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Alison Watts

Southern Illinois University Carbondale

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Non-Myopic Formation of Circle Networks

Alison Watts*
Department of Economics
Southern Illinois University
Carbondale, IL 62901
e-mail: wattsa@siu.edu

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Abstract: We examine the dynamic formation of networks by self-interested individuals who can form and sever links. We assume that agents are initially unconnected, that the cost of forming a first link exceeds its benefits, and that indirect links are valuable. We show that if agents are non-myopic then it is possible for a network shaped like a circle to form.

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1. Introduction

There have been several recent papers analyzing which network structures will arise in dynamic models of network formation where self-interested individuals can form and sever links; see Watts [2001], Jackson and Watts [1998], and Bala and Goyal [2000]. These papers all assume that agents are myopic, which is a common assumption in situations where agents have limited information about the payoffs and incentives of others. Unfortunately, myopic agents can easily end up stuck in inefficient network structures. Consider a situation where agents are initially unconnected and where the cost of forming a first link exceeds its benefits but thereafter additional links are valuable. Myopic agents will become stuck in the empty network even though they would all be better off in a connected network because no one wants to form the first link. One solution to this problem is to allow random mutations where at some time a link may randomly form to seed the network formation process; see Jackson and Watts [1998] for details. An alternative solution is to allow agents to be forward looking; this solution makes sense in contexts where individuals who care about the future are well-informed about the payoffs and incentives of others. Such agents may add the first link in anticipation of where the network formation process will lead. This forward-looking approach is the focus of the current paper.

Jackson and Wolinsky [1996] have developed a number of models for the static study of network stability and efficiency. (Dutta and Mutuswami [1997] have looked at this relationship in further detail.) We consider a dynamic version of Jackson and Wolinsky's [1996] connections model and show that if the cost of forming a first link exceeds its benefits and if agents are non-myopic then it is possible for a network shaped like a circle to form. After the circle has formed all agents receive a positive payoff and thus everyone is better off than in the myopic case where no links form. As in Aumann and Myerson [1988] we assume that agents meet in a specific order

called the order of play. The circle network will only form if the order of play allows the circle to form quickly and if the discounted future payoff from being a member of the circle network is large enough. As the number of players increases, it becomes more likely that the discounted future payoff is large and thus it becomes more likely that there exists a meeting order (order of play) for which the formation of a circle network is supported as a subgame perfect equilibrium.

The paper proceeds as follows. In Section 2 we provide the definitions comprising the basic model. In Section 3 we present the non-myopic network formation results.

2. Model

Networks

There are n agents, $\{1, 2, \dots, n\}$, who can form links or ties with each other. A pattern of links between agents is represented as a network of graph. We represent a direct connection between agents i and j in graph g as $ij \in g$. Payoffs are represented as in Jackson and Wolinsky's [1996] connections model. Agent i receives payoff $u_i(g)$ from being a member of network g , where this payoff equals

$$u_i(g) = \sum_{j \neq i} \delta^{t(ij)} - \sum_{j: ij \in g} c \quad (1)$$

Here $c > 0$ represents the cost of maintaining a direct connection, $t(ij)$ represents the number of direct links in the shortest path between agents i and j , and $\delta^{(ij)}$ represents the payoff i gets from being connected to agent j by $t(ij)$ links. If there is no path between i and j then we let $\delta^{(ij)} = 0$. We assume that $0 < \delta < 1$, so that agents value closer connections more than distant connections.

Network Dynamics

The players are initially unconnected; they meet over time and have the opportunity to form and sever links with each other. In each period, two players i and j meet; we represent this as $i:j$. If players i and j are unlinked, then they can form a direct link with each other if both players agree. At the same time any player can sever any of his existing ties with the restriction that no player can simultaneously form and sever a link (i.e., player i can either form a link with player j or sever any of his existing links but he cannot do both). As in Aumann and Myerson [1988], the players meet in a specific order called the order of play. After every pair of players has met, then the order of play starts over.

We assume that agents are forward seeking and that each agent discounts the future at rate $0 < d < 1$. If agent i knows that in periods $(t, t+1, t+2, \dots)$ he will be a member of networks $(g_t, g_{t+1}, g_{t+2}, \dots)$, respectively, then at the beginning of period t , agent i 's non-myopic payoff, u_i^t , will equal the summation of his discounted myopic payoffs

$$u_i^t = u_i(g_t) + d \cdot u_i(g_{t+1}) + d^2 \cdot u_i(g_{t+2}) + \dots$$

where $u_i(g)$ is defined by equation 1.

3. Non-Myopic Results

Watts [2001] shows that if agents are myopic ($d=0$) and if $c > \frac{1}{2}$, then no links will form even though all players may be better off in a connected network. However, we show, in Proposition 1, that if the agents are non-myopic then the agents may agree to form a network which is shaped like a circle. We also show that as the number of agents increases, it becomes easier for such a circle network to form.

Notice that the agents will only form a network which generates a positive payoff for each

network member, since any agent who receives a negative payoff would be better off if he severed some or all of his ties. The circle network guarantees that each agent receives a positive payoff, as long as the inequalities of Proposition 1 hold. Note also that the circle network may not be the only network which generates a positive payoff for each member. However, we choose to focus on the circle network because of its simplicity and its symmetry; after the circle has formed, each agent receives the same payoff.

Proposition 1 Assume $c > 0$, and assume n is even.¹ If inequalities (i) and (ii) hold true then there exists an order of play for which the formation of a circle network is supported as a subgame perfect equilibrium.

$$(i) \quad (c - \delta) < \left(\frac{1-d}{1+d} \right) \left(\sum_{t=2}^{n/2-1} \delta^t - \sum_{t=n/2+1}^{n-1} \delta^t \right) + \frac{d}{1+d} \left(\sum_{t=2}^{n/2-1} 2\delta^t + \delta^{n/2} \right)$$

$$(ii) \quad 2 \sum_{t=2}^a (\delta^t - \delta^{t+b}) - \left(\frac{\delta^{n/2}}{1-d} \right) - \left(\frac{2d}{1-d} \right) \sum_{t=2}^{n/2-1} (\delta^t + \delta - c) < (c - \delta) < (d\delta)^2 \left(\sum_{t=0}^{n/2-3} d^{2t} \delta + \frac{d^{n-4}}{1+d} \delta^{n/2-2} \right)$$

where $a = n/4$ and $b = (n/4 - 1)$ if n is divisible by 4 and $a = b = (n/4 - 1/2)$ if n is not divisible by 4.

Remark: As the number of players increases, it becomes easier to meet condition (i) and the second inequality of condition (ii). Thus it becomes more likely that there exists an order of play for which the formation of a circle network is supported as a subgame perfect equilibrium. (Assuming that the

¹The case where n is odd is similar, and is omitted for the sake of brevity.

second inequality of condition (ii) holds and that players value the future enough then the left hand side of condition (ii) is negative and thus the first inequality of condition (ii) holds true.)

Proof of Proposition 1

Assume $c > 0$ and assume n is even. Let each agent be assigned a number randomly, and let the order of play equal² (1:2, 2:3, 1:4, 3:5, 4:6,...,n:1, 2:4, 2:5, ..., (n-1):(n-3)). We show that a subgame perfect equilibrium exists where all players adopt the Grim Strategy.

The Grim Strategy is defined as follows. Each agent agrees to link with the first two players he meets, and each agent never severs a link as long as all other players cooperate. (Cooperate means that an agent links with the first two players he meets and never severs these ties.) However, if player i deviates, then as punishment every player $j \neq i$ severs all ties in the next period and refuses to form any new links for the rest of the game. Thus if player i deviates, his payoff will be 0 in all future periods.

To show that the Grim Strategy is subgame perfect we first show that the Grim Strategy will deter players from refusing to cooperate, then we check that the Grim Strategy will deter punishers from defecting on the punishment.

If agent 2 adopts the Grim Strategy his payoff will be

$$u_2^1(GS) = (\delta - c) + d \cdot (2\delta - 2c) + d^2 \cdot (2\delta + \delta^2 - 2c) + \dots \\ + (d^{n-1} + d^n + \dots) (2\delta - 2c + 2\delta^2 + \dots + 2\delta^{\frac{n}{2}-1} + \delta^{\frac{n}{2}})$$

²The name of the agents is not important to the proof, it is only important that the order of play have the correct pattern, namely that the circle forms as quickly as possible. Since the agents are assigned a number randomly, the order (1:2, 2:3,...,n:1) is identical to the order (1:3, 3:5,...,(n-1):2,...,(n-2):n, n:1). The pairs (2:4, 2:5,...,(n-1):(n-3)) can be in any order.

Player 2 will only adopt the Grim Strategy if his payoff is positive, $u_2^1(GS) > 0$, which implies that

$$(c - \delta) < d^2 \delta^2 \left(\sum_{t=0}^{\frac{n}{2}-3} d^{2t} \delta^t + \frac{d^{n-4}}{1+d} \delta^{\frac{n}{2}-2} \right)$$

must hold true.

It is easy to check that if everyone adopts the Grim Strategy then $u_2^1(GS) = \min_i u_i^{t_i}(GS)$, where t_i is the period in which player i should form his first link. Therefore, if everyone adopts the Grim Strategy then $u_i^{t_i}(GS) - u_2^1(GS) > 0$ for all i . Thus each agent is better off adopting the Grim Strategy than he would be if he refused all links.

Next we must make sure that no player i wants to deviate in any period. We will start with player 2. Assume player 2 deviates in period 1. The only way for player 2 to deviate, is for player 2 to refuse to link with player 1, which will generate a payoff of 0 for player 2. Since $u_2^1(GS) > 0$, player 2 will not deviate in the first period.

Next assume player 2 deviates in period 2. If player 2 deviates by severing his link with player 1 and refusing to link with player 3, then his payoff from deviating will be 0. (If player 2 deviates by keeping his link with player 1 and refusing to link with player 3, his payoff from deviating will be $(-c) < 0$.) If instead player 2 follows the Grim Strategy his payoff will be

$$\begin{aligned} u_2^2(GS) &= (2\delta - 2c) + d(2\delta + \delta^2 - 2c) + \dots \\ &+ (d^{n-2} + d^{n-1} + \dots)(2\delta - 2c + 2\delta^2 + \dots + 2\delta^{\frac{n}{2}-1} + \delta^{\frac{n}{2}}) \end{aligned}$$

which is larger than $u_2^1(GS) > 0$. Therefore player 2 will not deviate in period 2. Similar analysis shows that player 2 will not deviate in period $t \in \{3, \dots, (n-1)\}$ by severing both of his links. If player 2 deviates in period $t \in \{3, \dots, (n-1)\}$ by severing one link then it is easy to show that $u_2^t(GS) -$

$u_2^i(\text{cheat}) - u_2^i(\text{GS}) > 0$, where $u_2^i(\text{cheat})$ is player 2's payoff from deviating. Therefore player 2 will not deviate in period $t \in \{3, \dots, (n-1)\}$.

Next we check that player 2 will not want to deviate after the circle has formed. If player 2 deviates in period $t = n$, he will do so by either severing a link or adding a link. (If player 2 deviates by severing both ties his payoff from deviating will be 0 while his payoff from following GS is strictly positive.) First assume player 2 deviates by severing one link and thus changes the graph from a circle to a line. His payoff from deviating and severing one tie will equal $(-c + 2^2 + 2^3 + \dots + 2^{n-1})$. If instead player 2 follows the Grim Strategy his payoff will equal $(1+d+d^2+\dots)(2^2 - 2c + 2^2 + 2^3 + \dots + 2^{n/2-1} + 2^{n/2})$. Player 2 will not deviate if his payoff from deviating is smaller than his payoff from playing GS, which implies that

$$(c - 2) < \left(\frac{1-d}{1+d}\right) \left(\sum_{t=2}^{n/2-1} \delta^t - \sum_{t=n/2+1}^{n-1} \delta^t\right) + \frac{d}{1+d} \left(\sum_{t=2}^{n/2-1} 2\delta^t + \delta^{n/2}\right).$$

Second we check that player 2 will not deviate by adding a link. Player 2's payoff from deviating will be largest if he adds a link to player n (the player who is directly across from him in the circle). If player 2 links to n his payoff will equal $(3 - 3c + 4^2 + 4^3 + \dots + 4^{n/4})$ if n is divisible by 4 and will equal $(3 - 3c + 4^2 + 4^3 + \dots + 4^{(n-2)/4} + 2^{(n+2)/4})$ if n is not divisible by 4. Thus player 2 will not deviate if his payoff from deviating is smaller than his payoff from playing GS, which implies that

$$2 \sum_{t=2}^a (\delta^t - \delta^{t+b}) - \left(\frac{\delta^{n/2}}{1-d}\right) - \left(\frac{2d}{1-d}\right) \sum_{t=2}^{n/2-1} (\delta^t + \delta - c) < (c - 2)$$

where $a = n/4$ and $b = (n/4 - 1)$ if n is divisible by 4 and $a = b = (n/4 - 1/2)$ if n is not divisible by 4.

Next consider player $i \geq 2$. If player i deviates in period $t = t_i$ by severing all of his ties then

his payoff from deviation is 0; while his payoff from playing the Grim Strategy is $u_i^t(\text{GS}) - u_i^t(\text{GS}) > 0$. Therefore i will not deviate by severing all of his ties. Alternatively, player i could deviate by severing one tie and keeping his other tie; then $u_i^t(\text{GS}) - u_i^t(\text{cheat}) - u_i^t(\text{GS}) > 0$ for all $t \in \{s, \dots, (n-1)\}$ where s is the period in which i should form his second link. Thus i will not deviate in any period $t \in (n-1)$. We also know that i will not deviate in a period, $t = n$, after the circle has formed, since i 's gain from deviation will equal 2 's gain from deviation and we have already shown that 2 will not deviate.

Next we check that the Grim Strategy will deter punishers from defecting on the punishment. Assume player i deviates in period t , we will show that player j has no incentive to defect from punishment. If everyone else plays the Grim Strategy, then in period $t+1$, all remaining players will sever their existing ties. So if player j wishes to deviate from punishment then he will refuse to sever some of his ties. If player j was not linked to player i in period t , then all of j 's ties will be broken by the remaining agents in period $t+1$. So agent j has no incentive to deviate from punishment. (Agent j receives a payoff of 0 whether he deviates from punishment or not.) If agent j is linked to agent i in period t then it is possible that if j does not sever this tie that it will survive. However since everyone else plays the Grim Strategy it will be the only tie that survives. Agent j 's payoff from such a link equals $(-c)(1+d+d^2) < 0$. Thus agent j is better off if he does not deviate from punishment in period $t+1$ since then he receives a payoff of 0. Similar analysis shows agent j will not deviate from punishment in periods later than $t+1$.

Remark: Proposition 1 says that even if $c > \frac{1}{1+d}$, it is possible for a circle network to form, as long as the inequality given in Proposition 1 holds, and the order of play takes a certain form. Namely, the order of play must be such that the circle forms as quickly as possible, which will minimize the number of periods an agent receives a payoff of $(-c)$. If the order of play is such that it takes more

than n periods for the circle to form, then the inequality given in Proposition 1 may not be enough to ensure that the circle network will form.

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