9-11-2007

Anticipating Semilinear SPDEs (Mittag-Leffler Institute Workshop)

Salah-Eldin A. Mohammed
Southern Illinois University Carbondale, salah@sfde.math.siu.edu

Invited Talk; Mittag-Leffler Institute Workshop I: Stochastic PDE's; Royal Swedish Academy of Sciences; Stockholm, Sweden; September 11, 2007

Anticipating Semilinear SPDEs [a]

Salah Mohammed [b]

http://sfde.math.siu.edu/

Mittag-Leffler: September 11, 2007
Sweden

[a] Results to appear in JFA [M-Z]
[b] Department of Mathematics, SIU-C, Carbondale, Illinois, USA

Acknowledgment

- Joint work with T.S. Zhang (Manchester, UK).

Acknowledgment

- Joint work with T.S. Zhang (Manchester, UK).
- Research supported by NSF: DMS-0203368 and DMS-0705970.

Objective

Question:

Objective

Question:

Does the following anticipating stochastic evolution equation (see):

$$\left.\begin{aligned} dv(t) &= -Av(t)\,dt + F_0\big(v(t)\big)\,dt \\ &\qquad + Bv(t)\circ dW(t),\ t>0, \\ v(0) &= Y \end{aligned}\right\} \qquad (1)$$

admit a solution with a random initial condition $Y:\Omega\longrightarrow H$ in a Hilbert space H?

Objective

Question:

Does the following anticipating stochastic evolution equation (see):

$$\left.\begin{aligned} dv(t) &= -Av(t)\,dt + F_0\big(v(t)\big)\,dt \\ &\qquad\qquad + Bv(t)\circ dW(t),\ t>0, \\ v(0) &= Y \end{aligned}\right\} \qquad (1)$$

admit a solution with a random initial condition $Y:\Omega \longrightarrow H$ in a Hilbert space H?

Answer:

Objective

Question:

Does the following anticipating stochastic evolution equation (see):

$$\left.\begin{aligned} dv(t) &= -Av(t)\,dt + F_0\big(v(t)\big)\,dt \\ &\qquad\qquad + Bv(t)\circ dW(t),\ t>0, \\ v(0) &= Y \end{aligned}\right\} \qquad (1)$$

admit a solution with a random initial condition $Y:\Omega\longrightarrow H$ in a Hilbert space H?

Answer:

YES! (provided Y is sufficiently regular).

Strategy

- Replace Y in see (1) by a deterministic initial condition x in H and get the corresponding (equivalent) Itô see:

$$\left.\begin{aligned} du(t,x) &= -Au(t,x)\,dt + F\big(u(t,x)\big)\,dt \\ &\qquad + Bu(t,x)\,dW(t), \quad t>0 \\ u(0,x) &= x \in H \end{aligned}\right\} \tag{2}$$

with F a suitably modified non-linear drift.

Strategy

- Replace Y in see (1) by a deterministic initial condition x in H and get the corresponding (equivalent) Itô see:

$$\left.\begin{aligned} du(t,x) &= -Au(t,x)\,dt + F\big(u(t,x)\big)\,dt \\ &\qquad + Bu(t,x)\,dW(t), \quad t>0 \\ u(0,x) &= x \in H \end{aligned}\right\} \tag{2}$$

  with F a suitably modified non-linear drift.
- View the solution of the see (2) as a function (cocycle) $U(t,x,\omega)$ of three variables (t,x,ω) with Fréchet and Malliavin regularity in x and ω (resp.)

Strategy-Contd

- Consider the Stratonovich version of the Itô see (2):

$$\left.\begin{aligned} du(t,x) &= -Au(t,x)\,dt + F_0\big(u(t,x)\big)\,dt \\ &\qquad + Bu(t,x)\circ dW(t), \quad t>0 \\ u(0,x) &= x \in H \end{aligned}\right\} \tag{2'}$$

Strategy-Contd

- Consider the Stratonovich version of the Itô see (2):

$$\left.\begin{aligned} du(t,x) &= -Au(t,x)\,dt + F_0\big(u(t,x)\big)\,dt \\ &\qquad\qquad + Bu(t,x)\circ dW(t), \quad t>0 \\ u(0,x) &= x \in H \end{aligned}\right\} \qquad (2')$$

- *In the above semilinear see, is it justified to replace the deterministic initial condition x by an arbitrary random variable Y* (substitution theorem)?

Strategy-Contd

- Then get back the anticipating Stratonovich see (1) again:

$$\left.\begin{aligned} dU(t,Y) &= -AU(t,Y)\,dt + F_0\big(U(t,Y)\big)\,dt \\ &\qquad + BU(t,Y)\circ dW(t), \quad t>0 \\ U(0,Y) &= Y \end{aligned}\right\} \tag{1}$$

by taking $v(t) := U(t,Y),\ t \geq 0.$

Difficulties

- Affirmative answer for the above question is known for a wide class of finite-dimensional sde's via substitution theorems ([Nu.1-2], [M-S.2]).

Difficulties

- Affirmative answer for the above question is known for a wide class of finite-dimensional sde's via substitution theorems ([Nu.1-2], [M-S.2]).
- Known substitution theorems require a level of regularity of the cocycle $U(t, x, \omega)$ in t that is inconsistent with infinite-dimensionality of the stochastic dynamics (Cf. Theorem 3.2.6 [Nu.1], Theorem 5.3.4 [Nu.2]).

Difficulties

- Affirmative answer for the above question is known for a wide class of finite-dimensional sde's via substitution theorems ([Nu.1-2], [M-S.2]).
- Known substitution theorems require a level of regularity of the cocycle $U(t, x, \omega)$ in t that is inconsistent with infinite-dimensionality of the stochastic dynamics (Cf. Theorem 3.2.6 [Nu.1], Theorem 5.3.4 [Nu.2]).
- Existing substitution theorems work under restrictive finite-dimensional or (σ-)compactness constraints ([G-Nu-S], [A-I]).

Difficulties-Contd

- Failure of Kolmogorov's continuity theorem in infinite dimensions ([Mo.1], [Sk]).

Difficulties-Contd

- Failure of Kolmogorov's continuity theorem in infinite dimensions ([Mo.1], [Sk]).
- Failure of Sobolev inequalities in infinite dimensions.

Approach

- Construct Fréchet differentiable stochastic semiflow for the semilinear see (2) using a chaos-type expansion technique ([M-Z-Z]).

Approach

- Construct Fréchet differentiable stochastic semiflow for the semilinear see (2) using a chaos-type expansion technique ([M-Z-Z]).
- Develop global estimates on the semiflow generated by the spde.

Approach

- Construct Fréchet differentiable stochastic semiflow for the semilinear see (2) using a chaos-type expansion technique ([M-Z-Z]).
- Develop global estimates on the semiflow generated by the spde.
- Use ideas and techniques of the Malliavin calculus: Assume Malliavin regularity of the initial condition -rather than imposing finite-dimensional or compactness restrictions on the values of the initial random condition.

Approach

- Construct Fréchet differentiable stochastic semiflow for the semilinear see (2) using a chaos-type expansion technique ([M-Z-Z]).
- Develop global estimates on the semiflow generated by the spde.
- Use ideas and techniques of the Malliavin calculus: Assume Malliavin regularity of the initial condition -rather than imposing finite-dimensional or compactness restrictions on the values of the initial random condition.
- Use of Malliavin calculus techniques is necessary because the initial condition and the underlying stochastic dynamics are infinite-dimensional.

Motivation

Substitution theorem provides a dynamic characterization of stable/unstable manifolds for semilinear see's near hyperbolic/anticipating stationary states. (*Expect hyperbolicty to be a generic property rather than ergodicity of the invariant measure!*)

Motivation

Substitution theorem provides a dynamic characterization of stable/unstable manifolds for semilinear see's near hyperbolic/anticipating stationary states. (*Expect hyperbolicty to be a generic property rather than ergodicity of the invariant measure!*)

Techniques developed in this analysis yield similar substitution theorems for semiflows induced by sfde's.(–>)

Motivation

Substitution theorem provides a dynamic characterization of stable/unstable manifolds for semilinear see's near hyperbolic/anticipating stationary states. (*Expect hyperbolicty to be a generic property rather than ergodicity of the invariant measure!*)

Techniques developed in this analysis yield similar substitution theorems for semiflows induced by sfde's.(–>)

Global moment estimates on the cocycle and its derivatives are interesting in their own right.

Motivation

Substitution theorem provides a dynamic characterization of stable/unstable manifolds for semilinear see's near hyperbolic/anticipating stationary states. (*Expect hyperbolicty to be a generic property rather than ergodicity of the invariant measure!*)

Techniques developed in this analysis yield similar substitution theorems for semiflows induced by sfde's.(–>)

Global moment estimates on the cocycle and its derivatives are interesting in their own right.

Expect results in this talk to lead to regularity in distribution of the invariant manifolds for semilinear spde's and sfde's.

The Set-up

- $(\Omega, \mathcal{F}, P) :=$ Wiener space of all continuous paths $\omega : \mathbf{R} \to E, \omega(0) = 0$, where E is a real separable Hilbert space.

The Set-up

- $(\Omega, \mathcal{F}, P) :=$ Wiener space of all continuous paths $\omega : \mathbf{R} \to E$, $\omega(0) = 0$, where E is a real separable Hilbert space.
- Wiener shifts $\theta : \mathbf{R} \times \Omega \to \Omega$: Group of P-preserving ergodic transformations on $(\Omega, \mathcal{F}, P)$:

$$\theta(t, \omega)(s) := \omega(t+s) - \omega(t), \quad t, s \in \mathbf{R}, \ \omega \in \Omega.$$

The Set-up

- $(\Omega, \mathcal{F}, P) :=$ Wiener space of all continuous paths $\omega : \mathbf{R} \to E$, $\omega(0) = 0$, where E is a real separable Hilbert space.
- Wiener shifts $\theta : \mathbf{R} \times \Omega \to \Omega$: Group of P-preserving ergodic transformations on $(\Omega, \mathcal{F}, P)$:

$$\theta(t, \omega)(s) := \omega(t+s) - \omega(t), \quad t, s \in \mathbf{R},\ \omega \in \Omega.$$

- H := real (separable) Hilbert space, norm $|\cdot|_H$.

The Set-up

- $(\Omega, \mathcal{F}, P) :=$ Wiener space of all continuous paths $\omega : \mathbf{R} \to E, \omega(0) = 0$, where E is a real separable Hilbert space.
- Wiener shifts $\theta : \mathbf{R} \times \Omega \to \Omega$: Group of P-preserving ergodic transformations on $(\Omega, \mathcal{F}, P)$:

$$\theta(t, \omega)(s) := \omega(t+s) - \omega(t), \quad t, s \in \mathbf{R}, \omega \in \Omega.$$

- $H :=$ real (separable) Hilbert space, norm $|\cdot|_H$.
- $\mathcal{B}(H) :=$ Borel σ-algebra of H.

The Set-up

- $(\Omega, \mathcal{F}, P) :=$ Wiener space of all continuous paths $\omega : \mathbf{R} \to E, \omega(0) = 0$, where E is a real separable Hilbert space.
- Wiener shifts $\theta : \mathbf{R} \times \Omega \to \Omega$: Group of P-preserving ergodic transformations on $(\Omega, \mathcal{F}, P)$:

$$\theta(t, \omega)(s) := \omega(t + s) - \omega(t), \quad t, s \in \mathbf{R}, \omega \in \Omega.$$

- $H :=$ real (separable) Hilbert space, norm $|\cdot|_H$.
- $\mathcal{B}(H) :=$ Borel σ-algebra of H.
- $L(H) :=$ Banach space of all bounded linear operators $H \to H$ given the uniform operator norm $\|\cdot\|_{L(H)}$.

Set-up: Brownian Motion

- $W :=$ E-valued Brownian motion $W : \mathbf{R} \times \Omega \to E$ with separable covariance Hilbert space $K \subset E$, Hilbert-Schmidt embedding.

Set-up: Brownian Motion

- $W :=$ E-valued Brownian motion $W : \mathbf{R} \times \Omega \to E$ with separable covariance Hilbert space $K \subset E$, Hilbert-Schmidt embedding.
- $$W(t) = \sum_{k=1}^{\infty} W^k(t) f_k, \quad t \in \mathbf{R};$$
  $\{f_k : k \geq 1\} :=$ complete orthonormal basis of K; $W^k, k \geq 1$, standard independent one-dimensional Wiener processes ([D-Z.1], Chapter 4).

Set-up: Brownian Motion

- $W :=$ E-valued Brownian motion $W : \mathbf{R} \times \Omega \to E$ with separable covariance Hilbert space $K \subset E$, Hilbert-Schmidt embedding.
- $$W(t) = \sum_{k=1}^{\infty} W^k(t) f_k, \quad t \in \mathbf{R};$$
  $\{f_k : k \geq 1\} :=$ complete orthonormal basis of K; $W^k, k \geq 1$, standard independent one-dimensional Wiener processes ([D-Z.1], Chapter 4). Series converges absolutely in E but not necessarily in K.

Set-up: Brownian Motion

- $W :=$ E-valued Brownian motion $W : \mathbf{R} \times \Omega \to E$ with separable covariance Hilbert space $K \subset E$, Hilbert-Schmidt embedding.
- $$W(t) = \sum_{k=1}^{\infty} W^k(t) f_k, \quad t \in \mathbf{R};$$
  $\{f_k : k \geq 1\} :=$ complete orthonormal basis of K; $W^k, k \geq 1$, standard independent one-dimensional Wiener processes ([D-Z.1], Chapter 4). Series converges absolutely in E but not necessarily in K.
- (W, θ) is a *helix*:
  $$W(t_1 + t_2, \omega) - W(t_1, \omega) = W(t_2, \theta(t_1, \omega))$$

Set-up-contd

- $L_2(K, H) :=$ Hilbert space of all Hilbert-Schmidt operators $S : K \to H$, with norm

$$\|S\|_2 := \left[\sum_{k=1}^{\infty} |S(f_k)|_H^2\right]^{1/2}$$

Set-up-contd

- $L_2(K, H) :=$ Hilbert space of all Hilbert-Schmidt operators $S : K \to H$, with norm

$$\|S\|_2 := \left[\sum_{k=1}^{\infty} |S(f_k)|_H^2 \right]^{1/2}$$

- $F_0 : H \to H$ is C_b^1.

Set-up-contd

- $L_2(K,H) :=$ Hilbert space of all Hilbert-Schmidt operators $S : K \to H$, with norm

$$\|S\|_2 := \left[\sum_{k=1}^{\infty} |S(f_k)|_H^2\right]^{1/2}$$

- $F_0 : H \to H$ is C_b^1.
- $F := F_0 + \frac{1}{2}\sum_{k=1}^{\infty} B_k^2$, where $B_k \in L(H)$ are given by

$$B_k(x) := B(x)(f_k),\ x \in H,\ k \geq 1;\ \text{and} \sum_{k=1}^{\infty} \|B_k\|^2$$

converges.

Set-up: The Semilinear SEE

Consider the semilinear Itô stochastic evolution equation (see):

$$\left.\begin{aligned} du(t,x) &= -Au(t,x)\,dt + F\big(u(t,x)\big)\,dt \\ &\qquad + Bu(t,x)\,dW(t), \quad t>0 \\ u(0,x) &= x \in H \end{aligned}\right\} \tag{2}$$

in H.

Set-up: The Semilinear SEE

Consider the semilinear Itô stochastic evolution equation (see):

$$\left.\begin{aligned} du(t,x) &= -Au(t,x)\,dt + F\big(u(t,x)\big)\,dt \\ &\qquad\qquad + Bu(t,x)\,dW(t), \quad t>0 \\ u(0,x) &= x \in H \end{aligned}\right\} \qquad (2)$$

in H.

$A : D(A) \subset H \to H$ is a closed linear operator on H.

Set-up: The Semilinear SEE

Consider the semilinear Itô stochastic evolution equation (see):

$$\left.\begin{aligned} du(t,x) &= -Au(t,x)\,dt + F\big(u(t,x)\big)\,dt \\ &\qquad + Bu(t,x)\,dW(t), \quad t>0 \\ u(0,x) &= x \in H \end{aligned}\right\} \qquad (2)$$

in H.

$A : D(A) \subset H \to H$ is a closed linear operator on H.

Assume A has a complete orthonormal system of eigenvectors $\{e_n : n \geq 1\}$ with corresponding positive eigenvalues $\{\mu_n, n \geq 1\}$; i.e., $Ae_n = \mu_n e_n,\ n \geq 1$.

The Set-up-contd

Suppose $-A$ generates a strongly continuous semigroup of bounded linear operators $T_t : H \to H$, $t \geq 0$.

The Set-up-contd

Suppose $-A$ generates a strongly continuous semigroup of bounded linear operators $T_t : H \to H$, $t \geq 0$.

$F : H \to H$ is (Fréchet) C_b^1: F has a globally bounded Fréchet derivative $F : H \to L(H)$.

The Set-up-contd

Suppose $-A$ generates a strongly continuous semigroup of bounded linear operators $T_t : H \to H$, $t \geq 0$.

$F : H \to H$ is (Fréchet) C_b^1: F has a globally bounded Fréchet derivative $F : H \to L(H)$.

Suppose $B : H \to L_2(K, H)$ is a bounded linear operator. The stochastic integral in the see (2) is defined in the sense of ([D-Z.1], Chapter 4):

Standing Hypotheses

- *Hypothesis* (A_1): $\sum_{n=1}^{\infty} \mu_n^{-1} \|B(e_n)\|_{L_2(K,H)}^2 < \infty.$

Standing Hypotheses

- *Hypothesis (A$_1$)*: $\displaystyle\sum_{n=1}^{\infty} \mu_n^{-1} \|B(e_n)\|^2_{L_2(K,H)} < \infty.$

- *Hypothesis (B)*: $B : H \to L_2(K,H)$ extends to a bounded linear operator $B \in L(H, L(E,H))$; $\displaystyle\sum_{k=1}^{\infty} \|B_k\|^2 < \infty$, where $B_k \in L(H)$ is defined by

$$B_k(x) := B(x)(f_k), x \in H, k \geq 1.$$

Remarks

- Hypothesis (A_1) is implied by the following two requirements:

Remarks

- Hypothesis (A_1) is implied by the following two requirements:
  - (a) The operator $B : H \longrightarrow L_2(K, H)$ is Hilbert-Schmidt.

Remarks

- Hypothesis (A_1) is implied by the following two requirements:
  - (a) The operator $B : H \longrightarrow L_2(K, H)$ is Hilbert-Schmidt.
  - (b) $\liminf_{n \to \infty} \mu_n > 0$.

Remarks

- Hypothesis (A_1) is implied by the following two requirements:
  - (a) The operator $B : H \longrightarrow L_2(K, H)$ is Hilbert-Schmidt.
  - (b) $\liminf_{n\to\infty} \mu_n > 0$.
- Requirement (b) above is satisfied if $A = -\Delta$, where Δ is the Laplacian on a compact smooth d-dimensional Riemannian manifold M with boundary, under Dirichlet boundary conditions.

Remarks

- Hypothesis (A_1) is implied by the following two requirements:
  - (a) The operator $B : H \to L_2(K, H)$ is Hilbert-Schmidt.
  - (b) $\liminf_{n\to\infty} \mu_n > 0$.
- Requirement (b) above is satisfied if $A = -\Delta$, where Δ is the Laplacian on a compact smooth d-dimensional Riemannian manifold M with boundary, under Dirichlet boundary conditions.
- No restriction on $\dim M$ under (A_1) for spdes.

Mild Solutions

A mild solution of the semilinear see (2) is a family of $(\mathcal{B}(\mathbf{R}^+)\otimes\mathcal{F},\mathcal{B}(H))$-measurable, $(\mathcal{F}_t)_{t\geq 0}$-adapted processes $u(\cdot,x,\cdot):\mathbf{R}^+\times\Omega\to H,\ x\in H$, satisfying the following stochastic integral equation:

$$u(t,x,\cdot)=T_tx+\int_0^t T_{t-s}F(u(s,x,\cdot))\,ds + \int_0^t T_{t-s}Bu(s,x,\cdot)\,dW(s), \quad t\geq 0, \tag{2'}$$

([D-Z.1-2]).

Stratonovich Form

The Itô see (2) has the equivalent Stratonovich form

$$\left.\begin{aligned} du(t,x) &= -Au(t,x)\,dt + F\big(u(t,x)\big)\,dt \\ &\qquad - \frac{1}{2}\sum_{k=1}^{\infty} B_k^2 u(t,x)\,dt + Bu(t,x)\circ dW(t) \\ u(0,x) &= x \in H \end{aligned}\right\} \tag{3}$$

where $B_k \in L(H)$ are given by $B_k(x) := B(x)(f_k)$, $x \in H$, $k \geq 1$.

The Cocycle

Theorem 1:

Under Hypotheses (B) and (A_1), the see (2) (or (3)) admits a perfect jointly measurable C^1 cocycle (U, θ), $U : \mathbf{R}^+ \times H \times \Omega \to H$:

$$U(t_1 + t_2, \cdot, \omega) = U(t_2, \cdot, \theta(t_1, \omega)) \circ U(t_1, \cdot, \omega)$$

for all $t_1, t_2 \in \mathbf{R}^+$, all $\omega \in \Omega$.

The Cocycle

Theorem 1:

Under Hypotheses (B) and (A_1), the see (2) (or (3)) admits a perfect jointly measurable C^1 cocycle (U, θ), $U : \mathbf{R}^+ \times H \times \Omega \to H$:

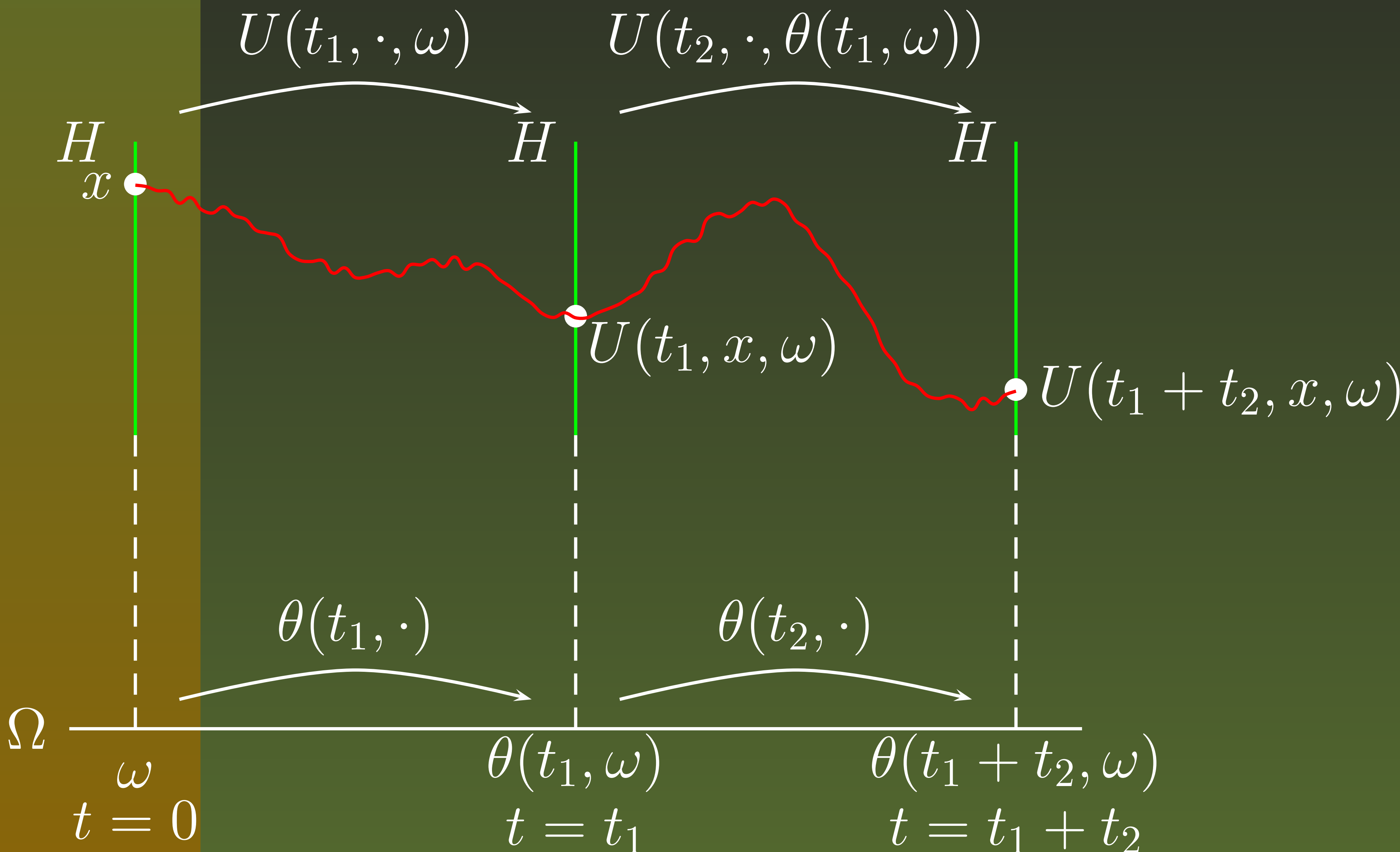
$$U(t_1 + t_2, \cdot, \omega) = U(t_2, \cdot, \theta(t_1, \omega)) \circ U(t_1, \cdot, \omega)$$

for all $t_1, t_2 \in \mathbf{R}^+$, all $\omega \in \Omega$.

Proof of Theorem 1:

([M-Z-Z], Theorem 1.2.6); cf. [F.1-2]. □

The Cocycle Property



Malliavin Regularity

For any integer $p \geq 2$, denote by $\mathbb{D}^{1,p}(\Omega, H)$ the Sobolev space of all $\mathcal{F}$-measurable random variables $Y : \Omega \to H$ which are p-integrable together with their Malliavin derivatives $\mathcal{D}Y$ ([Nu.1-2]).

Malliavin Regularity

For any integer $p \geq 2$, denote by $\mathbb{D}^{1,p}(\Omega, H)$ the Sobolev space of all $\mathcal{F}$-measurable random variables $Y : \Omega \to H$ which are p-integrable together with their Malliavin derivatives $\mathcal{D}Y$ ([Nu.1-2]).

We now state the main substitution theorem in this talk.

Substitution

Theorem 2: (The Substitution Theorem)

Assume Hypotheses (B) and (A_1). Let $U : \mathbf{R}^+ \times H \times \Omega \to H$ be the C^1 cocycle generated by the see (2). Let $Y \in \mathbb{D}^{1,4}(\Omega, H)$ be a random variable. Then $v(t) := U(t, Y)$, $t \geq 0$, is a mild solution of the (anticipating) Stratonovich see

Substitution

Theorem 2: (The Substitution Theorem)

Assume Hypotheses (B) and (A_1). Let $U : \mathbf{R}^+ \times H \times \Omega \to H$ be the C^1 cocycle generated by the see (2). Let $Y \in \mathbb{D}^{1,4}(\Omega, H)$ be a random variable. Then $v(t) := U(t, Y)$, $t \geq 0$, is a mild solution of the (anticipating) Stratonovich see

$$\left.\begin{aligned} dv(t) &= -Av(t)\,dt + F_0\big(v(t)\big)\,dt \\ &\qquad\qquad + Bv(t) \circ dW(t),\ t > 0, \\ v(0) &= Y \end{aligned}\right\} \qquad (1)$$

where $F_0 = F - \frac{1}{2}\sum_{k=1}^{\infty} B_k^2$.

Substitution Theorem-contd

In particular, if $Y \in \mathbb{D}^{1,4}(\Omega, H)$ *is a stationary point of the see (2) (or (3)), then* $U(t, Y) = Y\big(\theta(t)\big),\ t \geq 0$*, is a stationary solution of the (anticipating) Stratonovich see (1):*

Substitution Theorem-contd

In particular, if $Y \in \mathbb{D}^{1,4}(\Omega, H)$ *is a stationary point of the see (2) (or (3)), then* $U(t,Y) = Y\big(\theta(t)\big),\ t \geq 0$, *is a stationary solution of the (anticipating) Stratonovich see (1):*

$$\left.\begin{aligned} dY(\theta(t)) &= -AY(\theta(t))\,dt + F_0\big(Y(\theta(t))\big)\,dt \\ &\qquad + BY(\theta(t)) \circ dW(t), t > 0, \\ Y(\theta(0)) &= Y. \end{aligned}\right\} \quad (4)$$

Substitution Theorem-contd

Furthermore, assume that F_0 *is* C_b^2*. Then the linearized cocycle* $DU(t, Y)$ *is a mild solution of the linearized anticipating see*

Substitution Theorem-contd

Furthermore, assume that F_0 is C_b^2. Then the linearized cocycle $DU(t,Y)$ is a mild solution of the linearized anticipating see

$$\left.\begin{aligned} dDU(t,Y) &= -ADU(t,Y)\,dt \\ &\quad + DF_0\big(U(t,Y)\big)DU(t,Y)\,dt \\ &\quad + \big\{B \circ DU(t,Y)\big\} \circ dW(t),\ t>0, \\ DU(0,Y) &= \mathrm{id}_{L(H)}. \end{aligned}\right\} \tag{5}$$

Outline of Proof

- Construct a linear cocycle (Φ, θ) for the linear Itô see (with $F \equiv 0$):

Outline of Proof

- Construct a linear cocycle (Φ, θ) for the linear Itô see (with $F \equiv 0$):
  - Lift linear see to the Hilbert space $L_2(H)$.

Outline of Proof

- Construct a linear cocycle (Φ, θ) for the linear Itô see (with $F \equiv 0$):
  - Lift linear see to the Hilbert space $L_2(H)$.
  - Use chaos-type expansion in $L_2(H)$

Outline of Proof

- Construct a linear cocycle (Φ, θ) for the linear Itô see (with $F \equiv 0$):
  - Lift linear see to the Hilbert space $L_2(H)$.
  - Use chaos-type expansion in $L_2(H)$
  - Prove convergence of the expansion in $L^{2p}(\Omega, L_2(H))$ via repeated application of moment estimates of the Itô integral

Outline of Proof

- Construct a linear cocycle (Φ, θ) for the linear Itô see (with $F \equiv 0$):
  - Lift linear see to the Hilbert space $L_2(H)$.
  - Use chaos-type expansion in $L_2(H)$
  - Prove convergence of the expansion in $L^{2p}(\Omega, L_2(H))$ via repeated application of moment estimates of the Itô integral
- Use the linear cocycle to get a pathwise variational integral equation equivalent to the semilinear see.(–>)

Outline of Proof

- Construct a linear cocycle (Φ, θ) for the linear Itô see (with $F \equiv 0$):
  - Lift linear see to the Hilbert space $L_2(H)$.
  - Use chaos-type expansion in $L_2(H)$
  - Prove convergence of the expansion in $L^{2p}(\Omega, L_2(H))$ via repeated application of moment estimates of the Itô integral
- Use the linear cocycle to get a pathwise variational integral equation equivalent to the semilinear see.(–>)
- Derive moment estimates for the nonlinear cocycle, its Fréchet and Malliavin derivatives. (–>)

Outline of Proof-Contd

- Prove the substitution theorem when Y is replaced by its finite-dimensional projections Y_n: Use finite-dimensional projections to smooth out the semigroup T_t in t, and apply finite-dimensional substitution techniques.

Outline of Proof-Contd

- Prove the substitution theorem when Y is replaced by its finite-dimensional projections Y_n: Use finite-dimensional projections to smooth out the semigroup T_t in t, and apply finite-dimensional substitution techniques.
- Use moment estimates on the cocycle to rewrite each finite-dimensional anticipating Stratonovich integral in terms of a Skorohod integral plus a Lebesgue integral correction term involving Malliavin derivatives of the cocycle.

Outline of Proof-Contd

- Prove the substitution theorem when Y is replaced by its finite-dimensional projections Y_n: Use finite-dimensional projections to smooth out the semigroup T_t in t, and apply finite-dimensional substitution techniques.
- Use moment estimates on the cocycle to rewrite each finite-dimensional anticipating Stratonovich integral in terms of a Skorohod integral plus a Lebesgue integral correction term involving Malliavin derivatives of the cocycle.
- Take n to ∞ via the moment estimates on the cocycle, its Fréchet and Malliavin derivatives and dominated convergence. $\square$

REFERENCES

A-I Arnold, L., and Imkeller, P., Stratonovich calculus with spatial parameters and anticipative problems in multiplicative ergodic theory, *Stochastic Processes and their Applications,* Vol. 62 (1996), 19–54.(<–)

D-Z.1 Da Prato, G., and Zabczyk, J., *Stochastic Equations in Infinite Dimensions*, Cambridge University Press (1992).

D-Z.2 Da Prato, G., and Zabczyk, J., *Ergodicity for Infinite Dimensional Systems*, Cambridge University Press (1996).

REFERENCES-contd

G-Nu-S Grorud, A., Nualart, D., and Sanz-Solé, M., Hilbert-valued anticipating stochastic differential equations, *Annales de l'institut Henri Poincaré (B) Probabilités et Statistiques*, 30 no. 1 (1994), 133-161. (<–)

Ma Malliavin, P., Stochastic calculus of variations and hypoelliptic operators, *Proceedings of the International Conference on Stochastic Differential Equations, Kyoto*, Kinokuniya, 1976, 195-263.

Mo.1 Mohammed, S.-E.A., *Stochastic Functional Differential Equations*, Research Notes in Mathematics, no. 99, Pitman Advanced Publishing Program, Boston-London-Melbourne (1984).(<–)

REFERENCES-contd

Mo.2 Mohammed, S.-E. A., Non-Linear Flows for Linear Stochastic Delay Equations, *Stochastics,* Vol. l7 #3, (1987), 207–212.

M-S.1 Mohammed, S.-E. A., and Scheutzow, M. K. R., The Stable Manifold Theorem for Nonlinear Stochastic Systems with Memory, Part I: Existence of the Semiflow, *Journal of Functional Analysis*, 205, (2003), 271-305. Part II: The Local Stable Manifold Theorem, *Journal of Functional Analysis*, 206, (2004), 253-306.

REFERENCES-contd

M-S.2 Mohammed, S.-E. A., and Scheutzow, M. K. R., The stable manifold theorem for stochastic differential equations, *The Annals of Probability,* Vol. 27, No. 2, (1999), 615-652. (<–)

M-Z-Z Mohammed, S.-E. A., Zhang, T. S. and Zhao, H. Z., The stable manifold theorem for semilinear stochastic evolution equations and stochastic partial differential equations, Part 1: The Stochastic semifbw, Part 2: Existence of stable and unstable manifolds, pp. 98 (2006), *Memoirs of the American Mathematical Society* (to appear).(<–)

REFERENCES-contd

M-Z Mohammed, S.-E. A. and Zhang, T. S., The substitution theorem for semilinear stochastic partial differential equations, *Journal of Functional Analysis* (to appear) (preprint, 2007) (<–)

Nu.1 Nualart, D., *The Malliavin Calculus and Related Topics*, Probability and its Applications, Springer-Verlag (1995).

Nu.2 Nualart, D., *Analysis on Wiener space and anticipating stochastic calculus*, Springer LNM, 1690, Ecole d'Et'e de Probabilit'es de Saint-Flour XXV-1995, ed: P. Bernard (1995).(<–)

REFERENCES-contd

N-P Nualart, D., and Pardoux, E., Stochastic calculus with anticipating integrands, Analysis on Wiener space and anticipating stochastic calculus, *Probab. Th. Rel. Fields* , 78 (1988), 535-581.

Sk Skorohod, A. V., *Random Linear Operators*, Riedel 1984.(<–)