house cleaned at certain intervals, where the cleaning service may have multiple cleaners employed of different abilities or different work ethics. Another example is the International Trade Network established in 1996, which promotes sales of medical equipment and contains members of different specialties and qualities.

wants to exit the game (i.e. all payoffs must be non-negative) and no buyer would like to simultaneously sever her current link and form a new link. The set of prices which supports such a stable graph is called the set of active trading prices.

In Propositions 1 through 6 we look at which graphs are stable and at how the trade network influences the set of active trading prices. In Proposition 1, we find that if the number of buyers is greater than or equal to the number of high-quality sellers then each high-quality seller must be linked with a buyer or must be a member of an association that is linked with a buyer. However, it is possible for a high quality good to remain unsold in a stable network. Examples are given in Section 3 where a high-quality association forms but is only linked to one buyer, thus creating an excess supply of high-quality goods. Proposition 2 shows that if we have at least one high-quality seller and at least one low-quality seller then it is always possible for all low-quality sellers to be unlinked. Proposition 3 shows that if the number of buyers is less than or equal to the number of sellers then stability requires that all buyers be linked, however if the number of buyers is greater than the number of sellers than some buyers may choose to remain unlinked in order to avoid the cost of maintaining a link. In Proposition 4 we give conditions under which a stable graph can feature a sellers' association which contains all sellers, thus buyers are linked to an association of sellers of mixed quality.

The last two propositions examine the relationship between trade networks and prices. Proposition 5 shows that if the number of buyers is greater than the number of sellers and if the costs of forming an association and buyer-seller links are low enough, then a high-quality sellers' association with a good that remains unsold will always set price strictly above the associations' reservation price. Thus, a priori, sellers in the association will always expect a non-negligible positive payoff, which depends positively on the valuation of the high-quality good and negatively on the number of buyers. Finally Proposition 6 considers pricing in a stable graph with an unlinked seller. Here, the active sellers always set price strictly below the buyer's reservation price. Thus the existence of an unlinked seller (or outside option) creates competition which guarantees the buyers a non-negligible positive payoff.[3] This payoff is greater when the valuation of the low-quality good over the link costs per pair of buyer-seller is higher. This proposition motivates an important reason sellers have for forming an association. If sellers can create an association which eliminates unlinked sellers, then all sellers may be able to charge a higher price and thus may make themselves better off. Such an example of sellers making themselves better off by forming an association is given in Section 3.4.

There are a number of findings contrasting with conventional theories of trade between buyers and sellers. For brevity, we summarize but a few. First, due to price incentives, a sellers' association may naturally form in many circumstances, and may consist of members of mixed quality even under complete information. Second, a trade mismatch may occur in the absence of market or informational frictions, where a high quality good may remain unsold even without an economy-wide excess supply of goods. Third, given a set of active trading prices, there are generically multiple stable network configurations. Fourth, prices are affected by whether a seller remains inactive, but not by the number of inactive sellers. While the removal of an inactive seller affects both non-negligible net payoffs to sellers and buyers, eliminating an unlinked buyer only alters the effective minimum price (or the sellers' ability to earn a non-negligible net payoff). Fifth, to each sellers' association, an increase in its customer base can result in a price compression in the sense that the effective minimum price in the active trading range increases and the effective maximum price in the active trading range decreases. Finally, Propositions 5 and 6 show that the sellers' (resp. buyers') net payoffs are influenced by the valuation of the high-quality good and the number of buyers (resp. the valuation of the low-quality good), rather than the average quality, the quality differential or the relative market tightness (measured by the ratio of sellers to buyers).

[3] The industrial organization literature investigates the relationship between buyer-supplier contracts and outside supplier options, focusing on how sunk costs and supplier uncertainty may make an outside option more or less desirable to the buyer, see Helper and Levine (1992), Scheffman and Spiller (1992) and Riordan (1996). Alternatively, Proposition 6 shows how the outside supplier's network affects the buyer-supplier price.

The most closely related work to the present paper is that of Kranton and Minehart (2001) and Kranton and Minehart (2000). In their pivotal studies, Kranton and Minehart (2001) focus on when the non-cooperative formation of buyer-seller networks leads to the formation of efficient graphs,[4] while Kranton and Minehart (2000) examine the competitive equilibrium prices in buyer-seller networks. Our paper adds to theirs in two significant ways. First, we examine the case of heterogenous sellers and homogeneous buyers, while Kranton and Minehart (2000, 2001) consider the case of homogeneous sellers and heterogeneous buyers (specifically, buyers are heterogeneous in that they have different valuations in demand). Second, we introduce the possibility of links between sellers in the form of a sellers' association that contains a group of sellers, while Kranton and Minehart (2000, 2001) focus on links between individual buyers and sellers.

There is a trade network literature which concentrates on the value that trade networks create by decreasing search costs, reducing informational asymmetries, and facilitating cooperation.[5] In contrast, our paper focuses on the non-cooperative formation of trade links and the determination of price ranges which support active trade. Our work is also related to general theories of network formation and stability.[6] In particular, in our study of the endogenous formation of trade networks, players can form and

[4]Jackson (2001) gives a further analysis of the Kranton and Minehart (2001) efficiency results for the case where the cost of a link is incurred by both the buyer and the seller.

[5] For example, Helper and Levine (1992) show how alternative sources of trade alter the terms of trade. While Rauch (1996) allows buyer-seller links to eliminate the search for a buyer, Riordan (1996) considers links which facilitate cooperation and hence create value and Casella and Rauch (1997) allow family ties to reduce information asymmetries. For a recent survey of the trade network literature, see Rauch (2001).

[6] See Aumann and Myerson (1988), Jackson and Wolinsky (1996), Dutta and Mutuswami (1997), Chwe (2000) , Jackson and Watts (1999), Watts (2001), and Bala and Goyal (2000). Jackson and Wolinsky (1996) and Dutta and Mutuswami (1997) both concentrate on the tension between network stability and network efficiency, whereas Chwe (2000) studies how network structure influences coordination games. Aumann and Myerson (1988) were the first to take an explicit look at network formation in a strategic context where connections defined a communication structure that was applied to a cooperative game. Jackson and Watts (1999), Bala and Goyal (2000) and Watts (2001) examine the endogenous formation of networks when players can form and sever links.

sever links. However, our focus on the influences of sellers' quality differences and the patterns of sellers' associations is certainly very different from that in the existing literature.

The remainder of the paper is organized as follows. In Section 2 we present the model, while in Sections 3 and 4 we apply the model to specific examples. The general results can be found in Section 5. In Section 6 we extend the model in some natural ways, by allowing buyers to form multiple links and to form an association and by discussing the implications of active seller networking.

2. The Environment

Consider a trading economy with indivisible goods of high and low quality where agents have full information. The central feature of the model is to determine endogenously both the structures of trade networks and the ranges of product prices for active trade.

2.1. Agents

Denote the set of buyers as B and the set of sellers as $S = H \cup L$ where H and L are the set of high- and low-quality sellers, respectively. Let $\mu(X)$ represent the cardinality of integer set X. We assume that there are at least 2 buyers and 2 sellers, with at least one high-quality seller and one low-quality seller. Thus, we have: $\mu(B) \equiv \beta \geq 2$, $\mu(S) \equiv \sigma \geq 2$, $\mu(H) \equiv \sigma^H \geq 1$ and $\mu(L) \equiv \sigma^L \geq 1$, where $\sigma^H + \sigma^L = \sigma$. For illustration purposes, we will refer to a buyer as "she" and a seller as "he".

2.2. Production

Assume that every seller of type $\tau \in \{H, L\}$ produces exactly one unit of an indivisible good per period, where the good produced has quality τ. Goods are completely perishable and hence have no inventory value. Each buyer demands at most one unit of the good per period. Thus, if a buyer and seller are allowed to trade they will trade at most one unit.

2.3. Trade Network Structure

In contrast with studies of bilateral trade, we allow buyers and sellers to form an ***association*** which is denoted by A. An association is called,

(i) a ***buyers' association*** (denoted A^B) if $a \in B$ for all $a \in A^B$;

(ii) a ***sellers' association*** (denoted A^S) if $a \in S$ for all $a \in A^S$.

Each buyer and seller may belong to at most one association. An association, A, is called ***nondegenerate*** if $\mu(A) > 1$. The cost of maintaining an association of size $m \geq 1$ equals $(m-1)k$ for each association member, where $k \geq 0$. Thus, k is an individual association member's unit cost of maintaining a tie with every other member in the association.

Trade can occur only if there is a trade link between a buyers' association A^B and a sellers' association A^S. The cost of maintaining such a trade link for each association is $c \geq 0$. We assume that each association divides this cost evenly among its members. Note that if $\mu(A^S) > \mu(A^B)$ then not all sellers in A^S are guaranteed a trade with A^B. In this case, we assume that an individual seller $s \in A^S$ sells a product to A^B with probability $\mu(A^B)/\mu(A^S)$. Similarly, if $\mu(A^S) < \mu(A^B)$, then buyer $b \in A^B$ purchases a product from A^S with probability $\mu(A^S)/\mu(A^B)$.

A ***trade network*** is a graph, G, consisting of trade links between buyers' and sellers' associations. If A^S is linked with A^B, we write $(A^S, A^B) \in G$. We assume in the basic framework that each buyers' association has at most one link, with a relaxation of this assumption provided in Section 6.1. However, a sellers' association may have links to multiple buyers' associations.

2.4. Individual Payoffs

To each buyer, the consumption of one unit of the quality differentiated good yields a utility of q^i, for i = H, L (and the consumption of any additional units generates no utility). We assume that:

Assumption 1: $q^H > q^L > 2(c + k)$.

While the first inequality is trivial, the second inequality ensures that the value generated from trading a low-quality good is sufficient to cover the cost of maintaining a link between a buyers' association and a nondegenerate sellers' association of size 2.

Let the set of buyers' associations be represented by $\mathcal{A}^B = \{A_1^B,...,A_m^B\}$ and the set of sellers' associations by $A^S = \{A_1^S,...,A_n^S\}$ with G representing the trade network between the sets. Denote the price of a good produced by seller $s \in A_j^S$ as $p(A_j^S)$. We assume that all sellers in an association set a uniform price regardless of any differences in quality.

We now define the payoff for seller $s \in A_j^S$. Let B_j represent the set of buyers' associations that A_j^S is linked with in graph G and let m_j be the number of individual buyers in the set B_j. Seller s's net payoff, $v(B_j,A_j^S)$, is then given by:

$$v(B_j,A_j^S) = \min\left\{\frac{m_j}{\mu(A_j^S)}, 1\right\}p(A_j^S) - \frac{\mu(B_j)}{\mu(A_j^S)}c - \left[\mu(A_j^S)-1\right]k. \tag{1}$$

The first part of equation (1) represents the probability that seller s sells a product times the price he receives if his good is sold. The second part of (1) represents s's share of the trade link cost and s's share of the association cost, respectively. Let $p^L(A_j^S)$ be the price that sets $v(B_j,A_j^S) = 0$, which is conveniently referred to as ***sellers' reservation price*** in the sense of active market participation.

Next, we define the net payoff for a buyer, which is more complex due to different valuations of quality-differentiated goods. Assume that in graph G, buyer $b \in A_i^B$ maintains a single link with sellers' association A_j^S, and that A_j^S is linked with m_j buyers in G (m_j is the total number of buyers in all the buyers' associations that A_j^S is linked to). Denote b's payoff as $u(A_i^B, A_j^S)$, whose value is given by:

(i) if $m_j \leq \mu(A_j^S)$, then

$$u(A_i^B, A_j^S) = \frac{1}{\mu(A_j^S)}\left[\mu(A_j^S \cap H)q^H + (\mu(A_j^S \cap H))q^L\right] - p(A^S) - \frac{1}{\mu(A_i^B)}c - [\mu(A_i^B)-1]k; \tag{2a}$$

(ii) if $m_j \geq \mu(A_j^S)$, then

$$u(A_i^B, A_j^S) = \frac{1}{m_j}\left[\mu(A_j^S \cap H)q^H + \mu(A_j^S \cap L)q^L - \mu(A_j^S)p(A^S)\right] - \frac{1}{\mu(A_i^B)}c - [\mu(A_i^B)-1]k. \tag{2b}$$

In equations (2a) and (2b), the first part of each equation represents expected quality of a good purchased from A_j^S minus the expected price (in part (ii) under $m_j > \mu(A_j^S)$, buyer b is not guaranteed a purchase and thus only pays price p if she receives a good). The last part of (2a) and (2b) represents buyer b's share of the trade link cost and her share of the association cost, respectively. Let $p^U(A_j^S)$ be the price that sets

$u(A_i^B, A_j^S) = 0$, referred to as ***buyers' reservation price*** in the sense of active market participation.

2.5. The concept of Equilibrium

With regard to price determination, we use the standard cooperative game concept such that prices fall in the range where buyers and sellers are both willing to participate. We normalize the value of outside alternatives for both buyers and sellers to zero. We define prices for the case where there are no buyers' associations, as this is the focus of Sections 3-5. Since buyers are homogeneous and sellers are not, the formation of buyers' associations is not as interesting as the formation of sellers' associations which may contain members of different types. We will return to the case of buyers' associations in Section 6.2.

Fix a set of buyers $\{b_1,...,b_m\}$ and a set of sellers' associations $\{A_1^S,...,A_n^S\}$ so that each seller is a member of at most one association. A graph G, which links each buyer to at most one sellers' association, is ***stable*** if for all b_i linked to A_k^S in G and for all b_u unlinked in G,

$$\begin{aligned} &u(b_i,A_k^S) \geq u(b_i,A_j^S) \text{ and } 0 > u(b_u,A_j^S) \quad \text{for all } A_j^S \in \{A_1^S,...,A_n^S\}; \\ &u(b_i,A_k^S) \geq 0; \\ &v(b_i,A_k^S) \geq 0 \quad \text{for all } A_k^S \in \{A_1^S,...,A_n^S\}. \end{aligned} \tag{3}$$

We call the set of prices which supports a given stable graph the ***active trading prices***. Interpreting equation (3), G is stable if

(i) no buyer, who is linked to some sellers' association in G, wants to sever her current link or wants to simultaneously sever her link and form a new link;

(ii) no buyer, who is unlinked in G, wants to form a link;

(iii) no buyer would choose the non-consuming state and no seller would exit the game (i.e., no buyer or seller would prefer to sever all ties and receive a payoff of 0).

We assume that if a buyer is indifferent between forming a link and having no links then she chooses to form a link. We use this in the proof of Proposition 3 to rule out equilibria where there is at least one unlinked buyer and at least one linked buyer and where all buyers (linked and unlinked) receive a payoff of zero. The assumption that a buyer would always choose to sever a link when indifferent would also work. However, we assume a buyer will always form a link when indifferent for ease of exposition.

As in Kranton and Minehart (2001) we assume that only the buyer can initiate forming a link. Thus, under this definition of stable network, we have ruled out the possibility of active seller networking because sellers are not allowed to discriminate against or seek out buyers and thus sellers are not allowed to form and/or sever individual links with buyers. This simplification is innocuous, moreover, one can easily see that even with relatively "passive" sellers, a nondegenerate sellers' association may still emerge. This implies that the underlying mechanism must be direct price incentives rather than market power (which may influence prices indirectly as in the case of cartels). A discussion of active seller networking is provided in Section 6.3.

3. Trade Networks with Equal Number of Buyers and Sellers

It is sufficient to illustrate our model with 3 buyers and 3 sellers (i.e., $\beta = \sigma = 3$, $B = \{b_1, b_2, b_3\}$ and $S = \{s_1, s_2, s_3\}$). The consideration of 3 (rather than 2) sellers allows the number of high- and low-quality sellers to be different. In particular, we focus on two cases: (i) $\sigma^H = 2$ and $\sigma^L = 1$, with $s_1, s_2 \in H$ and $s_3 \in L$; (ii) $\sigma^H = 1$ and $\sigma^L = 2$, with $s_1 \in H$ and $s_2, s_3 \in L$. Again, for illustration purposes, we allow sellers to form associations but not buyers. There are four possible types of nondegenerate sellers' associations: the grand coalition $\{s_1, s_2, s_3\}$ and three different two-seller associations with either homogeneous quality types or heterogeneous quality types.

3.1. Formation of Trade Networks and Sellers' Association

In an economy with more high-quality than low-quality sellers (with $s_1, s_2 \in H$ and $s_3 \in L$), there are eight stable graphs. Recall that we denote (b_i, s_j) as a link between b_i and s_j. The eight stable graphs are: (i) $G=\{(b_1, s_1), (b_2, s_2), (b_3, s_3)\}$; (ii) $G=\{(b_1, s_1), (b_2, s_1), (b_3, s_2)\}$; (iii) $G=\{(b_1, \{s_1, s_2\}), (b_2, \{s_1, s_2\}), (b_3, s_3)\}$; (iv) $G=\{(b_1, s_1), (b_2, \{s_2, s_3\}), (b_3, \{s_2, s_3\})\}$; (v) $G=\{(b_1, \{s_1, s_2\}), (b_2, \{s_1, s_2\}), (b_3, \{s_1, s_2\})\}$;

(vi) $G=\{(b_1, \{s_1, s_2, s_3\}), (b_2, \{s_1, s_2, s_3\}), (b_3, \{s_1, s_2, s_3\})\}$; (vii) $G=\{(b_1, \{s_1, s_2\}), (b_2, s_3), (b_3, s_3)\}$; and, (viii) $G=\{(b_1, s_1), (b_2, s_1), (b_3, \{s_2, s_3\})\}$. When there are more low-quality than high-quality sellers ($s_1 \in H$ and $s_2, s_3 \in L$), we have an additional graph: (ix) $G=\{(b_1, s_1), (b_2, s_1), (b_3, s_1)\}$. These stable trade networks are compactly depicted in Figures 1A and 1B.

For illustration purposes, let us consider the case with $s_1, s_2 \in H$ and $s_3 \in L$. We show that the one-to-one link pattern, $G = \{(b_1, s_1), (b_2, s_2), (b_3, s_3)\}$, is stable. The net payoffs of buyers are:

$$u(\{b_i\}, \{s_i\}) = q^H - p_i - c \text{ for } i = 1,2 \text{ and } u(\{b_3\}, \{s_3\}) = q^L - p_3 - c, \tag{4}$$

where p_i denotes the price of a good produced by seller $i\in\{1,2,3\}$. The net payoffs of sellers are: $v(\{b_i\}, \{s_i\}) = p_i - c$ for $i = 1,2,3$. In order for no buyer or seller to want to sever her/his link and exit the game we must have $u(b_i,s_i) \geq 0$ and $v(b_i,s_i) \geq 0$ for all i which implies that $p_1, p_2 \in [c, q^H - c]$ and $p_3 \in [c, q^L - c]$. Moreover, no buyer should want to sever her current link and link with someone else. In order for b_1 not to want to sever her link and form a link with s_2 or s_3, we must have: $p_1 \leq \min\{ q^H - (1/2)(q^H - p_2), q^H - (1/2)(q^L - p_3) \}$; for b_2 not to want to sever her link and form a link with s_1 or s_3, we need: $p_2 \leq \min\{ q^H - (1/2)(q^H - p_1), q^H - (1/2)(q^L - p_3) \}$; similarly, for b_3 not to want to sever her link and form a link with s_1 or s_2, we need: $p_3 \leq \min\{ q^L - (1/2)(q^H - p_1), q^L - (1/2)(q^H - p_2) \}$. Thus, active trading prices must satisfy:

$$\begin{aligned} c &\leq p_1 \leq (1/2) \min\{ q^H + p_2, 2q^H - q^L + p_3, 2q^H - 2c\} \\ c &\leq p_2 \leq (1/2) \min\{ q^H + p_1, 2q^H - q^L + p_3, 2q^H - 2c\} \\ c &\leq p_3 \leq (1/2) \min\{ 2q^L - q^H + p_1, 2q^L - q^H + p_2, 2q^L - 2c\}. \end{aligned} \tag{5}$$

Such active trading prices exist as long as Assumption 1 is met, so G is stable.

Similarly, one can prove that the remaining cases are stable and can derive the active trading prices for these cases. Representative proofs can be found in the Appendix and the active trading prices for all graphs are summarized in Tables 1A and 1B. Note that in the proofs and tables, when sellers' association $\{s_i,\ldots,s_j\}$ is formed we represent the association's corresponding price as $p_{i\ldots j}$. Thus the association sells all goods (even those of different qualities) at the same price. Also note that we assume that an unlinked seller sets price equal to c which is his break even price.

Next, we briefly describe the set of stable graphs. In graphs (i), (iii), (iv), and (vi) of Figure 1 all goods are sold, while in each of the remaining graphs there is a mismatch in that not all goods are sold. The graphs in the first group satisfy the "efficiency" criterion in the sense that active trading enables all goods to be consumed. Of these efficient stable graphs, it is possible to have a one-to-one link pattern (case (i) of Figures 1A and 1B) and also possible for buyers to be linked to a sellers' association which is either of high quality (case (iii) of Figure 1A), low quality (case (iv) of Figure 1B) or mixed quality (case (iii) of Figure 1B, (iv) of 1A, and (vi) of 1A and 1B). We thus conclude that nondegenerate sellers' associations can naturally arise in stable networks to achieve first-best allocations, where the associations may be of various sizes containing members of uniformly high, uniformly low or mixed quality.

3.2. Mismatch in Stable Trade Networks

A "mismatch" occurs whenever a graph with equal numbers of buyers and sellers has some goods which are unsold (see cases (ii), (v), (vii), (viii), and (ix) in Figures 1A and 1B). The first type of mismatch occurs when a sellers' association has fewer buyers linked to it than it has members; this situation results in an excess supply of goods. For example, consider case (viii) of Figures 1A and 1B. Here, the sellers' association has an average quality which is lower than that of the single seller, and the association has an excess supply of goods while the single high-quality seller has an excess demand for his good. As shown in the Appendix, in order for this type of trade network to be stable, one requires, in addition to Assumption 1, that the value of the high-quality good be much larger than the cost of maintaining a trade link. As a consequence, multiple buyers are willing to maintain a link with the single high-quality buyer since the expected payoff from maintaining such a link is large, causing the mismatch. However, it is also possible for a sellers' association to have an excess supply of goods even if the sellers' association has a higher average quality than that of the single seller (see case (vii) of Figures 1A and 1B). This may seem counter-intuitive at first glance, yet such a graph can be stable if the low quality good is sufficiently valued and if the sellers' association maintains a high price. Here, buyers 2 and 3 prefer a 1/2 chance of buying from the single low-quality seller to buying from the association with certainty, while buyer 1 prefers buying from the association with certainty to having a 1/3 chance of buying from the single low-quality seller.

The second type of mismatch features an inactive seller (cases (ii) and (v) in Figures 1A and 1B) or even two inactive sellers (case (ix) in Figure 1B). An immediate observation is that an inactive seller is always of low-quality. (For a general analysis see Propositions 1 and 2 in Section 5.) Such a graph with an inactive seller is stable only if the quality differential is large enough. To see this recall that an inactive low-quality seller sets his price equal to c, thus if a buyer forms a link with this seller he will receive a payoff of q^L-c. This graph is stable if each buyer is currently receiving a payoff greater than or equal to q^L-c. So for instance, in graph (ii) of Figure 1, buyers 1 and 2 each receive a payoff of $(1/2)(q^H - p_1 - c)$. Stability requires that this payoff be greater than or equal to q^L-c (to keep these buyers from severing their current ties and linking with the inactive seller), which is true if the quality differential is large enough. Thus graphs with inactive sellers require additional conditions to Assumption 1.

3.3. Characterization of Active Trading Prices

In Tables 1A and 1B, we list the active trading price ranges for every stable graph. From these ranges, we can determine the minimum possible price, p_i^{min}, and the maximum possible price, p_i^{max}, that each sellers' association i can charge so that the graph remains stable. These ***effective minimum prices*** and ***effective maximum prices*** can be found in the Appendix.

Next, we compare the effective minimum and maximum prices to the sellers' reservation price, p^L, and the buyer's reservation price, p^U. Focusing on the effective minimum prices, we find that the effective minimum price may exceed the corresponding sellers' reservation price. For example, consider case (ii) of Figure 1A. Here, two buyers are linked with one high-quality seller, the third buyer is linked with another high-quality seller while the low-quality seller remains inactive. The stability of this trade network requires that the second high-quality seller always sets a price higher than his reservation price.

If he sets his price equal to his reservation price then one of the two buyers linked with the first high-quality seller would prefer to link with seller 2 in order to receive a higher net payoff. This would not occur if the second seller were of low quality (case (ii) in Figure 1B). Intuitively, with an inactive seller of identical (low) quality, an active seller cannot set the price higher than the reservation level, or its buyer would prefer to link with the inactive seller.

The second type of interior effective minimum pricing is when there are two associations of sellers, one of higher average quality than the other. If the quality differential is large enough, then the effective minimum price of the higher-quality association will be larger than its reservation price (see cases (iii), (vii), and (viii) of Figures 1A and 1B). If the high-quality association sets its price at the reservation value, then the buyers linked to the low-quality association will prefer to sever their ties and form links with the high-quality association and thus the graph will not be stable.

The third type of stable trade networks with interior minimum pricing occurs when a nondegenerate sellers' association has an idle good (cases (vii) and (viii) in Figures 1A and 1B). Here the high minimum price keeps the graph stable by preventing buyers who are not linked to the association from severing their ties and forming ties with the association. When the quality differential is sufficiently large, this high price is necessary in case (vii) of Figure 1, where the sellers' association provides goods of higher quality on average. In case (viii) of Figure 1, the sellers' association provides goods of lower quality on average thus the association has an interior minimum price only if the quality differential is sufficiently small.

We next turn to effective maximum prices where an interior maximum price implies that the effective maximum price is below the buyer's reservation price. In contrast to the case of interior minimum prices, the case of interior maximum prices is much more straightforward. The only crucial factor now is whether or not there are inactive sellers. When there are inactive sellers (either one low-quality inactive seller as in cases (ii) and (v) in Figures 1A and 1B or more than one inactive sellers as in case (ix) in Figure 1B), an active seller cannot pin any of his customers down at their reservation utility level. This is because any of his customers would otherwise sever their link and form a link with the initially inactive seller to receive a higher net payoff.

3.4. Gains to Sellers from Forming a Sellers' Association

Finally, we provide an example showing that "price incentives" are crucial for the formation of a nondegenerate sellers' association. Here, we contrast case (ii) with case (vi) in Figure 1A. In case (ii), there are three buyers and three sellers where the first two sellers are linked to buyers but the third seller is inactive. In case (vi), all sellers are part of an association which is linked to all buyers. By examining the effective minimum and maximum prices (see the Appendix), we find that despite identical effective minimum prices in both cases, the case with the sellers' association has a higher effective maximum price for all three goods as long as the quality differential is not too large (specifically, $4q^L \geq q^H + 6c$). This result occurs because an inactive seller always sets his price as low as possible and so if the active sellers want to keep their customers they cannot set prices too high. However, if the sellers form an association which includes this inactive seller, then the sellers are able to eliminate the stiff competition of an inactive seller and thus can set prices higher. Therefore, if buyers and sellers have equal bargaining power (if price is split evenly between the effective minimum and maximum prices) or if sellers have greater bargaining power and if the cost of forming an association is not too big, then all sellers are strictly better off in the association.

4. Trade Networks with Unequal Numbers of Buyers and Sellers

In this section, we consider unequal numbers of buyers and sellers. To allow for quality differentiation, we study cases with at least one high-quality seller and at least one low-quality seller.

4.1. More Buyers Than Sellers

We begin by discussing the case with three buyers and two sellers where the first seller (s_1) is of high-quality and the second seller (s_2) is of low-quality seller. There are six different stable networks: (i) $G=\{(b_1, s_1), (b_2, s_1), (b_3, s_2)\}$; (ii) $G=\{(b_1, s_1), (b_2, s_1), (b_3, s_1)\}$; (iii) $G=\{(b_1, \{s_1, s_2\}), (b_2, \{s_1, s_2\}), (b_3,$

$\{s_1, s_2\})\}$; (iv) $G=\{(b_1, s_1), (b_2, s_2), (b_3, s_2)\}$; (v) $G=\{(b_1, s_1), (b_2, s_2)\}$; (vi) $G=\{(b_1, \{s_1, s_2\}), (b_2, \{s_1, s_2\})\}$. These networks are depicted in Figure 2 with the corresponding active trading prices given in Table 2.

We focus on contrasting the stable trade network patterns and corresponding active trading prices with those of Section 3, leaving all detailed proofs in the Appendix. One difference from Section 3 regarding the patterns of trade networks is that every stable graph requires an additional condition to Assumption 1 (which can be found in the Appendix). The requirement of additional conditions is due to the excess demand for goods. Specifically, if multiple buyers are linked to the single high-quality seller then q^H must be large enough to entice the buyers to maintain their links in the presence of excess demand. Similarly, if multiple buyers are linked to the low-quality seller, then q^L must be sufficiently large. A second noticeable difference is that it is now possible to have a stable graph with an unlinked buyer. This result is only possible if the number of buyers is strictly greater than the number of sellers. If the number of buyers is less than or equal to the number of sellers, then the existence of an unlinked buyer implies that there is an idle good which is priced strictly below $q^i - c$ ($i = H, L$). In this case, the idle buyer always has incentive to form a link to the seller of this idle good. (See Proposition 3 for a formal proof in general.)

Next, we compare the current active trading prices to those of Section 3. First, notice that prices are influenced by the fact that an idle seller exists, but not by the number of idle sellers. To see this consider the case where all buyers are linked to one high-quality seller. In Section 3, the corresponding graph has two unlinked low-quality sellers (case (ix) of Figure 1B), while in section 4.1 the corresponding graph has only one unlinked low-quality seller (case (ii) of Figure 2). By comparing the active trading prices for the two graphs (given in Tables 1 and 2), we see that the active trading prices are identical. The existence of just one idle seller gives each buyer an alternative source from which to purchase the product, thus active sellers must lower their prices in order to keep their existing customers. Since each buyer only wishes to purchase one item, the existence of an additional idle seller does not influence a buyer's behavior and so prices do not fall further.

So what happens if an idle seller is removed from the market? Here, we compare the graph where two buyers are linked to the same high-quality seller and a third buyer is linked to a low-quality seller (case (i) in Figure 2) to the identical graph where one additional idle low-quality seller is added (case (ii) in Figure 1B). Comparing the effective minimum and maximum prices of the two graphs (see the Appendix), we find that both the effective minimum and maximum prices are higher for the case without the idle seller. Thus, removing an idle seller lessens competition and allows the remaining sellers to price higher.

On the contrary, what happens if we eliminate an unlinked buyer? Consider case (i) of Figure 1B where each buyer is linked to one seller. We compare this to case (v) of Figure 2 where the third seller is eliminated and the third buyer is idle. Here, the effective maximum prices are the same for both graphs. However, the effective minimum price is higher for the case with the idle buyer. Intuitively, to sustain the graph with the idle buyer, the active sellers cannot set prices too low or the idle buyer will decide that it is better to compete with another buyer for the product rather than to remain idle.

4.2. More Sellers Than Buyers

In this subsection, we consider the case of two buyers and three sellers. If there are more high quality than low-quality sellers (with $s_1, s_2 \in H$ and $s_3 \in L$), then there are five different stable trade network patterns:(i) $G=\{(b_1, s_1), (b_2, s_2)\}$; (ii) $G=\{(b_1, \{s_1, s_2\}), (b_2, \{s_1, s_2\})\}$; (iii) $G=\{(b_1, s_1), (b_2, \{s_2, s_3\})\}$;(iv) $G=\{(b_1, \{s_1, s_2, s_3\}), (b_2, \{s_1, s_2, s_3\})\}$; (v) $G=\{(b_1, \{s_1, s_2\}), (b_2, s_3)\}$. When there are more low quality than high-quality sellers (with $s_1 \in H$ and $s_2, s_3 \in L$), we have an additional link pattern: (vi) $G=\{(b_1, s_1), (b_2, s_1)\}$. These trade networks are depicted in Figures 3A and 3B and the corresponding active trading prices are given in Tables 3A and 3B.

In contrast to the economy with more buyers than sellers described in Section 4.1, the majority of the stable graphs here feature an emergence of a sellers' association. This is due to the pressure from the "excess supply" of goods that forces the sellers to pool themselves to secure the demand. Notice that very few of these cases require an additional condition beyond Assumption 1 (additional conditions are listed in the Appendix). Therefore, the presence of the excess supply of goods makes it easier to set active trading prices for each stable graph.

We now compare the current active trading prices to those of Section 3. First, we illustrate that adding a buyer with whom an idle seller can link results in lower prices. Consider case (i) of Figure 1B where each buyer is linked to one seller. We compare this to case (i) of Figure 3B where the third buyer is removed leaving the third seller idle. By comparing the effective maximum and minimum prices (see the Appendix), we find that both prices are higher for the case with the idle seller. As an idle seller sets his price as low as possible, eliminating an idle seller by adding a new buyer decreases competition and hence allows the remaining sellers to raise their prices.

We next look at what happens if we increase the number of buyers purchasing from a seller's association. Here, we compare case (ii) of Figure 3B where two buyers are linked to a seller's association of size two and the third seller is idle, to case (v) of Figure 1B where there is an additional buyer linked to the seller's association. The effective minimum price of the goods sold by the seller's association is lower for the two buyer case, while the effective maximum price is higher (see the Appendix). The effective minimum price is higher when there are more buyers linked to the seller's association since maintaining such links is costly, and thus the seller's association must raise their minimum price to cover this increased cost. However, the maximum price is lower when there are more buyers, because more buyers lead to excess demand. As a result, the seller's association must lower its maximum price to prevent buyers from linking with the idle seller.

5. General Properties for the Formation of Trade Networks

From the discussion in Sections 3 and 4 above, we can draw some general properties of trade networks and prove them formally. Throughout this section, we assume that the buyers' association is degenerate so that $\mu(A^B) = 1$. We define an ***inactive seller*** (respectively, ***inactive buyer***) to be a seller (buyer) who has no links to buyers (sellers). We also define an ***idle good*** to be a good unsold. Notably, an idle good could either be a good produced by an inactive seller or be a good produced by a sellers' association where the number of buyers linked to the association is strictly less than the number of association members (thus the association is unable to sell all of its goods).

The four propositions in Section 5.1 characterize the patterns of trade networks. Propositions 1-3 establish conditions under which a stable network may feature inactive sellers or inactive buyers, while Proposition 4 examines the possibility for all sellers of mixed quality to form a sellers' association. In Section 5.2, Propositions 5 and 6 characterize the active trading prices supporting stable trade networks.

5.1. Stable Network Properties

Proposition 1: *If the number of buyers is greater than or equal to the number of high-quality sellers (i.e., $\mu(B) \geq \mu(H)$), then there does not exist a stable network containing any inactive sellers of high quality.*

Proof: We prove this proposition by contradiction. Assume that G is a stable graph containing $s \in S \cap H$ who is inactive. If a buyer $b \in B$ severs her link with A^S to form a link with the inactive seller s, then the net gain equals $\Delta = q^H - 2c - u(b, A^S)$, where the inactive seller's price is fixed at the minimum, c. It remains to show that $\Delta>0$ and thus that G is unstable. First notice that if A^S is empty then $\Delta = q^H - 2c > 0$, thus all buyers must be linked. Second assume that the average quality of a good purchased from A^S is $q<q^H$. Then $u(b, A^S) \leq q - p(A^S) - c < q^H - 2c$ and thus $\Delta>0$. (Note that $u(b, A^S) = q - p(A^S) - c$ if A^S is linked with one buyer and is less than or equal to $q - p(A^S) - c$ if A^S is linked with multiple buyers.) Therefore, if G is stable, then A^S must consist of only high-quality sellers and similarly all buyers must be linked to high-quality sellers' associations. By assumption $\mu(B) \geq \mu(H)$ and by assumption there exists at least one high-quality seller who is inactive, thus there must exist an association, say A^S, which maintains links with m buyers such that $m>\mu(A^S)$. The stability of G requires that no seller in A^S wishes to sever a link, thus we need $v(., A^S) \geq 0$, or, equivalently, $\min\{m/\mu(A^S), 1\}p(A^S) \geq mc/\mu(A^S) + [\mu(A^S)-1]k$, or, $p(A^S) \geq \max\{\mu(A^S)/m, 1\}\{mc/\mu(A^S) + [\mu(A^S)-1]k\}$. This implies $u(\{b\}, A^S) \leq q^H - \max\{\mu(A^S)/m,$

$1\}\{mc/\mu(A^S) + [\mu(A^S)-1]k\}$ - c, and thus that $\Delta \geq \max\{\mu(A^S)/m,1\}\{mc/\mu(A^S) + [\mu(A^S)-1]k\} - c \geq$ $[\max\{\mu(A^S)/m, 1\}m/\mu(A^S) - 1]c > 0$, which implies G is not stable. Q.E.D.

Proposition 2: *If $\mu(H) \geq 1$ and $\mu(L) \geq 1$, then a stable network may feature inactive low-quality sellers when the quality differential is sufficiently large.*

Proof: We consider two different cases, $\mu(H)>\mu(B)$ and $\mu(H)\leq\mu(B)$, and show that in both cases a graph with inactive low-quality sellers is stable when the quality differential is sufficiently large. First, assume $\mu(H) > \mu(B)$. Consider the graph where each buyer is linked to a different high-quality seller and all low-quality sellers and remaining high-quality sellers are inactive. If each seller sets price equal to c then no buyer has incentive to sever her current tie or to sever her current tie and link to someone else and no seller has incentive to sever his tie, thus the graph is stable. Second, consider $\mu(H) \leq \mu(B)$. We show that the following graph G is stable. Let all low-quality sellers be inactive. Let each high-quality seller be linked to either m or m+1 buyers where m and m+1 are the integers closest to $\mu(B)/\mu(H)$. (If $\mu(B)/\mu(H)$ is an integer then let each high-quality seller be linked to $m+1 = \mu(B)/\mu(H)$ buyers.) Let each seller linked to m (resp. m+1) buyers set the same price p_m (resp. p_{m+1}). Then, the net payoff of a buyer b is: $(1/i)(q^H - p_i) - c$, for $i \in \{m , m+1\}$, depending on which type of high-quality seller she is linked to. In order to ensure active participation of all sellers we must have $p_i \geq i\,c$, where $i \in \{m ,m+1\}$. To ensure active participation of buyers we must have $(1/i)(q^H - p_i) - c \geq 0$ for $i \in \{m ,m+1\}$. If the graph is stable then no buyer can want to sever her current link and link with an inactive low-quality seller, i.e., $(1/i)(q^H - p_i) - c \geq q^L - 2c$ for $i \in \{m ,m+1\}$. Additionally, no buyer can want to sever her current link and form a link with another high-quality seller, thus we must have $(1/m)(q^H - p_m) - c \geq [1/(m+2)](q^H - p_{m+1}) - c$ and $[1/(m+1)](q^H - p_{m+1}) - c \geq [1/(m+1)](q^H - p_m) - c$. If we let $p_m = p_{m+1} + \epsilon$ where $\epsilon>0$ and p_{m+1} satisfies:

$$(m+1)c \leq p_{m+1} \leq \min\{ q^H - (m+1)(q^L - c), q^H - (m+1)c \}, \tag{6}$$

then all conditions are met. So active trading prices exists as long as: $q^H \geq (m+1)q^L \geq \mu(B)q^L$ and $q^H \geq 2(m+1)c \geq 2\mu(B)c$. By Assumption 1 we know that $q^L \geq 2c$, thus active trading prices exist as long as the quality differential is sufficiently large (specifically, $q^H - q^L \geq mq^L$). Q.E.D.

Proposition 3: (i) *If $\mu(B) \leq \mu(S)$, then there does not exist a stable network containing any inactive buyers.* (ii) *If $\mu(B) > \mu(S)$, then there exists a stable network containing at least one inactive buyer.*

Proof: We prove part (i) of the proposition by contradiction. Assume to the contrary that $\mu(B) \leq \mu(S)$ and that there exists a stable network, G, with at least one buyer, say b_1, who is unlinked and thus does not purchase a product. Since $\mu(B) \leq \mu(S)$, there must also exist at least one seller, say s_1, who does not sell his product. If s_1 is unlinked then s_1 sets his price at c and by Assumption 1, b_1 would gain from linking to s_1 and thus G is not stable. If s_1 is linked but is not selling his product, then he must be part of a seller's association. Since we assumed that G is stable and that some buyers are already linked to this association these buyers must expect to receive a payoff $u \geq 0$. (If $u<0$ then the buyers would sever their ties to this association). Since s_1 is not selling his product, b_1 would also expect payoff $u \geq 0$ if he linked with this seller's association. Thus b_1 will form the link and so G is not stable.[7]

Next we prove part (ii) of the proposition. Assume $\beta=\mu(B) > \mu(S)=\sigma$. We show that graph G = $\{(b_1,s_1), (b_2,s_2),\ldots,(b_\sigma, s_\sigma), (b_{\sigma+1}),\ldots,(b_\beta)\}$ is stable. To show that G is stable we need to find at least one active trading price vector which supports G. By Assumption 1, we can select $0< \epsilon < c/2$ such that $q^L \geq 2(c+\epsilon)$. Let $p(s_j) = q^j - c - \epsilon$, where and $q^j = q^H$ if $s_j \in H$ and $q^j = q^L$ if $s_j \in L$. To prove that G is stable, we must show that no buyer wants to sever a link and/or form a new link and that no seller wants to sever a link. By construction, $v(b_j,s_j) = p(s_j) - c = q^j - 2c - \epsilon > 0$; thus no seller wants to sever a link. Similarly, $u(b_j,s_j) = q^j - p(s_j) - c = \epsilon > 0$ for $0 \leq j \leq \sigma$; thus no buyer wishes to sever a link. If a linked buyer, say j,

[7] Note that we assume here that if a buyer is indifferent between forming a link or having no links then she forms a link. Alternatively, we could assume that if a buyer is indifferent then she always severs the link. Using this assumption in the above proof we would just need to change "$u \geq 0$" to "$u>0$" and "$u<0$" to "$u \leq 0$".

severs her current tie and simultaneously links with seller $k \neq j$ (who, by construction, is already linked to another buyer), then buyer j's payoff would equal $1/2q^k - 1/2p(s_k) - c = \epsilon - 1/2c < 0$; thus buyer j will stay linked to seller j. Lastly, we check that no initially unlinked buyer wishes to form a link with a seller, say seller j. Forming such a link would give the buyer a payoff of $1/2q^j - 1/2p(s_j) - c = \epsilon - 1/2c < 0$; thus the buyer will not form the link. Q.E.D.

The results in Propositions 1-3 suggest that high-quality sellers can never remain inactive in a stable trade network, whereas lower quality sellers can be inactive if the quality differential is sufficiently large. Additionally, a buyer can remain inactive only when there is global excess demand.

Proposition 4: *A stable network may feature a sellers' association as the grand coalition of all sellers only if the costs of maintaining links (both between traders and within the association) are low compared to q^H and q^L.*

Proof: Consider a graph where every buyer is linked to a sellers' association containing all sellers. There are two cases depending on whether the number of buyers β is greater than or smaller than the number of sellers σ. First assume that $\beta \leq \sigma$. Denoting the price of the good as p, the net payoff of $b \in B$ is:

$$u(\{b\},\{S\}) = (\sigma^H/\sigma)q^H + (\sigma^L/\sigma)q^L - p - c. \tag{7}$$

To ensure active participation by buyers $u(\{b\},\{S\})$ must be non-negative. Since the number of buyers is less than the number of sellers, no seller is guaranteed to make a sale. To ensure active participation of sellers we must have: $(\beta/\sigma)(p-c) - (\sigma-1)k \geq 0$. Thus, active trading prices must satisfy:

$$(\sigma^H/\sigma)q^H + (\sigma^L/\sigma)q^L - c \geq p \geq c + [\sigma(\sigma-1)/\beta]k. \tag{8}$$

This constraint can be met only if

$$(\sigma^H/\sigma)q^H + (\sigma^L/\sigma)q^L \geq 2c + [\sigma(\sigma-1)/\beta]k, \tag{9}$$

and thus holds true for q^H and q^L significantly larger than c and k.

Next, we must check the case where $\beta \geq \sigma$, and thus not all buyers are guaranteed of purchasing the product. Now, we have $u(\{b\},\{S\}) = (\sigma^H/\beta)q^H + (\sigma^L/\beta)q^L - (\sigma/\beta)p - c$ and active trading prices must satisfy: $(\sigma^H/\sigma)q^H + (\sigma^L/\sigma)q^L - (\beta/\sigma)c \geq p \geq c + (\sigma-1)k$. This requires

$$(\sigma^H/\sigma)q^H + (\sigma^L/\sigma)q^L \geq [(\beta+\sigma)/\sigma]c + (\sigma-1)k, \tag{10}$$

which again holds true only if q^H and q^L are significantly larger than c and k. Q.E.D.

The significance of Proposition 4 is the emergence of a stable trade network with a nondegenerate sellers' association containing producers of mixed quality. Interestingly, under Assumption 1, (9) holds true as long as k is sufficiently small, regardless of the magnitude of c. Yet, (10) requires both k and c to be small. Thus, with excess demand, the formation of a grand-coalition sellers' association requires low costs of maintaining links not only within the association but also between buyers and sellers. The later condition is needed so that in an economy with excess demand, the single sellers' association will agree to link with all buyers.

5.2. Active Trading Price Properties

In the next two propositions, we characterize the active trading prices supporting a stable network. In particular, we are interested in when an active price will be set strictly above the sellers' reservation price p^L or strictly below the buyers' reservation price p^U. While the former implies a non-negligible positive surplus for sellers, the latter implies a non-negligible positive surplus for buyers.

Proposition 5: *Let $c=k=0$ and $\beta=\mu(B)\geq\mu(S)\geq 3$. Then a high-quality seller's association A with at least one idle good always sets price $p(A) \geq [1/(\beta-1)]\, q^H > p^L(A)$.*

Proof: Consider a stable graph G, which contains a high-quality seller's association, say A, which has at least one idle good. Since there exists an idle good and since $\beta \geq \sigma$, we know that there exists at least one buyer, say b_1, who is not guaranteed a purchase in graph G. Thus b_1 must be one of m' buyers who is linked to sellers' association A', where $m' > \mu(A')$. (Note that it is possible that A' is empty and thus that

b_1 is linked to no one.) Let q' be the average quality of a good purchased from A'. Then buyer b_1's expected payoff is $u(b_1, A') = [\mu(A')/(m')][q' - p(A')]$.

Next we compare $u(b_1, A')$ to the payoff b_1 would receive if she severed her tie to A' and linked to A. Let the number of buyers that A is already linked be $m > 0$ (as A is a high-quality association, we know from Proposition 1 that A must be linked with at least one buyer). Since A has an idle good we know that if b_1 links with A, b_1 purchases a high quality good with certainty. So $u(b_1, A) = q^H - p(A)$. If G is stable, then we must have $u(b_1, A') \geq u(b_1, A)$, which implies $p(A) \geq q^H - [\mu(A')/(m')][q' - p(A')]$. By assumption, $m' \leq \beta\text{-}1$ and $\mu(A')/(m') \leq (m'-1)/m' \leq (\beta-2)/(\beta-1)$. Thus, $q^H - [\mu(A')/(m')][q' - p(A')] \geq q^H - [(\beta-2)/(\beta-1)]q^H = [1/(\beta-1)]q^H$, and, hence, $p(A) \geq [1/(\beta-1)]q^H > 0 = p^L(A)$. Q.E.D.

Proposition 6: *Let G be a stable graph with at least one inactive seller. Then any active association, A, sets price $p(A) \leq p^U(A) - (q^L - 2c) < p^U(A)$.*

Proof: Consider an active sellers' association A. Let b_1 be one of m buyers linked to A and q^A be the average quality of a good purchased from A. Then, buyer b_1's expected payoff becomes $u(b_1, A) = \min\{\mu(A)/m, 1\}[q^A - p(A)] - c$. Buyer b_1's reservation price sets $u(b_1, A) = 0$, thus implying $p^U(A) = q^A - \max\{m/\mu(A), 1\}c$.

Let s_1 be an inactive seller in G of quality q^i, $i \in \{H,L\}$. (If $\mu(B)>\mu(H)$ then from Proposition 1 we know that i=L.) If b_1 severs her link to A and links to s_1, her payoff would be $u(b_1,s_1) = q^i - 2c$ (since an inactive seller always sets price equal to c). By the stability of G we must have $u(b_1, A) \geq u(b_1,s_1)$ which implies that $p(A) \leq q^A - \max\{m/\mu(A), 1\}(q^i - c)$. This inequality can be rewritten as $p(A) \leq p^U(A) - \max\{m/\mu(A), 1\}(q^i - 2c) \leq p^U(A) - (q^L - 2c)$, where the last inequality is a result of the fact that $q^i \geq q^L$ and that $\max\{m/\mu(A), 1\} \geq 1$. Q.E.D.

From Proposition 5, we learn that a high-quality sellers' association with an idle good always receives a non-negligible payoff even if the cost of forming an association or a buyer-seller link is zero.

This result implies that the sellers' non-negligible payoff is not the result of the sellers trying to cover the cost of forming an association or a buyer-seller link. Instead, a high price is necessary to keep other buyers from linking with this association. Furthermore, Proposition 5 suggests that such non-negligible payoffs to sellers increase with the valuation of the high-quality good (q^H) rather than the average quality or the quality differential; moreover, they decrease with the number of buyers (β) as opposed to the relative market tightness measured by the ratio of sellers to buyers.

Proposition 6 suggests that every sellers' association in an economy with at least one inactive seller always sets its price below its buyers' reservation price. So the existence of an inactive seller guarantees that linked buyers will receive a non-negligible positive payoff. Furthermore, the higher the valuation of the low-quality good over the buyer-seller link costs (q^L - 2c), the greater will the buyer's non-negligible positive payoff be. Since this payoff is mainly driven by the outside option facing each buyer in the presence of inactive low-quality sellers, it is not influenced by the average quality, the quality differential or the relative market tightness.

6. Extensions and Concluding Remarks

In this section, we extend the basic framework by first allowing buyers to form multiple links and by second allowing buyers to form an association. We conclude this section by discussing the implications of active seller networking, which present interesting avenues for future research.

6.1. Multiple Links by Buyers

For illustrative purposes, assume that each buyer is allowed to have at most two links. Consider the case where there are two buyers and three sellers and where the first two sellers are high quality and the third is of low quality. Rather than examining all possible graphs (as this becomes quite repetitive without adding much intuition), we consider the following interesting graph and show that it is stable: $G = \{(b_1,s_1), (b_1,s_2), (b_2,s_2), (b_2,s_3)\}$. This graph is chosen because each buyer maintains exactly two links, one with a seller who has no other links and one with a seller who has multiple links.

Recall that each buyer demands at most one unit of the good. Thus, when a buyer has multiple links she will buy from the seller that maximizes her net payoff. If one of these sellers has m links then the buyer receives the product from this seller with probability 1/m. If this buyer has a second link, she can tell the second seller she will purchase the product with probability 1-(1/m). If graph G is stable, it must be that no buyer wants to sever one of her links, that no buyer wants to simultaneously sever one of her links and form a new link, and that no buyer or seller wants to simultaneously sever all of her/his links and become inactive. Thus, the more links a buyer has, the more stability conditions there are to check. We show (in the Appendix) that graph G is stable with active trading prices satisfying:

$$
\begin{aligned}
&c \le p_1 \le \min\{q^H - 2c,\ q^H - q^L + p_3\};\\
&2c \le p_2 \le \min\{p_1,\ q^H - q^L + p_3 - 2c\};\\
&c \le p_3 \le \min\{q^L - 2c,\ q^L - q^H + p_1\};\\
&p_1 + p_2 \le 2q^H - 4c;\\
&p_2 + p_3 \le (1/2)q^H + q^L - 2c.
\end{aligned}
\tag{11}
$$

Thus active trading prices exist if Assumption 1 plus $q^L \ge 3c$ and $q^H \ge 4c$ hold true. Therefore, if the cost of maintaining a link is not too high, it is possible to have a stable graph with multiple links by buyers.

6.2. Formation of Buyers' Association

To study the possibility of the formation of a ***buyers' association***, we focus on the case where there are more buyers than sellers. Specifically, there will be three buyers, one high-quality seller (s_1) and one low-quality seller (s_2). Again, we do not list all the possible stable graphs but instead focus on the following graph and show that it is stable: $G = \{(\{b_1,b_2\},s_1), (b_3, s_2)\}$.

In order for graph G to be stable, it must be that no buyers' association would like to sever its link and form a new one, that no individual buyer would like to leave the association and form a new link, and that no buyer or seller wants to become inactive. We show (in the Appendix) that G is stable and that active trading prices must satisfy:

$$\begin{aligned} &c \leq p_1 \leq \min\{q^H - (2/3)(q^L - p_2),\ q^H - q^L + p_2,\ q^H - q^L + p_2 + c - 2k,\ q^H - c - 2k\} \\ &c \leq p_2 \leq \min\{ q^L - (1/3)(q^H - p_1) - c + C,\ q^L - c\} \end{aligned} \tag{12}$$

where $C = c$ if $k > (1/6)c$ and $C = (1/3)c + 2k$ if $k \leq (1/6)c$. It is noted that this requires no additional conditions to Assumption 1, suggesting a strong possibility for a buyers' association to form whenever there is an excess demand for goods (since $\mu(B) > \mu(S)$).

It is interesting to compare these prices to the active trading prices of Case (i) of Table 2 where both b_1 and b_2 are individually tied to s_1 while b_3 is linked with s_2 as in the present case. First, notice that the addition of the buyers' association decreases the number of links that s_1 must maintain and so the effective minimum price that s_1 needs to break even is smaller with the buyers' association. Second, if k is small relative to c, then the effective maximum price of p_1 is larger under the buyers' association than without it. Since forming an association allows buyers to maintain fewer links to $s_{1,}$ the buyers are therefore willing to pay a higher price if the cost of forming the association is low.

6.3. Active Seller Networking

Throughout the paper, we assume as do Kranton and Minehart (2001) that only the buyer can initiate forming a link. Sellers can close up shop and thus sell to no one, but sellers are not allowed to discriminate against or seek out buyers and thus sellers are not allowed to form and/or sever individual links with buyers. What if the seller can sever individual links? The definition of a stable network now has an additional condition (to those listed in Section 2), namely, in a stable network the seller (or sellers' association) must not want to sever any of his links. If the seller can pass on the buyer-seller link costs (in the form of a higher price) to the buyer, then the set of stable networks found in Sections 3-5 will remain unchanged. A simple way to model this is to set the seller's cost of maintaining a link equal to 0 and to set the buyer's link cost equal to 2c (thus the buyer pays all link costs). It is easy to see that this assumption will quantitatively change the active trading price ranges of Tables 1-3 but will not qualitatively change the price ranges and will not change the set of stable networks.[8] That is, the qualitative results (of Sections 3-5) concerning the configuration of stable networks and active trading prices remain valid.

What happens if we allow active seller networking while at the same time allowing both buyers and sellers to maintain multiple links? This may be particularly relevant if buyers are heterogeneous (as in Kranton and Minehart (2001)). In this case, there exists an incentive for a seller of high quality to maintain links with buyers who value high quality, while additionally maintaining links to buyers who do not value quality in order to ensure sale. Alternatively, one could also motivate active seller networking by allowing buyers and sellers to bargain over prices in the active trading range (as in Cho (2002)). For instance, graphs in which there is excess demand (resp. excess supply) for a good should lead to more bargaining power for the sellers (resp. buyers) and thus prices should end up in the upper end (resp. lower end) of the active trading range. Allowing for such bargaining would make it beneficial for sellers to maintain multiple links to buyers since doing so would allure more customers.

[8] For instance, in Table 1A case (i) the "c" term on the lefthand side of the inequality will now equal zero, while the "c" term on the righthand side of the inequality will now equal 2c.

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Appendix

This Appendix, organized by the respective sections, verifies the emergence of trade networks and derives active trading prices for representative cases (with the remaining available upon request).

Proof of Selected Cases in Section 3 with Two High-Quality and One Low-Quality Sellers:

Case (ii) $G=\{(b_1, s_1), (b_2, s_1), (b_3, s_2)\}$:

In this case, the low-quality seller s_3 is inactive, whereas one of the two high-quality sellers (say, s_1) is linked to two buyers. The net payoffs of buyers are:

$$u(\{b_i\}, \{s_1\}) = (1/2)(q^H - p_1) - c \quad \text{for } i = 1,2 \quad \text{and} \quad u(\{b_3\}, \{s_2\}) = q^H - p_2 - c \tag{A1}$$

Since two buyers are linked to a single seller with a seller remaining inactive, the determination of price in this case is not as trivial as in Case (i). To begin, we may observe that in order for b_1 or b_2 not to sever the link to form a link with either s_2 or s_3, the following condition must be met: $p_1 \le 2 \min \{ (1/2)q^H - (1/2)(q^H - p_2), (1/2)q^H - q^L + p_3) \}$. Since s_3 in the initial graph is inactive, we can replace p_3 by its minimum value, c, to obtain: $p_1 \le \min \{ p_2, q^H - 2q^L + 2c \}$. In order for b_3 not to sever the link to form a link with either s_1 or s_3, we need: $p_2 \le \min \{ q^H - (1/3)(q^H - p_1), q^H - q^L + p_3) \}$, or, $p_2 \le \min \{ (1/3)(2q^H + p_1), q^H - q^L + c \}$. Additionally, the net payoffs of buyers and sellers must be positive. Hence, active trading prices must satisfy:

$$\begin{aligned} &2c \le p_1 \le \min \{ p_2, q^H - 2q^L + 2c, q^H - 2c\}; \\ &c \le p_2 \le \min \{ (1/3)(2q^H + p_1), q^H - q^L + c, q^H - c\} \end{aligned} \tag{A2}$$

which requires Assumption 1 and $q^H - 2q^L \ge 4c$ to hold true.

Case (iii) $G=\{(b_1, \{s_1, s_2\}), (b_2, \{s_1, s_2\}), (b_3, s_3)\}$:

This case features a sellers' associations of size 2, consisting of both high-quality sellers: $A^S = \{s_1, s_2\}$. The net payoffs of buyers are:

$$u(\{b_i\}, \{s_1, s_2\}) = q^H - p_{12} - c \quad \text{for } i = 1,2 \quad \text{and} \quad u(\{b_3\}, \{s_3\}) = q^L - p_3 - c. \tag{A3}$$

The presence of the sellers' associations of size 2 implies the reservation value of s_1 and s_2 becomes c+k. In order for b_1 or b_2 not to sever the link with $\{s_1, s_2\}$ to form a link with s_3, the following condition must be met: $p_{12} \le q^H - (1/2)(q^L - p_3)$, or, $p_{12} \le (1/2)(2q^H - q^L + p_3)$. In order for b_3 not to sever the link to form a link with the sellers' association, we need: $p_3 \le q^L - (2/3)(q^H - p_{12})$. Additionally, the net payoffs of buyers and sellers must be positive. Thus, active trading under this network occurs when,

$$\begin{aligned} &c + k \le p_{12} \le \min\{q^H - c, (1/2)(2q^H - q^L + p_3)\}; \\ &c \le p_3 \le \min\{q^L - c, q^L - (2/3)(q^H - p_{12})\}. \end{aligned} \tag{A4}$$

which require no additional condition to Assumption 1.

Proof of Selected Cases in Section 3 with One High-Quality and Two Low-Quality Sellers:

Case (ix) $G=\{(b_1, s_1), (b_2, s_1), (b_3, s_1)\}$:

In this case, both low-quality sellers, s_2 and s_3, are inactive. The net payoffs of buyers are:

$$u(\{b_i\}, \{s_1\}) = (1/3)(q^H - p_1) - c \quad \text{for } i = 1,2,3. \tag{A5}$$

Since both s_2 and s_3 remain inactive in the initial graph, their prices are set at c. For any buyer not to sever the link with the high-quality seller s_1 to form a link with either s_2 or s_3, we need: $p_1 \leq 3[(1/3)q^H - (q^L - c) = q^H - 3q^L + 3c$. Also, the net payoffs of buyers and sellers must be positive. So the active trading prices are:

$$3c \leq p_1 \leq \min\{ q^H - 3q^L + 3c, q^H - 3c\}; \; p_2 = p_3 = c, \tag{A6}$$

which requires: $q^H \geq 3q^L$.

Summary of Additional Assumptions Required for Stable Trade Networks in Section 3:

Case	$s_1, s_2 \in H$ and $s_3 \in L$	$s_1 \in H$ and $s_2, s_3 \in L$
(i)	none	none
(ii)	$q^H - 2q^L \geq 4c$	$q^H \geq 2q^L$
(iii)	none	none
(iv)	none	none
(v)	$q^H - (3/2)q^L \geq 3c+k$	$q^H - 2q^L \geq 3c + k$
(vi)	none	none
(vii)	$q^L \geq 4c$	$q^L \geq 4c$
(viii)	$q^H \geq 4c$	$q^H \geq 4c$
(ix)	n/a	$q^H \geq 3q^L$

Summary of Minimum and Maximum Pricing for Stable Trade Networks in Section 3:

Solving the simultaneous equation system for each applicable lower and upper bounds for the active trading prices listed in Tables 1A and 1B, we obtain the respective minimum pricing and maximum pricing for each stable trade network.

Case	**$s_1, s_2 \in H$ and $s_3 \in L$**	
	Minimum Pricing	**Maximum Pricing**
(i)	$p_1^{min} = \max\{c, q^H-2(q^L-c)\} \geq c = p_1^L$	$p_1^{max} = q^H-c = p_1^U$
	$p_2^{min} = \max\{c, q^H-2(q^L-c)\} \geq c = p_2^L$	$p_2^{max} = q^H-c = p_2^U$
	$p_3^{min} = c = p_3^L$	$p_3^{max} = q^L-c = p_3^U$
(ii)	$p_1^{min} = 2c = p_1^L$	$p_1^{max} = q^H-2(q^L-c) < q^H-2c = p_1^U$
	$p_2^{min} = 2c > c = p_2^L$	$p_2^{max} = q^H-(q^L-c) < q^H-c = p_2^U$
	$p_3^{min} = c$ (inactive)	$p_3^{max} = c$ (inactive)

	Minimum Pricing	Maximum Pricing
(iii)	$p_{12}^{min} = \max\{c+k,\ q^H-(3/2)(q^L-c)\} \geq c+k = p_{12}^L$	$p_{12}^{max} = q^H-c = p_{12}^U$
	$p_3^{min} = c = p_3^L$	$p_3^{max} = q^L-c = p_3^U$
(iv)	$p_1^{min} = c = p_1^L$	$p_1^{max} = q^H-c = p_1^U$
	$p_{23}^{min} = \max\{c+k,\ 3/2c-q^H+1/2q^L\} \geq p_{23}^L$	$p_{23}^{max} = (1/2)(q^H+q^L)-c = p_{23}^U$
(v)	$p_{12}^{min} = (3/2)c+k = p_{12}^L$	$p_{12}^{max} = q^H-(3/2)(q^L-c) < q^H-(3/2)c = p_{12}^U$
	$p_3^{min} = c$ (inactive)	$p_3^{max} = c$ (inactive)
(vi)	$p_{123}^{min} = c+2k = p_{123}^L$	$p_{123}^{max} = (1/3)(2q^H+q^L)-c = p_{123}^U$
(vii)	$p_{12}^{min} = \max\{c+2k, q^H-(1/2)(q^L-2c)\} \geq c+2k = p_{12}^L$	$p_{12}^{max} = q^H-c = p_{12}^U$
	$p_3^{min} = \max\{2c, q^L-3q^H+3(c+2k)\} \geq 2c = p_3^L$	$p_3^{max} = q^L-2c = p_3^U$
(viii)	$p_1^{min} = \max\{2c, 3(c+2k)-(1/2)(q^H+3q^L)\} \geq 2c = p_1^L$	$p_1^{max} = q^H-2c = p_1^U$
	$p_{23}^{min} = \max\{c+2k, (1/2)q^L+c\} \geq c+2k = p_{23}^L$	$p_{23}^{max} = (1/2)(q^H+q^L)-c = p_{23}^U$

Case **$s_1 \in H$ and $s_2, s_3 \in L$**

Case	Minimum Pricing	Maximum Pricing
(i)	$p_1^{min} = \max\{c,\ q^H-2(q^L-c)\} \geq c = p_1^L$	$p_1^{max} = q^H-c = p_1^U$
	$p_2^{min} = c = p_2^L$	$p_2^{max} = q^L-c = p_2^U$
	$p_3^{min} = c = p_3^L$	$p_3^{max} = q^L-c = p_3^U$
(ii)	$p_1^{min} = 2c = p_1^L$	$p_1^{max} = q^H-2(q^L-c) < q^H-2c = p_1^U$
	$p_2^{min} = c = p_2^L$	$p_2^{max} = c < q^L-2c = p_2^U$
	$p_3^{min} = c$ (inactive)	$p_3^{max} = c$ (inactive)
(iii)	$p_{12}^{min} = \max\{c+k,\ (1/2)(3c+q^H-2q^L)\} \geq c+k = p_{12}^L$	$p_{12}^{max} = (1/2)(q^H+q^L)-c = p_{12}^U$
	$p_3^{min} = c = p_3^L$	$p_3^{max} = q^L-c = p_3^U$
(iv)	$p_1^{min} = \max\{c,\ q^H+2(c+k-q^L)\} \geq c = p_1^L$	$p_1^{max} = q^H-c = p_1^U$
	$p_{23}^{min} = \max\{c+k,\ (3/2)c - (3/2)q^H+q^L\} \geq p_{23}^L$	$p_{23}^{max} = q^L-c = p_{23}^U$
(v)	$p_{12}^{min} = (3/2)c+k = p_{12}^L$	$p_{12}^{max} = (1/2)(q^H-2q^L+3c) < (1/2)(q^H+q^L-3c) = p_{12}^U$
	$p_3^{min} = c$ (inactive)	$p_3^{max} = c$ (inactive)
(vi)	$p_{123}^{min} = c+2k = p_{123}^L$	$p_{123}^{max} = (1/3)(q^H+2q^L)-c = p_{123}^U$

(vii) p_{12}^{min} = max{(1/2)q^H+c, c+2k} ≥ c+2k = p_{12}^L p_{12}^{max} = (1/2)(q^H+q^L)-c = p_{12}^U
p_3^{min} = max{3c+6k-(3/2)q^H-(1/2)q^L, 2c} ≥ 2c = p_3^L p_3^{max} = q^L-2c = p_3^U

(viii) p_1^{min} = max{2c, q^H+3(c+2k-q^L)} ≥ 2c = p_1^L p_1^{max} = q^H-2c = p_1^U
p_{23}^{min} = max{c+2k, q^L-(1/2)q^H+c} ≥ c/2+k = p_{23}^L p_{23}^{max} = q^L-c = p_{23}^U

(ix) p_1^{min} = 3c = p_1^L p_1^{max} = q^H-3(q^L-c) < q^H-3c = p_1^U
p_2^{min} = c (inactive) p_2^{max} = c (inactive)
p_3^{min} = c (inactive) p_3^{max} = c (inactive)

Proof of Selected Cases in Section 4 with More Buyers Than Sellers:

Case (v) G={(b_1, s_1), (b_2, s_2)}:

The net payoffs of buyers are: u({b_1}, {s_1}) = q^H - p_1 - c; u({b_2}, {s_2}) = q^L - p_2 - c; u({b_3}, {}) = 0. For b_3 not to form a link with s_1 or s_2, we need: 0 > (1/2)(q^H - p_1) - c and 0 > (1/2)(q^L - p_2) - c. (The inequality must be strict since we assume that if a buyer is indifferent between adding a link and being unlinked then she always adds the link.) For b_1 and b_2 not to sever their ties, we need: q^H - p_1 - c ≥ 0 and q^L - p_2 - c ≥ 0. The above conditions imply that b_1 (respectively b_2) will never sever her tie and link with s_2 (resp. s_1) since doing so would give b_1 (respectively b_2) a negative payoff. Thus, active trading prices satisfy:

$$q^H - 2c < p_1 \le q^H - c \text{ and } c \le p_1; \qquad q^L - 2c < p_2 \le q^L - c \text{ and } c \le p_2 \qquad (A7)$$

which requires that c>0.

Summary of Additional Assumptions Required for Stable Trade Networks in Section 4:

Case	$s_1 \in H$ and $s_2 \in L$
(i)	$q^H \ge 4c$
(ii)	$q^H \ge 3q^L$
(iii)	$q^H + q^L \ge 6c+2k$
(iv)	$q^L \ge 4c$
(v)	c>0
(vi)	c>0

Case	$s_1, s_2 \in H$ and $s_3 \in L$	$s_1 \in H$ and $s_2, s_3 \in L$
(i)	none	none
(ii)	none	$q^H - q^L \ge 2k$
(iii)	none	none
(iv)	$2q^H + q^L \ge 6c+9k$	$q^H + 2q^L \ge 6c+9k$
(v)	$2q^H - 3q^L \ge 6c+2k$	none
(vi)	n/a	$q^H > 2(q^L + c)$

Summary of Minimum and Maximum Pricing for Stable Trade Networks in Section 4:

Case — $s_1 \in H$ and $s_2 \in L$

Case	Minimum Pricing	Maximum Pricing
(i)	$p_1^{min} = \max\{2c, q^H-3(q^L-c)\} \geq 2c = p_1^L$ $p_2^{min} = \max\{c, q^L-q^H+2c\} \geq c = p_2^L$	$p_1^{max} = q^H-2c = p_1^U$ $p_2^{max} = q^L-c = p_2^U$
(ii)	$p_1^{min} = 3c = p_1^L$ $p_2^{min} = c = p_2^L$	$p_1^{max} = q^H-3(q^L-c) < q^H-3c = p_1^U$ $p_2^{max} = c$ (inactive)
(iii)	$p_{12}^{min} = (3/2)c+k = p_{12}^L$	$p_{12}^{max} = (1/2)(q^H+q^L) - (3/2)c = p_{12}^U$
(iv)	$p_1^{min} = \max\{c, q^H-q^L+2c\} \geq c = p_1^L$ $p_2^{min} = \max\{2c, q^L-3q^H+3c\} \geq 2c = p_2^L$	$p_1^{max} = q^H-c = p_1^U$ $p_2^{max} = q^L-2c = p_2^U$
(v)	$p_1^{min} = \max\{c, q^H-2c\} \geq c = p_1^L$ $p_2^{min} = \max\{c, q^L-2c\} \geq c = p_2^L$	$p_1^{max} = q^H-c = p_1^U$ $p_2^{max} = q^L-c = p_2^U$
(vi)	$p_{12}^{min} = \max\{c+k, (1/2)(q^H+q^L-3c)\} \geq c+k = p_{12}^L$	$p_{12}^{max} = (1/2)(q^H+q^L) - c = p_{12}^U$

Case — $s_1, s_2 \in H$ and $s_3 \in L$

Case	Minimum Pricing	Maximum Pricing
(i)	$p_1^{min} = c = p_1^L$ $p_2^{min} = c = p_2^L$ $p_3^{min} = c = p_3^L$	$p_1^{max} = q^H-q^L+c < q^H-c = p_1^U$ $p_2^{max} = q^H-q^L+c < q^H-c = p_2^U$ $p_3^{max} = c = p_3^U$ (inactive)
(ii)	$p_{12}^{min} = c+k = p_{12}^L$ $p_3^{min} = c$ (inactive)	$p_{12}^{max} = q^H-q^L+c < q^H-c = p_{12}^U$ $p_3^{max} = c$ (inactive)
(iii)	$p_1^{min} = \max\{c, 2c+4k-q^L\} \geq p_1^L$ $p_{23}^{min} = \max\{c+2k, (1/2)(q^L-q^H)+c\} \geq p_{23}^L$	$p_1^{max} = q^H-c = p_1^U$ $p_{23}^{max} = (1/2)(q^H+q^L)-c = p_{23}^U$
(iv)	$p_{123}^{min} = c+3k = p_{123}^L$	$p_{123}^{max} = (1/3)(2q^H+q^L)-c = p_{123}^U$
(v)	$p_{12}^{min} = \max\{c+2k, q^H-q^L+c\} \geq p_{12}^L$ $p_3^{min} = \max\{c, (1/2)q^L-q^H+c+2k\} \geq p_3^L$	$p_{12}^{max} = q^H-c = p_{12}^U$ $p_3^{max} = q^L-c = p_3^U$

Case	**$s_1 \in H$ and $s_2, s_3 \in L$** **Minimum Pricing**	**Maximum Pricing**
(i)	$p_1^{min} = c = p_1^L$ $p_2^{min} = c = p_2^L$ $p_3^{min} = c$ (inactive)	$p_1^{max} = q^H - q^L + c < q^H - c = p_1^U$ $p_2^{max} = c = p_2^U$ $p_3^{max} = c$ (inactive)
(ii)	$p_{12}^{min} = c + k = p_{12}^L$ $p_3^{min} = c$ (inactive)	$p_{12}^{max} = (1/2)(q^H - q^L) + c < (1/2)(q^H + q^L) + c = p_{12}^U$ $p_3^{max} = c$ (inactive)
(iii)	$p_1^{min} = \max\{c, q^H - 2q^L + 2c + 4k\} \geq p_1^L$ $p_{23}^{min} = c + 2k = p_{23}^L$	$p_1^{max} = q^H - c = p_1^U$ $p_{23}^{max} = q^L - c = p_{23}^U$
(iv)	$p_{123}^{min} = c + 3k = p_{123}^L$	$p_{123}^{max} = (1/3)(q^H + 2q^L) - c = p_{123}^U$
(v)	$p_{12}^{min} = \max\{c + 2k, (1/2)(q^H - q^L) + c\} \geq p_{12}^L$ $p_3^{min} = \max\{c, c + 2k - (1/2)q^H\} \geq p_3^L$	$p_{12}^{max} = (1/2)(q^H + q^L) - c = p_{12}^U$ $p_3^{max} = q^L - c = p_3^U$
(vi)	$p_1^{min} = 2c = p_1^L$ $p_2^{min} = c$ (inactive) $p_3^{min} = c$ (inactive)	$p_1^{max} = (1/2)q^H - q^L + c < (1/2)q^H - c = p_1^U$ $p_2^{max} = c$ (inactive) $p_3^{max} = c$ (inactive)

Proof of Stable Graphs in Section 6:

Multiple Links by Buyers with $G = \{(b_1, s_1), (b_1, s_2), (b_2, s_2), (b_2, s_3)\}$:

The net payoffs of buyers are:

$$u((\{b_1\},\{s_1\}), (\{b_1\},\{s_2\})) = q^H - \min\{p_1, (1/2)(p_1 + p_2)\} - 2c$$

$$u((\{b_2\},\{s_2\}), (\{b_2\},\{s_3\})) = \max\ \{q^L - p_3, (1/2)(q^H + q^L - p_2 - p_3)\} - 2c \tag{A8}$$

We must check that no buyer wants to sever a link and that no buyer would like to simultaneously sever a link and form a new link. First note that in order for b_1 not to sever her link to s_2 we must have at least $\min\{p_1, (1/2)p_1 + (1/2)p_2\} = (1/2)(p_1 + p_2)$ and in order for b_2 not to sever her link to s_2 we must have at least: $\max\ \{q^L - p_3, (1/2)(q^H + q^L - p_2 - p_3)\} = (1/2)(q^H + q^L - p_2 - p_3)$.

Next we systematically check all links. In order for b_1 not to sever her link to s_1 we must have: $p_1 \leq q^H - 2c$. In order for b_1 not to sever her link to s_1 and form a link with s_3 we must have: $p_1 \leq q^H - q^L + p_3$. In order for b_1 not to want to sever her link to s_2 or sever this link and form one with s_3 we must have: $p_2 \leq \min\ \{p_1,\ q^H - q^L + p_3\}$. In order for b_2 not to want to sever her link to s_2 or sever this link and form one with s_1 we must have: $p_2 \leq q^H - q^L + p_3 - 2c$. In order for b_2 not to want to sever her link to s_3 or sever this link and form one with s_1 we must have: $p_3 \leq \min\ \{q^L - 2c, q^L - q^H + p_1\}$. Additionally, no buyer or seller wants to sever all of her/his links. Combining these results, we obtain the set of active trading prices which must satisfy (11).

Formation of Buyers' Association with $G = \{(\{b_1,b_2\},s_1), (b_3, s_2)\}$

In graph G with a buyers' association $\{b_1,b_2\}$, the net payoffs of buyers are:

$$u(\{b_1,b_2\},\{s_1\}) = (1/2)(q^H - p_1) - (1/2)c - k$$
$$u(\{b_3\}, \{s_2\}) = q^L - p_2 - c. \tag{A9}$$

We must check that no buyers' association would like to sever its link and form a new one and that no individual buyer would like to leave the association and form a new link. First, we check that the association $\{b_1,b_2\}$does not want to sever its link to s_1 and form one with s_2, which requires that: $p_1 \leq q^H - (2/3)(q^L - p_2)$.

Next, we check that no individual buyer would like to leave the buyers' association, sever her current link and form a new one. When a buyer contemplates such a change she acts independently and assumes that all other buyer and seller links remain. Thus, if b_1 severs her link with s_1 then she simultaneously severs her association with b_2; the link between b_2 and s_1 is assumed to remain. In order for b_1 not to sever her link with s_1 and form a link with s_2 we must have: $q^H - p_1 - c - 2k \geq q^L - p_2 - 2c$. It is also possible that when b_1 links with s_1 that she simultaneously forms an association with b_3. Thus we also must have: $q^H - p_1 - c - 2k \geq q^L - p_2 - c -2k$. The association between b_1 and b_3 will only form if b_3 agrees. Buyer b_3 agrees to the association if belonging to the association decreases her total cost which implies that: $(1/2)c + k \leq c$. Thus, in order for b_1 not to sever her link with s_1 and form a link with s_2 we must have: $q^H - p_1 - c - 2k \geq q^L - p_2 - \min \{2c, c -2k\}$. The same condition will prevent b_2 from severing her link.

In order for b_3 not to sever her link and link with s_1 we need: $q^L - p_2 - c \geq (1/3)(q^H - p_1) - (1/3)c - 2k$. However, b_1 and b_2 will only agree to this association if: $(1/3)c + 2k \leq (1/2)c + k$. Additionally all net payoffs for buyers and sellers should be positive. Hence, from the above arguments, active trading prices must satisfy (12).

Table 1: Stable Trade Network with Three Buyers and Three Sellers

Case A: $H = \{s_1, s_2\}$, $L = \{s_3\}$

	Trade Network Patterns	Active Trading Price Ranges
i	$\{(b_1, s_1), (b_2, s_2), (b_3, s_3)\}$	$c \leq p_1 \leq (1/2) \min \{ q^H + p_2, 2q^H - q^L + p_3, 2q^H -2c\}$ $c \leq p_2 \leq (1/2) \min \{ q^H + p_1, 2q^H - q^L + p_3, 2q^H -2c\}$ $c \leq p_3 \leq (1/2) \min \{ 2q^L - q^H + p_1, 2q^L - q^H + p_2, 2q^L -2c\}$
ii	$\{(b_1, s_1), (b_2, s_1), (b_3, s_2)\}$	$2c \leq p_1 \leq \min \{ p_2, q^H - 2q^L + 2c\}$ $c \leq p_2 \leq \min \{ (1/3) (2q^H + p_1), q^H - q^L + c\}$ $p_3 = c$
iii	$\{(b_1, \{s_1, s_2\}), (b_2, \{s_1, s_2\}), (b_3, s_3)\}$	$c + k \leq p_{12} \leq \min \{ (1/2) (2q^H - q^L + p_3), q^H -c\}$ $c \leq p_3 \leq \min \{ q^L - (2/3) (q^H - p_{12}), q^L -c\}$
iv	$\{(b_1, s_1), (b_2, \{s_2, s_3\}), (b_3, \{s_2, s_3\})\}$	$c \leq p_1 \leq \min \{ (1/3) (2q^H - q^L + 2p_{23}), q^H -c\}$ $c +k \leq p_{23} \leq (1/2) \min \{ q^L + p_1, q^H + q^L -2c\}$
v	$\{(b_1, \{s_1, s_2\}), (b_2, \{s_1, s_2\}), (b_3, \{s_1, s_2\})\}$	$(3/2)c + k \leq p_{12} \leq q^H - (3/2)q^L + (3/2)c$ $p_3 = c$
vi	$\{(b_1, \{s_1, s_2, s_3\}), (b_2, \{s_1, s_2, s_3\}), (b_3, \{s_1, s_2, s_3\})\}$	$c + 2k \leq p_{123} \leq (1/3) (2q^H + q^L) - c$
vii	$\{(b_1, \{s_1, s_2\}), (b_2, s_3), (b_3, s_3)\}$	$c + 2k \leq p_{12} \leq \min \{q^H - (1/3)q^L + (1/3)p_3, q^H - c\}$; $2c \leq p_3 \leq \min \{q^L - 2(q^H - p_{12}), q^L - 2c\}$
viii	$\{(b_1, s_1), (b_2, s_1), (b_3, \{s_2, s_3\})\}$	$2c \leq p_1 \leq \min \{2p_{23} - q^L, q^H - 2c\}$ $c +2k \leq p_{23} \leq \min \{(1/6)(q^H +3q^L + 2p_1), (1/2)(q^H + q^L) - c\}$

Table 1: Stable Trade Network with Three Buyers and Three Sellers

Case B: $H = \{s_1\}$, $L = \{s_2, s_3\}$

Trade Network Patterns		Active Trading Price Ranges
i	$\{(b_1, s_1), (b_2, s_2), (b_3, s_3)\}$	$c \le p_1 \le (1/2) \min \{ 2q^H - q^L + p_2, 2q^H - q^L + p_3, 2q^H - 2c\}$ $c \le p_2 \le (1/2) \min \{ 2q^L - q^H + p_1, q^L + p_3, 2q^L - 2c\}$ $c \le p_3 \le (1/2) \min \{ 2q^L - q^H + p_1, q^L + p_2, 2q^L - 2c\}$
ii	$\{(b_1, s_1), (b_2, s_1), (b_3, s_2)\}$	$2c \le p_1 \le q^H - 2q^L + 2c$ $p_2 = p_3 = c$
iii	$\{(b_1, \{s_1, s_2\}), (b_2, \{s_1, s_2\}), (b_3, s_3)\}$	$c + k \le p_{12} \le (1/2) \min \{ q^H + p_3, q^H + q^L - 2c\}$ $c \le p_3 \le \min \{ (1/3) (2q^L - q^H + 2p_{12}), q^L - c\}$
iv	$\{(b_1, s_1), (b_2, \{s_2, s_3\}), (b_3, \{s_2, s_3\})\}$	$c \le p_1 \le \min \{ (1/3) (3q^H - 2q^L + 2p_{23}), q^H - c\}$ $c + k \le p_{23} \le \min \{ (1/2) (2q^L - q^H + p_1), q^L - c\}$
v	$\{(b_1, \{s_1, s_2\}), (b_2, \{s_1, s_2\}), (b_3, \{s_1, s_2\})\}$	$(3/2)c + k \le p_{12} \le ½(q^H - 2q^L + 3c)$ $p_3 = c$
vi	$\{(b_1, \{s_1, s_2, s_3\}), (b_2, \{s_1, s_2, s_3\}), (b_3, \{s_1, s_2, s_3\})\}$	$c + 2k \le p_{123} \le (1/3) (q^H + 2q^L) - c$
vii	$\{(b_1, \{s_1, s_2\}), (b_2, s_3), (b_3, s_3)\}$	$c + 2k \le p_{12} \le \min\{(1/2)(q^H+q^L) - c, (1/2)q^H+(1/6)q^L+(1/3)p_3\}$ $2c \le p_3 \le \min\{ q^L - 2c, 2p_{12} - q^H\}$
viii	$\{(b_1, s_1), (b_2, s_1), (b_3, \{s_2, s_3\})\}$	$2c \le p_1 \le \min \{ q^H - 2q^L + 2p_{23}, q^H - 2c\}$ $c + 2k \le p_{23} \le \min \{ (1/3) (3q^L - q^H + p_1), q^L - c\}$
ix	$\{(b_1, s_1), (b_2, s_1), (b_3, s_1)\}$	$3c \le p_1 \le q^H - 3q^L + 3c$ $p_2 = p_3 = c$

Table 2: Stable Trade Network with Three Buyers and Two Sellers

Trade Network Patterns		Active Trading Price Ranges
i	$\{(b_1, s_1), (b_2, s_1), (b_3, s_2)\}$	$2c \leq p_1 \leq \min \{ q^H - q^L + p_2, q^H -2c\}$ $c \leq p_2 \leq \min \{ q^L - (1/3)(q^H - p_1), q^L -c\}$
ii	$\{(b_1, s_1), (b_2, s_1), (b_3, s_1)\}$	$3c \leq p_1 \leq \min \{ q^H - 3q^L + 3c, q^H -3c\}$ $p_2 = c$
iii	$\{(b_1, \{s_1, s_2\}), (b_2, \{s_1, s_2\}), (b_3, \{s_1, s_2\})\}$	$(3/2)c + k \leq p_{12} \leq (1/2)(q^H + q^L) - (3/2)c$
iv	$\{(b_1, s_1), (b_2, s_2), (b_3, s_2)\}$	$c \leq p_1 \leq \min \{ q^H - (1/3)(q^L - p_2), q^H -c\}$ $2c \leq p_2 \leq \min \{ q^L - q^H + p_1, q^L -2c\}$
v	$\{(b_1, s_1), (b_2, s_2)\}$	$q^H - 2c < p_1 \leq q^H - c$ and $c \leq p_1$ $q^L - 2c < p_2 \leq q^L - c$ and $c \leq p_2$
vi	$\{(b_1, \{s_1, s_2\}), (b_2, \{s_1, s_2\})\}$	$(1/2)(q^H + q^L - 3c) < p_{12} \leq (1/2)(q^H + q^L - 2c)$ and $c+k \leq p_{12}$

Table 3: Stable Trade Network with Two Buyers and Three Sellers

Case A: $H = \{s_1, s_2\}$, $L = \{s_3\}$

Trade Network Patterns		Active Trading Price Ranges
i	$\{(b_1, s_1), (b_2, s_2)\}$	$c \le p_1 \le \min \{(1/2)(q^H + p_2), q^H - q^L + c\}$ $c \le p_2 \le \min \{(1/2)(q^H + p_1), q^H - q^L + c\}$ $p_3 = c$
ii	$\{(b_1, \{s_1, s_2\}), (b_2, \{s_1, s_2\})\}$	$c+k \le p_{12} \le q^H - q^L + c$ $p_3 = c$
iii	$\{(b_1, s_1), (b_2, \{s_2, s_3\})\}$	$c \le p_1 \le \min \{ (1/2)(q^H - q^L) + p_{23}, q^H - c\}$ $c + 2k \le p_{23} \le \min \{ (1/2)(q^L + p_1), (1/2)(q^H + q^L) - c\}$
iv	$\{(b_1, \{s_1, s_2, s_3\}), (b_2, \{s_1, s_2, s_3\})\}$	$c + 3k \le p_{123} \le (1/3)(2q^H + q^L) - c$
v	$\{(b_1, \{s_1, s_2\}), (b_2, s_3)\}$	$c + 2k \le p_{12} \le \min \{ q^H - (1/2)q^L + p_3, q^H - c\}$ $c \le p_3 \le \min \{q^L - q^H + p_{12}, q^L - c\}$

Case B: $H = \{s_1\}$, $L = \{s_2, s_3\}$

Trade Network Patterns		Active Trading Price Ranges
i	$\{(b_1, s_1), (b_2, s_2)\}$	$c \le p_1 \le q^H - q^L + c$ $p_2 = p_3 = c$
ii	$\{(b_1, \{s_1, s_2\}), (b_2, \{s_1, s_2\})\}$	$c+k \le p_{12} \le (1/2)(q^H - q^L) + c$ $p_3 = c$
iii	$\{(b_1, s_1), (b_2, \{s_2, s_3\})\}$	$c \le p_1 \le \min \{q^H - q^L + p_{23}, q^H - c\}$ $c + 2k \le p_{23} \le \min \{ q^L - (1/2)(q^H - p_1), q^L - c\}$
iv	$\{(b_1, \{s_1, s_2, s_3\}), (b_2, \{s_1, s_2, s_3\})\}$	$c + 3k \le p_{123} \le (1/3)(q^H + 2q^L) - c$
v	$\{(b_1, \{s_1, s_2\}), (b_2, s_3)\}$	$c + 2k \le p_{12} \le \min \{ (1/2)q^H + p_3, (1/2)(q^H + q^L) - c\}$ $c \le p_3 \le \min \{ p_{12} - (1/2)(q^H - q^L), q^L - c\}$
vi	$\{(b_1, s_1), (b_2, s_1)\}$	$2c \le p_1 \le (1/2)q^H - q^L + c$ $p_2 = p_3 = c$

FIGURE 1: Patterns of Trade Networks with $\mu(B)=3$, $\mu(S)=3$

Figure 1A: Let i = H. Cases (i) – (viii) are stable
Figure 1B: Let i = L. Cases (i) – (ix) are stable

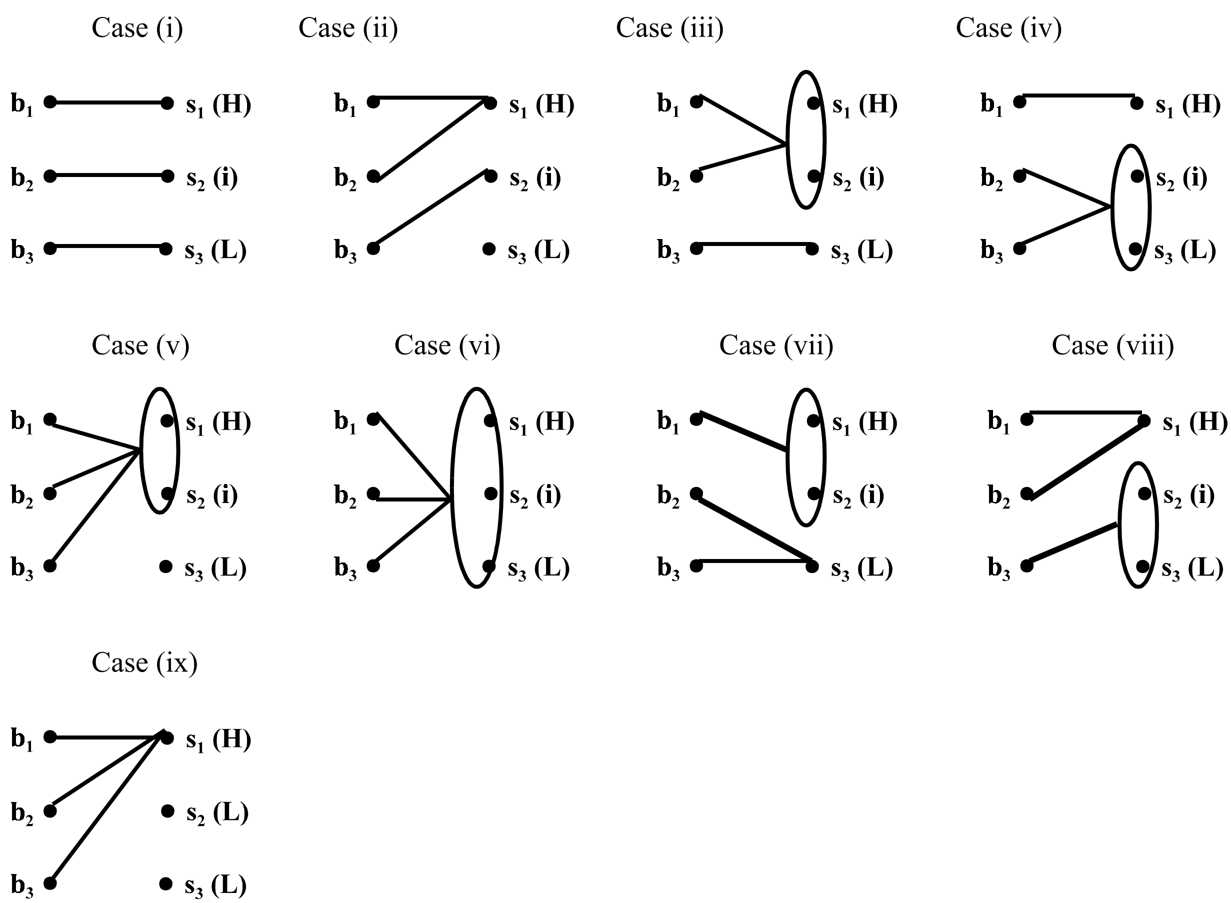

FIGURE 2: Patterns of Trade Networks with μ(B)=3, μ(S)=2 and μ(H)=μ(L)=1

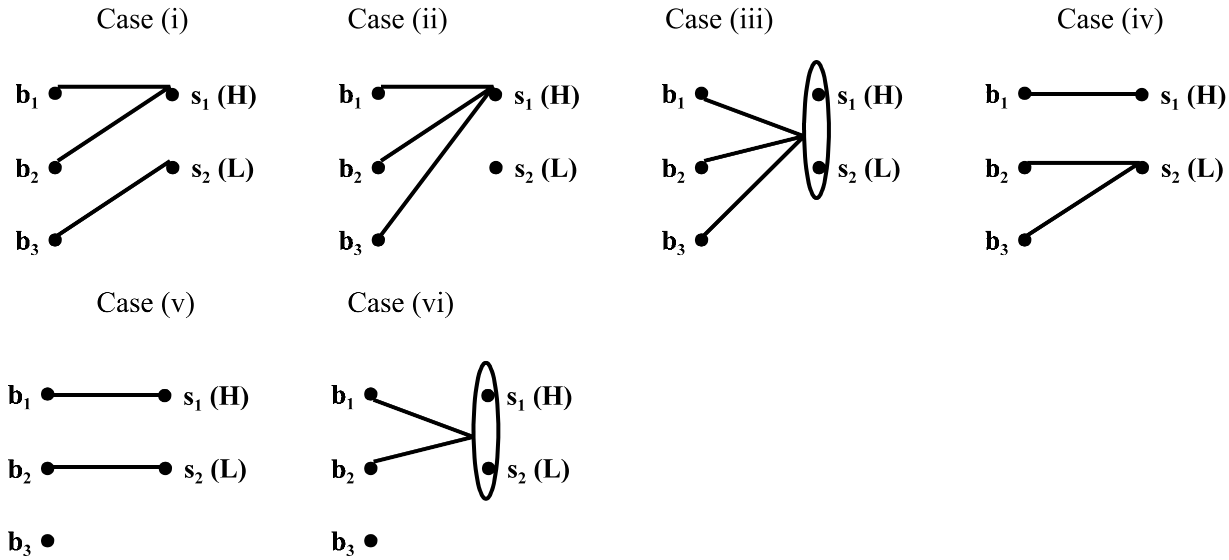

FIGURE 3: Patterns of Trade Networks with μ(B)=2, μ(S)=3

Figure 3A: Let i = H. Cases (i) – (v) are stable.
Figure 3B: Let i = L. Cases (i) – (vi) are stable.