MISCELLANEOUS.

NEW POPULAR WORKS IN MATHEMATICS.

We spoke at length in the last *Open Court* of the high character of the educational work of the great English mathematician DE MORGAN. The Open Court Publishing Company has, in the interest of exact and sound popular education, recently selected one of his most characteristic productions for re-publication It is entitled *The Study and Difficulties of Mathematics*, a work of some 284 duodecimo pages, treating in a lively and fascinating style of all the main difficulties of elementary mathematics from counting and systems of numeral notation to quadratic equations and the theory of proportion.

The book might appropriately be termed *The Spirit of Mathematics*. The principles of arithmetic, algebra, geometry, etc., are not presented as dead rules and mechanical systems, but as things of natural growth, which can be followed with the same delight and with more accuracy even than the development of a flower or a plant. The artificial and formal rigor of presentation which is the ideal of the finished scholar and master of a province but is so confusing and disheartening to the tyro, is here replaced by the treatment of simple specific types, from which the abstract generalisations are *collected* as in any other science. This is the real method of learning ; formal, rigorous proof is unintelligible and an actual hindrance, a stumbling-block, until experience has made the mind familiar with the general extent of the propositions to be proved. Demonstration clinches knowledge, but it is useless in the discovery and in the inculcation of knowledge ; we count our chickens after they are hatched. Many students can give faultless proofs of propositions in mathematics over which they have no practical power and with which they have really no acquaintance. Their education has begun at the wrong end.

To all such unfortunate beings, who are desirous of correcting the sins of their youth and of resuming in an economical manner the study of a science which lies at the basis of every rigorous world-conception, as well as to all elementary teachers who are seeking light and fresh views, and to all students who have grown "stale" in the mental athleticism of certain modern mill-systems, we cordially recommend this little book of De Morgan. What formerly made them frown will here make them smile ; the joy of knowledge will supplant the pain of ignorance. They will not find everything in the book, but what they do find they will understand, retain, and be able to put to profitable intellectual use. They will find advice on how to pursue their studies in the higher branches, they will obtain an insight into the philosophy of the subject, its history and development ; they will

discover that science is not an ogre designed by the Evil One to overawe and frighten mortals, but a thing of confidence and beauty that has come from little and simple things and is destined to work their salvation.

The work has been brought down to date by a few notes on modern text-books of algebra, logic, the philosophy of mathematics, and pangeometry. A picture of De Morgan is given as a frontispiece, an index has been supplied; the type is distinct and the binding pretty.[1]

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The Mathematical Essays and Recreations[2] of Professor Schubert, of Hamburg, Germany, is a collection of six articles bearing the following titles: (1) "The Definition and Notion of Number"; (2) "Monism in Arithmetic"; (3) "On the Nature of Mathematical Knowledge"; (4) "Magic Squares"; (5) "The Fourth Dimension"; (6) "The History of the Squaring of the Circle."

The first three articles are concerned with the construction of arithmetic as a monistic science, all the consequences of which flow as a matter of pure logic from a few simple principles. Number is defined as the result of counting. Fractional numbers, negative numbers, irrational numbers, imaginary numbers, complex numbers, are all extensions of the primitive result, made according to what Hankel calls the "principle of permanence," and Schubert the principle of no exception," which means that the operations conducted with them and the rules governing them shall be so treated as to form no exception to the operations and rules springing from the original real results. Arithmetic thus takes the general shape of a system of mental fictions, which have consistency and coherency among themselves, and of which the results admit of interpretation and application to real facts, but which have no actual counterparts in reality. It is what philosophers call a nominalistic view of science. It has its æsthetic and logical advantages, and high didactic value to the mature student and to the teacher. After one has reached the heights, there is no satisfaction comparable to that of a broad survey of the land below. Professor Schubert is one of the most successful teachers of Germany, and as there are few treatises in English that give this point of view, his sketch of monistic arithmetic will be found exceedingly suggestive.

The article on the "Fourth Dimension" is popular and shows clearly what is meant by "dimension" in science and what the legitimate function of a "fourth dimension" is in mathematics; of the claims of spiritualism to this beautiful and convenient concept, it disposes definitively. The article on "Magic Squares" is a pleasing recreation. That on the "Squaring of the Circle" gives the history of one of the most instructive and interesting episodes in the history of human thought. Both these essays are very complete popular accounts of their subjects, —more complete perhaps than any generally accessible accounts in English.

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The new *Introduction to Algebra*[3] by Dr. G. Chrystal, professor of mathematics in the University of Edinburgh, can hardly be called a popular book, but it deserves mention for its many innovations in elementary text-book writing and for its sound qualities of independence and common sense. In the year 1886

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[1] Published by the Open Court Publishing Co., 324 Dearborn Street, Chicago (London: Kegan Paul, Trench & Co.) Pp. 288. Price, $1 25 (5s.).

[2] Chicago: The Open Court Publishing Co. London: Kegan Paul, Trench, Trübner & Co. Pp. 148. Price, cloth; 75c. (3s.); paper, 25c. (1s. 6d.).

[3] London: Adam & Charles Black. New York: The Macmillan Company. Pp. 437. Price, $1.25.

Professor Chrystal wrote a little text-book of "elementary algebra" which filled two volumes of nearly 600 pages each. It broke completely with English tradition. Algebra, instead of being presented as a mere jumble of disconnected rules, was set forth as a coherent science, all the principles of which were deduced systematically from a few fundamental laws : it was made a science of pure form ; it was brought into connection with the remaining branches of mathematical knowledge, and no device was scorned which subserved the ends of illustration and clearness. It was filled with citations of the sources and historical notes, and so became both a manual and general reference book of the highest order.

Professor Chrystal is an algebraist κατ' ἐξοχήν. Not only is much algebra, and all of algebra, a pre-requisite in his world-view to entrance into Heaven, but it is also necessary to entrance into the Infinitesimal Calculus. But the world rebelled. While his book was successful it was felt in the abysmal subconscious depths of the average timorous citizen that 1,100 pages of matter, not to be read as a novel was too much, even for salvation ; and that if the same view was taken of every science, a life-time would be absolutely insufficient even as a *preparation* for the Nirvâna of knowledge. And so Professor Chrystal wrote his *Introduction to Algebra* "for the use of Secondary Schools and technical Colleges," that common people might gain some conception of the shape that algebraic science has been taking in the last century.

The book is naturally, even familiarly, written. One is struck by the author's insistence on practical points of view, by his genetic conception of education, by his easy introduction of modern notions. Examination puzzles are eschewed ; only seldom are its readers required to

> "wisely tell what hour o' th' day
> The clock doth strike by Algebra."

But the most notable feature of the book is its constant use of graphical illustration. It seems astonishing that this most powerful engine of education should not have been the common possession of elementary teachers a century ago. It is now really time for the schools of the Pithecanthropoi to adopt it. And yet Professor Chrystal must apologize for the unusually large amount of space he has devoted to its employment!

The book, in fine, deserves to be widely used. Where school-boards still insist on the retention of pre-historic treatises, where publishing companies still make a business of exploiting ignorance, independent teachers should surround themselves with and study such books as Professor Chrystal's. These books exist, and they should be used, by stealth if necessary. T. J. McCORMACK.

RECENT PHILOSOPHICAL PUBLICATIONS.

The philosophy of Nietzsche, which has for years been the dominant fad in Germany, has spread beyond the borders of the Fatherland, and been exciting increased interest in foreign countries. The Macmillan Company are now publishing a translation of *The Works of Nietzsche*, under the editorship of Alexander Tille, lecturer at the University of Glasgow, and based on the final German edition published by Naumann of Leipsic. Although Nietzsche's intellectual career naturally and logically terminated in insanity, his productions constitute one of the most remarkable phenomena in recent philosophy. He is disconnected, bizarre, freaky, erratic, but interesting and highly suggestive. His works are, owing to their highly condensed, epigrammatic, and elliptic style, exceedingly difficult to trans-