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Abstract

We propose tests for normality of the error components in panel data models, using estimates of skewness and kurtosis in the components.

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1. Introduction

The one-way error components model with individual effects is:

$$y_{it} = x_{it}'\beta + u_{it}, \quad i = 1, ..., n, \quad t = 1, ..., T, \tag{1}$$

where y_{it} is the dependent variable for individual i at time t, and x_{it} is a vector of observables for which we assume that $x_i, i = 1, ..., n$, are independent and identically distributed (IID) with finite eighth moments and non-singular variance-covariance matrix. The variable $u_{it} = \mu_i + \nu_{it}$ is the model's error, composed of two components. The unobserved individual effects $\mu_1, ..., \mu_n$ are the permanent component, random and IID with zero population mean and finite eighth moments. The unobserved random variables ν_{it} are the transitory component, zero-mean and IID across i and t, with finite eighth moments. Also, μ_i is independent of ν_i for all i, and both μ_i and ν_{it} are independent of x_{it} for all i and T.

We are interested in the distribution of the error components. The normal distribution is usually invoked to obtain the exact null distribution of hypothesis tests for the parameters β, hence departures from normality are clearly an important issue for economic inference when samples are sufficiently small. Normality is also invoked to obtain the exact distribution of tests for individual effects, and Blanchard and Mátyás (1996) examine the consequences of non-normal error components for the performance of such tests.

As noted by Baltagi (1998), in economic panel data modelling it can be important to adjust for departures of error components from normality. For example, Horowitz and Markatou (1996) study the dynamics of worker earnings, and for this purpose they need to estimate the joint distribution of the T-vector y_i, conditional on the T-vector x_i. In this case one needs estimates of the error component distributions.

The present work proposes tests for normality of the error component distributions. We obtain formulas for the skewness and kurtosis of the error components, as functions of the moments and cross-moments of the errors. We use these formulas to estimate skewness and kurtosis in the error components, and to test for non-normality. The tests are straightforward to implement and have convenient chi square asymptotic distributions.

2. Moments of Error Components

Denoting by F_μ and F_ν the (cumulative) distribution functions for the two unobserved error components, we compute higher order moments of these distributions, up to fourth order. Since, by assumption, the first moments of μ and ν are both zero, e.g. $\int z\,dF_\mu(z) = \int z\,dF_\nu(z) = 0$, it follows that, for each such distribution F, the quantity $\phi_k \equiv \int z^k dF(z)$ is, for $k \geq 2$, the centered k-th moment. For positive integers j and k we let

$$\psi_j \equiv E\,u_{it}^j\,, \quad \psi_{j,k} \equiv E\,u_{is}^j u_{it}^k\,, \tag{2}$$

for any given i and distinct s and t. Due to the assumed structure of the error components, ψ_j and $\psi_{j,k}$ are invariant to the choice of i, s and t. The second moments of the error components, which are well known functions of the error distribution (see Hsiao 1986, Baltagi 1995), can be expressed as: $\phi_{\mu,2} = \psi_{1,1}$ and $\phi_{\nu,2} = \psi_2 - \psi_{1,1}$.

For the third moments of the error components, we have first:

$$\psi_{\mu,3} = \psi_{1,2}, \tag{3}$$

which is true since $\psi_{1,2} = Eu_{is}u_{it}^2 = E(\mu_i + \nu_{is})(\mu_i + \nu_{it})^2$, and applying the zero-mean and independence properties of the error components, $\psi_{1,2} = E\mu_i^3 = \psi_{\mu,3}$. For the remaining error component, we have:

$$\psi_{\nu,3} = \psi_3 - \psi_{1,2}. \tag{4}$$

To verify this, we have $\psi_3 \equiv Eu_{it}^3 = E(\mu_i + \nu_{it})^3$, and applying the zero-mean and independence properties of error components, we deduce that $Eu_{it}^3 = \phi_{\mu,3} + \phi_{\nu,3}$, and combining this fact with (3) yields (4).

To further interpret skewness in the error components, we note first from (3) that for each individual i, non-zero skewness $\phi_{\mu,3}$ in the permanent component induces correlation between the error u_{is} at time s and the squared errors u_{it}^2 at remaining times. A measure of this nonlinear dependence is the correlation $\frac{\psi_{1,2}}{\sqrt{\psi_2(\psi_4-\psi_2^2)}}$ between u_{is} and u_{it}^2. This correlation is obviously determined by $\psi_{1,2}, \psi_2$ and ψ_4, and we have $\psi_{1,2} = \phi_{\mu,3}, \psi_2 = \phi_{\mu,2} + \phi_{\nu,2}$, and $\psi_4 = \phi_{\mu,4} + \phi_{\nu,4} + 6\phi_{\nu,2}\phi_{\mu,2}$. Hence, while the skewness $\phi_{\mu,3}$ of the permanent component contributes to nonlinear dependence via $\psi_{1,2}$, the skewness $\phi_{\nu,3}$ of the transitory component has no effect on this dependence. We also note that, since third moments can be negative, and since the sum of error component third moments equals the error third moment, it is possible to have skewness in the error components but no skewness in the error itself.

For the fourth moments, we have first:

$$\psi_{\mu,4} = \psi_{1,3} - 3\,\psi_{1,1}(\psi_2 - \psi_{1,1}). \tag{5}$$

This holds since $\psi_{1,3} = E(u_{is}u_{it}^3)$, and by expanding terms and applying zero-mean and independence assumptions, we obtain $\psi_{1,3} = E(\mu_i^4) + 3E(\mu_i^2)E(\nu_{it}^2)$, and hence conclude (5). For the second error component:

$$\phi_{\nu,4} = \psi_4 - \psi_{1,3} - 3\,\psi_{1,1}(\psi_2 - \psi_{1,1}), \tag{6}$$

which is true since, first, $\psi_4 = E\mu_i^4 + E\nu_{it}^4 + 6\,E\mu_i^2\,E\nu_{it}^2$, and combining this with (5) yields (6).

To further interpret the fourth moments and kurtosis in the error components, we recall that, for normally distributed variables z, $\phi_{z,4} = 3\,\phi_{z,2}^2$ and so the kurtosis $\frac{\phi_{z,4}}{(\phi_{z,2})^2}$ equals 3. Non-normal levels of kurtosis are those less than three (platykurtotic) and greater than three (leptokurtotic), and we deduce from (5) that the permanent component μ has normal kurtosis if and only if

$$\psi_{1,3} - 3\,\psi_2\psi_{1,1} = 0, \tag{7}$$

while from (6) we deduce that the transitory component ν has normal kurtosis if and only if

$$\psi_4 - \psi_{1,3} - 3\,\psi_2(\psi_2 - \psi_{1,1}) = 0. \tag{8}$$

According to (7), if μ has normal kurtosis then the covariance $\psi_{1,3}$ between u_{is} and u_{it}^3 is positive and given by the product of variances for the error and its permanent component. According to (8), with normal kurtosis in ν there may or may not be positive covariance between u_{is} and u_{it}^3, and the sign of the covariance is determined by the relative magnitudes of the quantities ψ_2, ψ_4 and $\psi_{1,1}$.

3. Normality Tests

To test for the normality of error components, we first obtain estimates of the moments ψ_j and cross-moments ψ_{jk} for the relevant j and k. With $\hat{\beta}$ any weakly consistent estimator of β (such as those described in Hsiao 1986 and Baltagi 1995), the residuals are $\hat{u}_{it} = y_{it} - x_{it}'\hat{\beta}$, and we obtain

$$\hat{\psi}_j = \frac{1}{nT}\sum_{i=1}^{n}\sum_{t=1}^{T}\hat{u}_{it}^j, \tag{9}$$

and

$$\hat{\psi}_{j,k} = \frac{1}{nT(T-1)}\sum_{i=1}^{n}\sum_{s\neq t}\hat{u}_{is}^j\hat{u}_{it}^k, \tag{10}$$

for each j and k. We require estimates of the variances and covariances for the statistics $\hat{\psi}_j$ and $\hat{\psi}_{ij}$, and in the interest of simplicity we propose the following form:

$$C_{a;b} = n^{-1} \sum_{i=1}^{n} (a_i - \bar{a})(b_i - \bar{b}), \tag{11}$$

where a_i and b_i are quantities of type $T^{-1} \sum_{t=1}^{T} \hat{u}_{it}^{j}$ or $(T(T-1))^{-1} \sum_{s \neq t} \hat{u}_{is}^{j} \hat{u}_{it}^{k}$, and $\bar{a}$ and $\bar{b}$ are the sample averages of a_i and b_i. As a and b are each identified by a specific j or (j,k), we use the notation $C_{j_a;j_b}$, $C_{j_a;j_a,j_b}$, etc., for $C_{a;b}$.

To test for skewness in the error components we introduce the ratios:

$$z_{\mu,1} = \frac{\hat{\psi}_{1,2}}{\sqrt{C_{1,2;1,2}}}, \qquad z_{\nu,1} = \frac{\hat{\psi}_3 - \hat{\psi}_{1,2}}{\sqrt{C_{3;3} + C_{1,2;1,2} - 2\, C_{3;1,2}}}, \tag{12}$$

where the numerator of the ratio $z_{\mu,1}$ estimates the skewness (3) of the error's permanent component, and the denominator estimates (consistently) the standard deviation of the statistic defined by the numerator. The numerator of $z_{\nu,1}$ estimates the skewness (4) of the transitory component, and the denominator, once again, estimates (consistently) the standard deviation of the statistic defined by the numerator.

To test for non-normal kurtosis, we introduce the ratios:

$$z_3 = \frac{\hat{\psi}_{1,3} - 3\, \hat{\psi}_2 \hat{\psi}_{1,1}}{\sqrt{C_{1,3;1,3} + 9\, (\hat{\psi}_2^2 C_{1,1;1,1} \;+\; \hat{\psi}_{1,1}^2 C_{2;2} + 2\, \hat{\psi}_2 \hat{\psi}_{1,1} C_{2;1,1}) - 6\, (\hat{\psi}_2 C_{1,3;1,1} + \hat{\psi}_{1,1} C_{1,3;2})}}, \tag{13}$$

$$z_4 = \frac{\hat{\psi}_4 - \hat{\psi}_{1,3} - 3\hat{\psi}_2(\hat{\psi}_2 - \hat{\psi}_{1,1})}{\sqrt{h}}, \tag{14}$$

where

$$\begin{gathered} h = C_{4;4} + C_{1,3;1,3} - 2\, C_{4;1,3} + \\ 9\, \Big((\hat{\psi}_2 - \hat{\psi}_{1,1})^2 C_{2;2} + \hat{\psi}_2^2 (C_{2;2} + C_{1,1;1,1} - 2\, C_{2;1,1}\,) + 2\, \hat{\psi}_2 (\hat{\psi}_2 - \hat{\psi}_{1,1})(C_{2;2} - C_{2;11}) \Big) - \\ 6\, \Big(\hat{\psi}_2 (\, 2\, (C_{2;4} - C_{2;1,3}) + C_{1,3;1,1} - C_{4;1,1}) + \hat{\psi}_{1,1} (C_{2;1,3} - C_{2;4}) \Big)\,. \end{gathered}$$

In (5) and (6), the numerator of the ratio $z_{\mu,2}$ estimates a quantity which equals 0 if and only if the error's permanent component has normal kurtosis, while the numerator of $z_{\nu,2}$ estimates a quantity which equals 0 if and only if the transitory component has normal kurtosis. The denominators of these ratios estimate (consistently) the standard deviation of the statistics defined by the numerators.

We have four null hypotheses of interest, denoted $H_{\mu,1}$, $H_{\mu,2}$, $H_{\nu,1}$, $H_{\nu,2}$, the first two signifying zero skewness and normal kurtosis, respectively, in μ, and the second two signifying zero skewness and normal kurtosis in ν. Under $H_{\mu,1}$, the statistic $z_{\mu,1}$ is unit normal asymptotically, and under $H_{\mu,2}$ the statistic $z_{\mu,2}$ is asymptotically unit normal, with analogous results for $z_{\nu,1}$ and $z_{\nu,2}$. Facilitating these facts are the consistency of the estimator $\hat{\beta}$ and the consistency of the variance estimators appearing in the denominators of ratios $z_{\mu,1}, z_{\mu,2}, z_{\nu,1}, z_{\nu,2}$. We obtain the consistent variance estimators using standard asymptotic approximations for smooth functions of statistics converging to constants (as for example in van der Vaart 1998, p. 33).

To tests each of the four null hypotheses, we propose to use the relevant z^2 statistic, rejecting if the statistic exceeds the chi square critical value. Asymptotically, the statistics $z_{\mu,1}$ and $z_{\mu,2}$ are independent under joint hypothesis $H_{\mu,1} \cap H_{\mu,2}$, while $z_{\nu,1}$ and $z_{\nu,2}$ are independent under the joint hypothesis $H_{\nu,1} \cap H_{\nu,2}$ (verified using standard asymptotic approximations to smooth functions of normal statistics). To test the first joint hypothesis we use $z^2_{\mu,1} + z^2_{\mu_2}$, asymptotically chi square (with 2 degrees of freedom), and we proceed analogously with the second joint null hypothesis. Interestingly, the statistics $z_{\mu,1}$ and $z_{\nu,1}$ are *not* asymptotically independent under $H_{\mu,1} \cap H_{\nu,1}$.

it is not possible to test both the permanent and transitory error components for

We have performed simulations (omitted for brevity) to corroborate the asymptotic distributions of the tests. For all tests the theory shows good accuracy even in fairly small samples ($n \geq 50$), when testing at the 1 percent level. Some distortion is noticeable when testing at the 5 percent level in samples as large as $n = 500$.

4. Conclusion

The present work provides estimators and tests of skewness and kurtosis of the error components. A legitimate question is whether the aim of such methods points in a direction of more general economic interest than the technical aspects of error component models. We would say 'yes', to the extent that economists want to further the prospect of fully specified probability models of panel data.

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